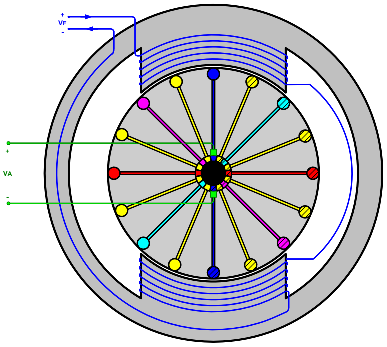
ECET 3500

DC Machines

†

Introduction

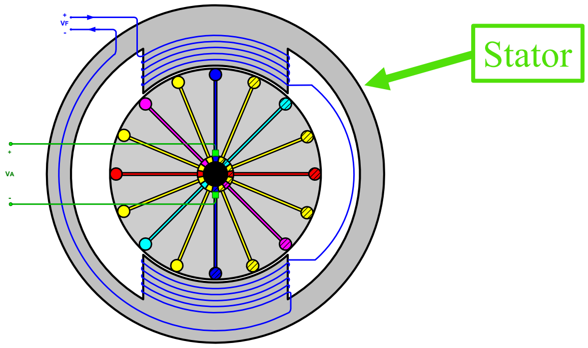
DC Machine Construction



The construction of the **DC machine** can be divided into two fundamental components;

- **Stator**
- **Rotor**

DC Machine Construction

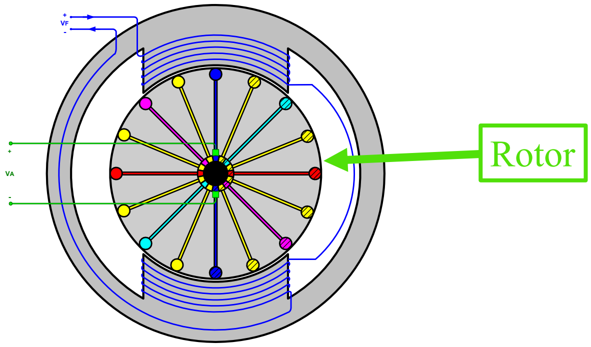


Stator

The **stator** is the stationary (outer) portion of the machine that provides the **primary magnetic field** required for operation.

This field will be referred to as the "**stator field**".

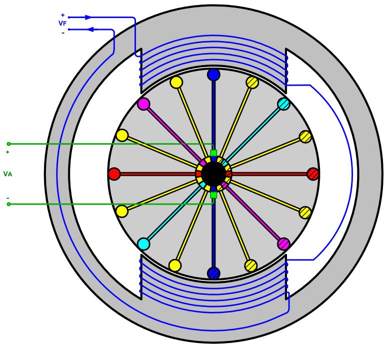
DC Machine Construction



Rotor

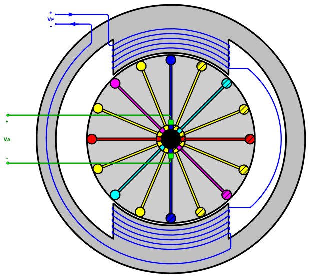
The **rotor** is the rotational (center) portion of the machine that provides the mechanism for **energy conversion** (elec→mech or mech→elec) as it rotates within the stator field.

DC Machine Construction



Although they are **magnetically coupled together**, the stator and the rotor can initially be considered individually to obtain a basic understanding of the machine's operation.

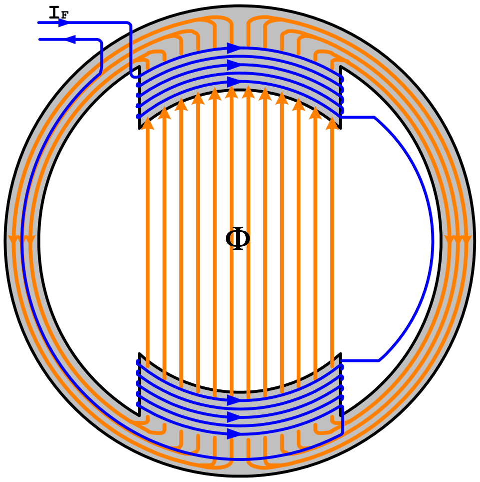
DC Machine Construction



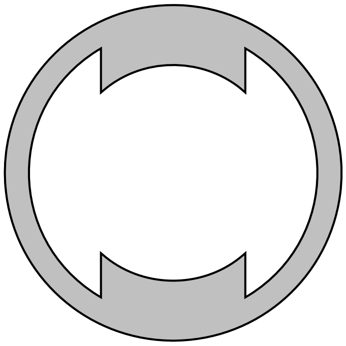
Note – The following presentation covers the construction of a **conceptual**, **2-pole**, **DC machine**.

Although the construction of an actual DC machine may vary from the conceptual machine shown, the operational mechanisms and characteristics will be similar.

DC Machines – Stator Construction



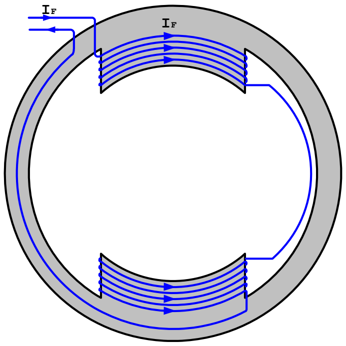
Stator Construction



The **stator** functions as the primary field source for the DC machine.

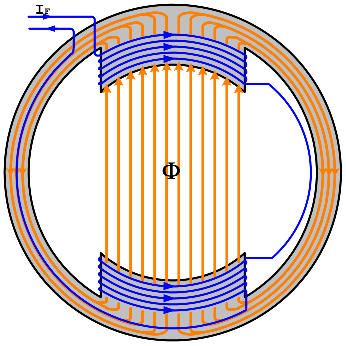
The stator can either utilize **electro-magnets** or **permanent magnets** to create this field.

Electro-Magnet Stator



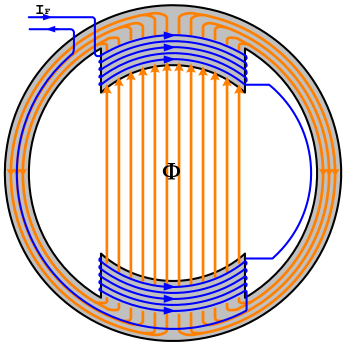
The stator shown above utilizes a single “**field winding**”, one-half of which is wrapped around each of the magnetic pole faces that are placed on opposite sides of the stator housing.

Electro-Magnet Stator



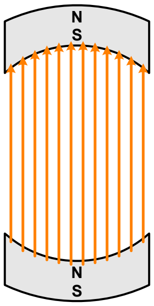
The two halves of the winding are wrapped such that the overall field winding creates a net **magnetic flux** that (ideally) passes through the rotor region in a linear manner.

Electro-Magnet Stator



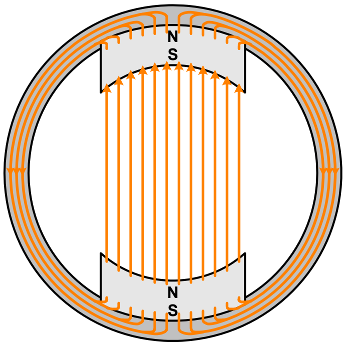
Although the flux lines will be constant in magnitude and direction when the field winding is sourced with a steady-state DC current (***I_F***), the magnitude of the **flux** can be varied by varying the magnitude of the field current.

Permanent Magnet Stator



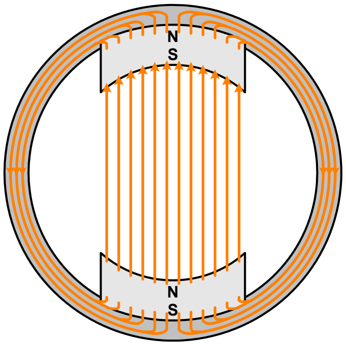
Note that the same **stator field** could be obtained by placing two **permanent magnets** on opposite sides of the rotor region as shown above.

Permanent Magnet Stator



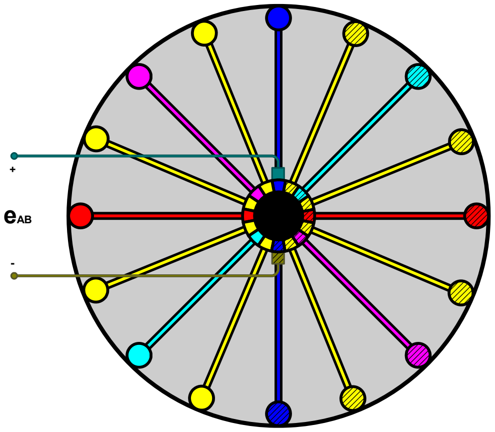
Permanent magnets are often used in small DC machines to simplify the construction of the machine and to eliminate the power consumption of the field winding.

Permanent Magnet Stator

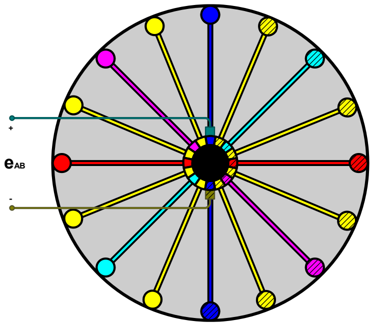


Although **larger DC machines** can be constructed using permanent magnets, they are costly, more difficult to assemble, and they lack ability for “field” control, which is often desired for industrial applications.

DC Machines – Rotor Construction

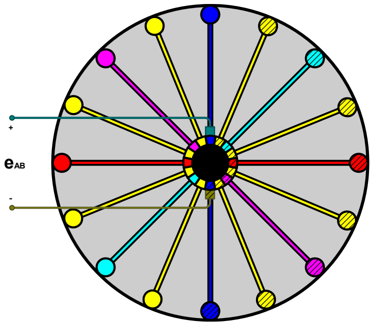


Rotor Construction



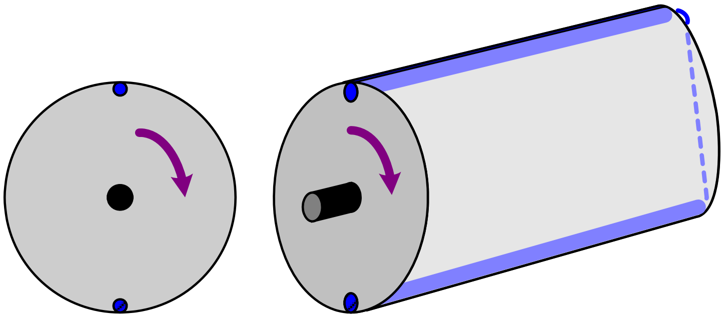
The **rotor** of the DC machine is very complex, both in terms of its construction and in terms of its operation.

Rotor Construction



Because of this, we will begin this discussion with a simple rotor and then add-in components or modify the rotor step-by-step until the overall rotor for a DC machine is realized.

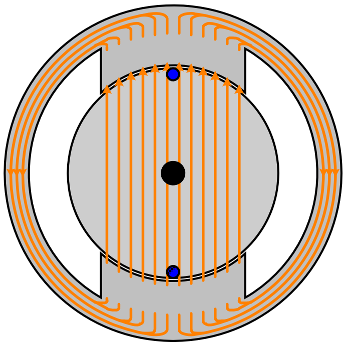
Rotor Construction



To begin, a **cylindrical rotor** is attached to a central shaft that provides an axis of rotation.

Embedded into the rotor are **two conductive bars**, the rear-ends of which are shorted together.

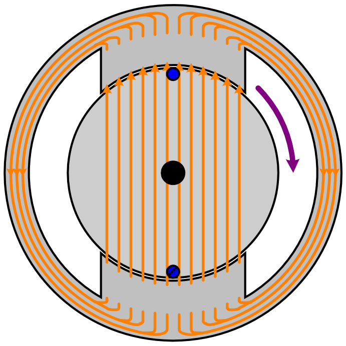
Stator Field – Rotor Interaction



The above figure shows the **rotor** placed within the **stator field** with the rotor conductors arbitrarily aligned vertically.

Note that the stator's "field coil" has been removed to simplify the figure.

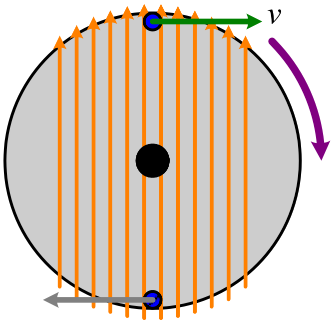
Stator Field – Rotor Interaction



This analysis will begin with the "**generator**" mode of operation, during which:

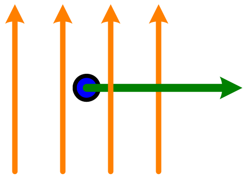
→ an **external force** is being applied to the shaft in order to rotate the rotor at a **constant** speed.

Stator Field – Rotor Interaction



As the rotor rotates, the **conductors** embedded within the rotor will cut across the lines of stator flux.

Stator Field – Rotor Interaction

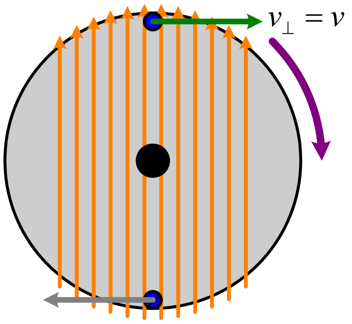


And as the conductors cut the across flux lines, a **voltage** (***e***) is induced across the conductors:

$$e = B \cdot l \cdot v_{\perp}$$

where $v_{\perp}$ is the component of the conductors’ velocities that are orthogonal to the flux lines.

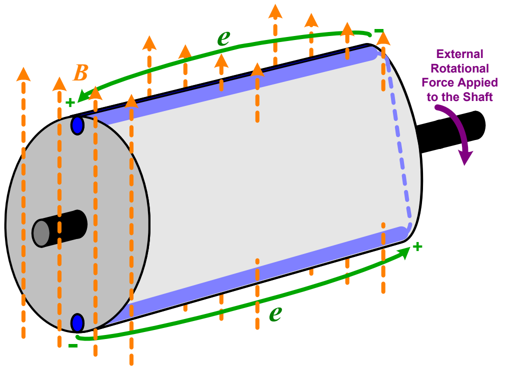
Stator Field – Rotor Interaction



For this analysis, the vertically-aligned position shown in the above figure will be referenced as the **0°** position.

At this position, the **velocity vectors** of the rotor conductors are **orthogonal** to the lines of flux.

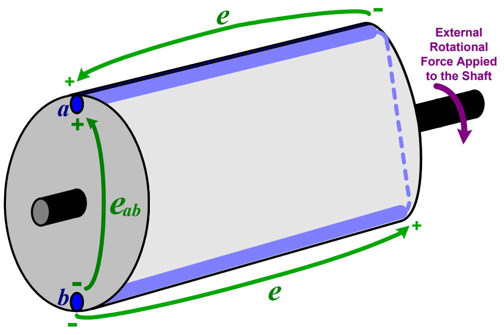
Stator Field – Rotor Interaction



Note that the individual conductors are cutting across the flux lines in **opposite directions**.

Because of this, the **polarities** of the voltage-rises induced across the conductors will be in **opposite directions**.

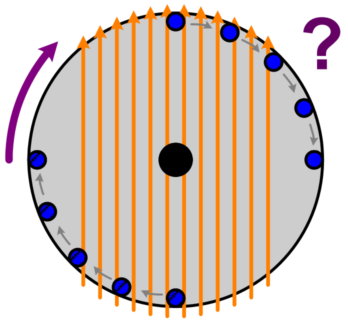
Stator Field – Rotor Interaction



The opposing polarity relationship results in a **net voltage rise** across the two conductors, e_{ab}, that is the sum of the two individual conductor voltages:

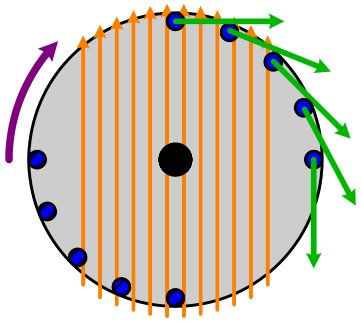
$$e_{ab} = e + e = 2\cdot e$$

Conductor Voltage vs. Rotor Position



Although the opposing polarity relationship will hold true regardless of the rotor’s orientation, we must also consider the manner in which the rotor’s rotation affects the magnitudes of the conductor voltages.

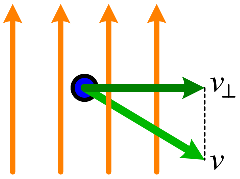
Conductor Voltage vs. Rotor Position



velocity vector of conductor shown at various rotational positions

If the rotor is rotating at a **constant speed**, the instantaneous velocity vector of the conductors will be constant in magnitude and have a **direction** that is **tangent** to the surface of the rotor.

Conductor Voltage vs. Rotor Position

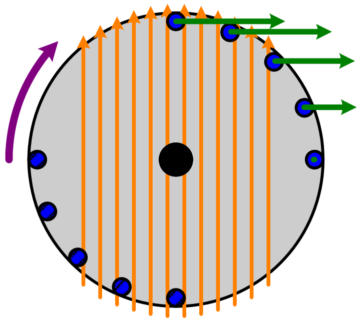


But the **voltage** induced across the conductors:

$$e = B \cdot l \cdot v_\perp$$

is proportional to the component of the velocity vector that is **orthogonal** to the flux lines.

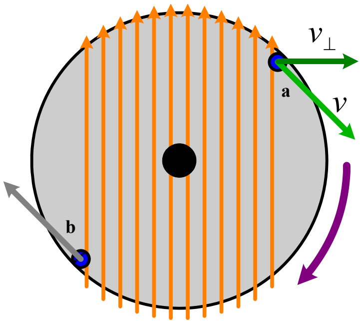
Conductor Voltage vs. Rotor Position



orthogonal component of velocity vector shown at various rotational positions

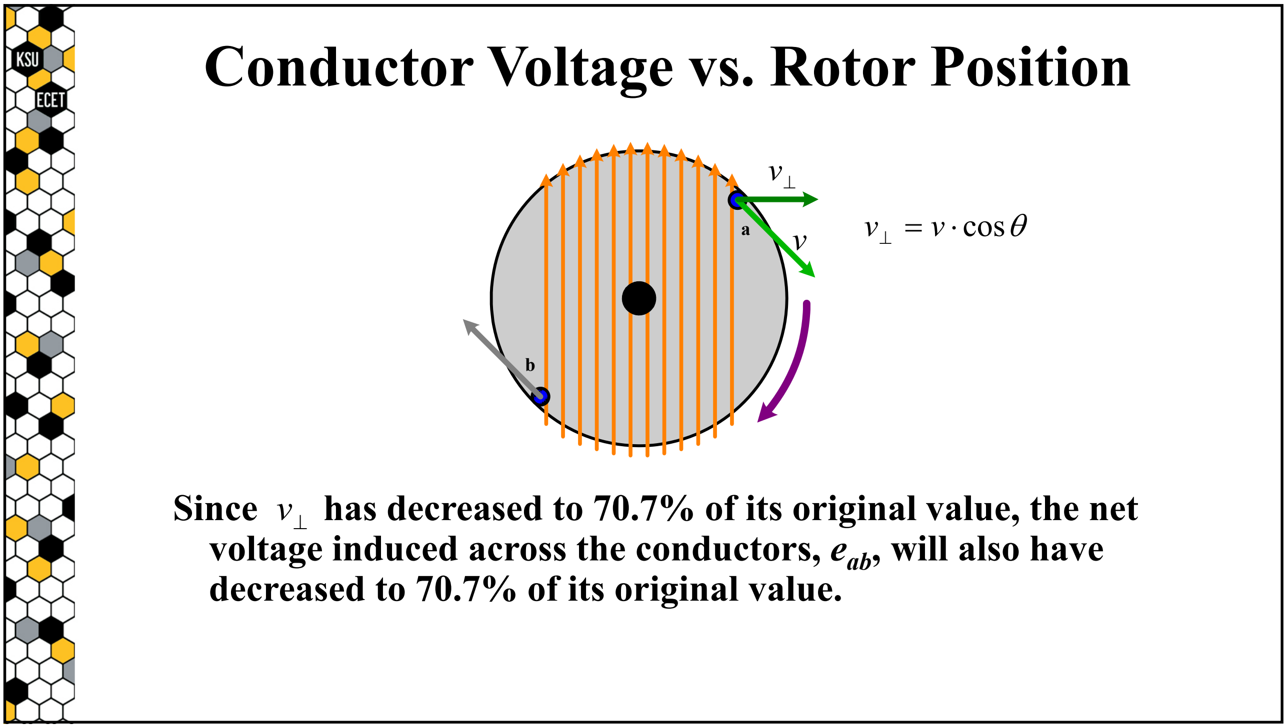
Even though the magnitude of the tangential velocity vector is constant, the <u>orthogonal</u> component of the velocity varies with rotor position, in-turn causing the conductor voltages to vary with rotor position.

Conductor Voltage vs. Rotor Position



$$v_\perp = v \cdot \cos\theta$$

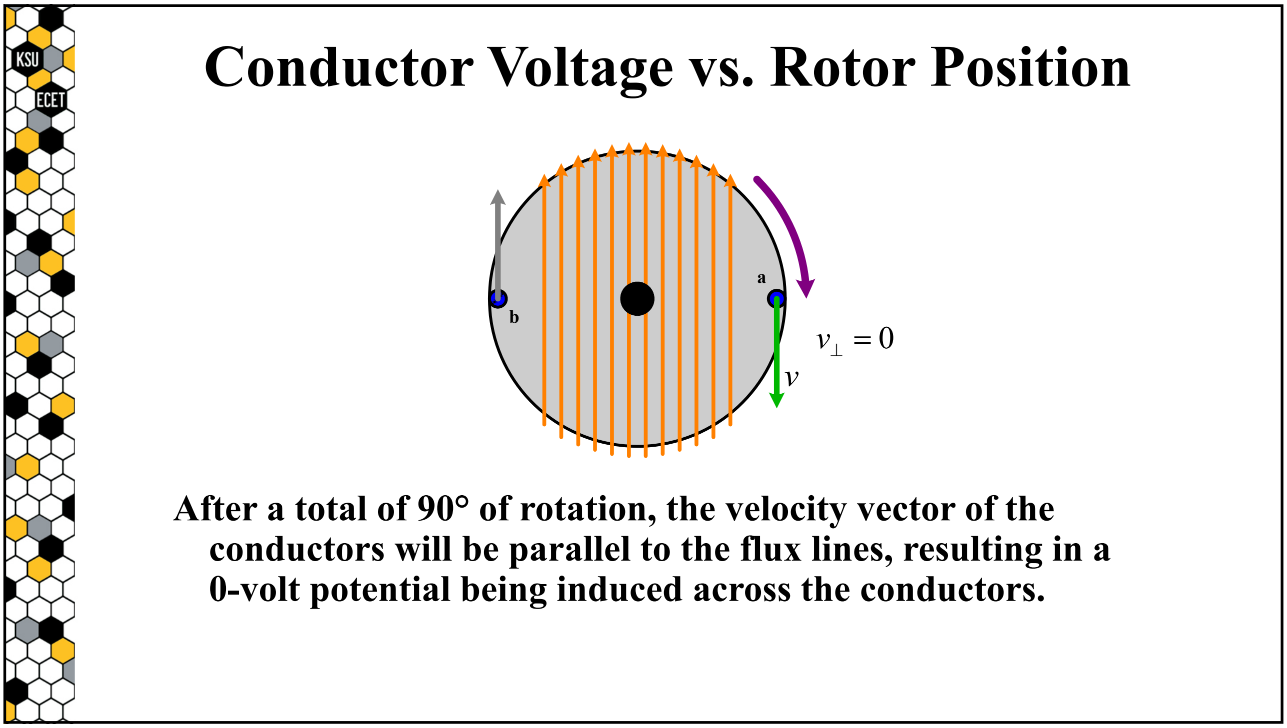
For example, after 45° of rotation, the orthogonal component of the velocity vector, $v_\perp$, has decreased to 70.7% of the value it had while in the 0° position.



Conductor Voltage vs. Rotor Position

$$v_{\perp} = v \cdot \cos\theta$$

Since $v_{\perp}$ has decreased to 70.7% of its original value, the net voltage induced across the conductors, e_{ab}, will also have decreased to 70.7% of its original value.

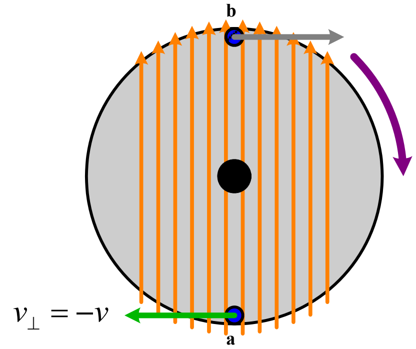


Conductor Voltage vs. Rotor Position

$$v_{\perp} = 0$$

After a total of 90° of rotation, the velocity vector of the conductors will be parallel to the flux lines, resulting in a 0-volt potential being induced across the conductors.

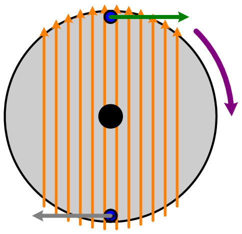
Conductor Voltage vs. Rotor Position



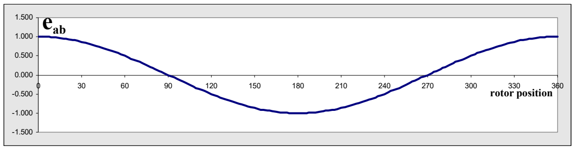
After a total of 180° of rotation, $v_\perp$ is back to its peak value, but in the "negative" direction.

Since the velocity vector is in the negative direction, the polarity of e_{ab} will be negative.

Conductor Voltage vs. Rotor Position



If the voltage e_{ab} is characterized through one complete revolution of the rotor, the voltage will vary in a sinusoidal manner:

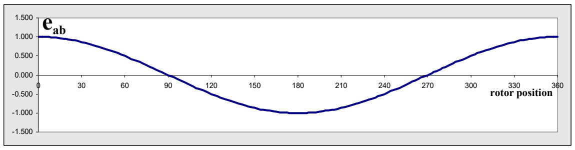


Voltage Induced in the Rotor

Although we have conceptually introduced a method for generating an AC voltage, the topic at-hand is **DC Machines**.

Thus, we need a method for converting the voltage from AC to DC (without using an external rectifier or other electronics-based circuit).

But there’s also another issue that we must address:
how do we access the voltage generated by the rotor conductors?

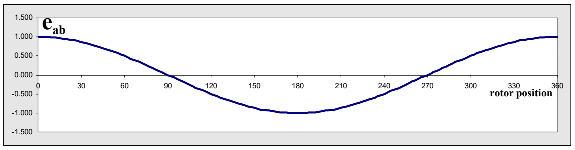


Stationary Connection to Rotor

As discussed, the sinusoidally varying voltage, e_{ab}, is the voltage present across the front-ends of the rotating rotor conductors.

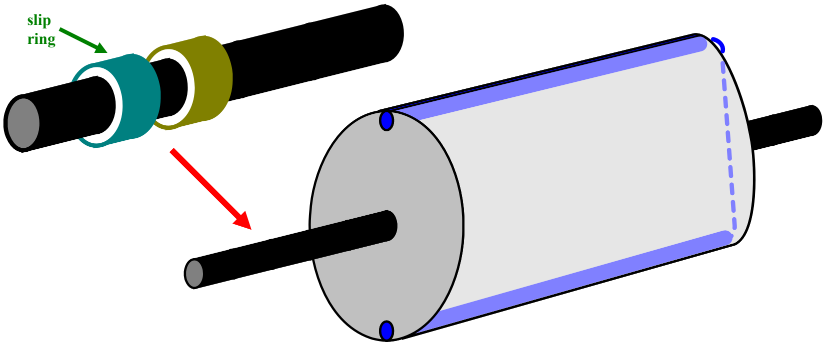
Thus, from a practical aspect, a stationary, external, electrical connection to the rotating conductors must first be provided.

To accomplish this task, we will initially utilize a pair of **slip rings** and **brushes** to provide this connection, and then we will modify in components in order to convert the voltage to DC.



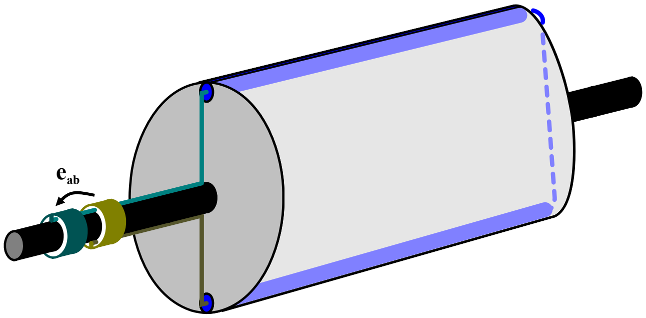
Slip Rings

A pair of conductive **slip rings** will be mounted on the shaft of the rotor such that they are electrically insulated from each other and from the steel shaft.



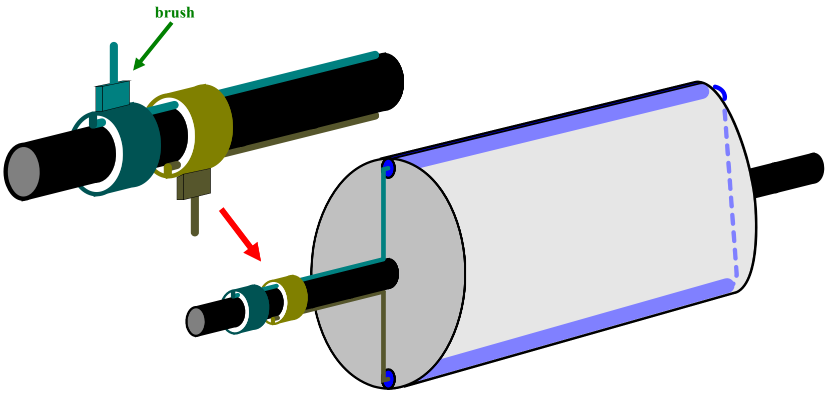
Slip Rings

The front ends of the rotor conductors will then be connected to the slip rings (one conductor per ring) by wires that are run along the shaft of the rotor, in-turn causing the conductor voltage e_{ab} to appear across the rings.



Brushes

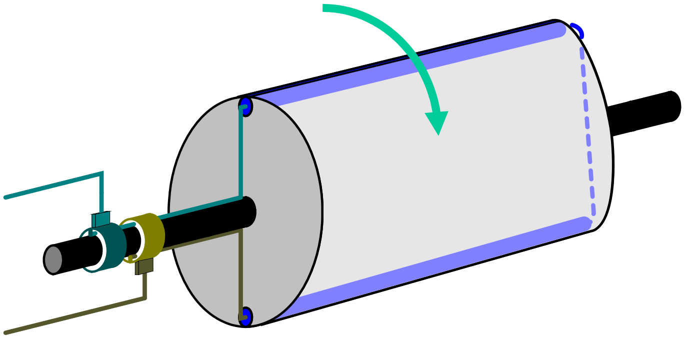
Stationary **brushes** are then mounted in a spring-loaded housings such that one brush is pressed against each ring, and an external conductor is bonded to each of the brushes.



Brushes

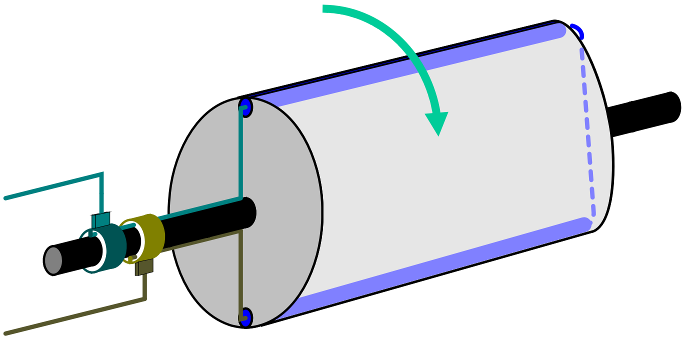
Since the **brushes** are only being pressed against the slip rings, they **can easily be replaced**.

Thus, the are constructed from a soft material, allowing them to wear down over time without damaging the rotating slip rings.



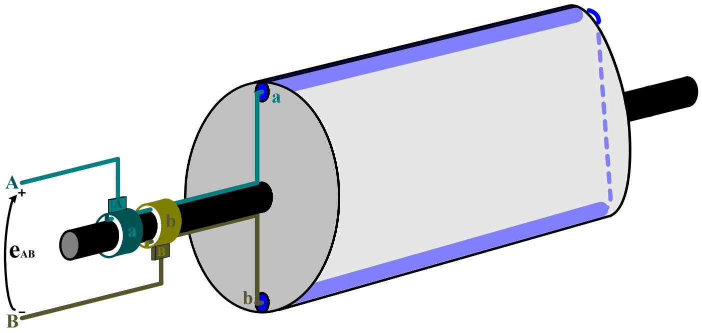
Brush – Slip Ring Connection

As long as the brushed remain pressed against the slip rings, the **contact-based connection** between the stationary brushes and the rotating slip rings provides the necessary external electrical connection to the rotating rotor conductors.



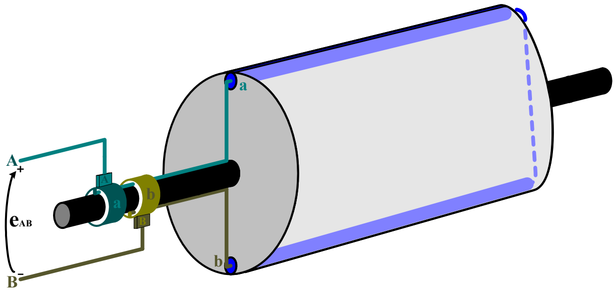
Brush Terminal Voltage

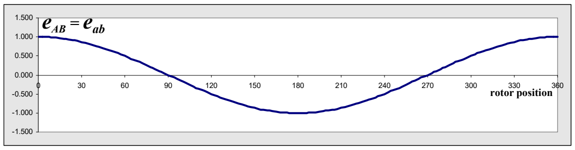
And since the brushes are always connected to their respective slip rings, the **conductor voltage** e_{ab} that appears across the rings will also **appear across the brush terminals**.



Brush Terminal Voltage

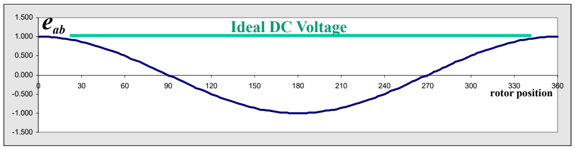
Brush Terminal Voltage $e_{AB} = e_{ab}$





Mechanical Rectifier

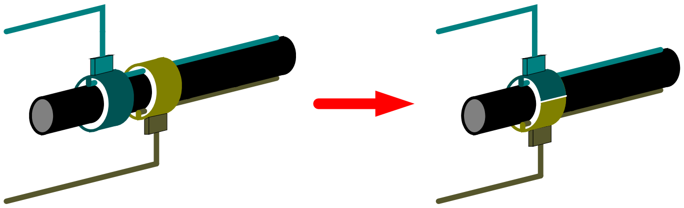
Now that a stationary connection has been provided to the rotor conductors, the next step to converting the AC voltage into a DC voltage is to <u>rectify</u> the waveform.



Mechanical Rectifier

Now that a stationary connection has been provided to the rotor conductors, the next step to converting the AC voltage into a DC voltage is to rectify the waveform.

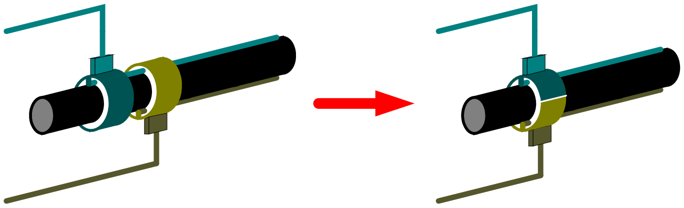
A simple modification to the brush – slip ring connection can be made to perform this task.



Mechanical Rectifier

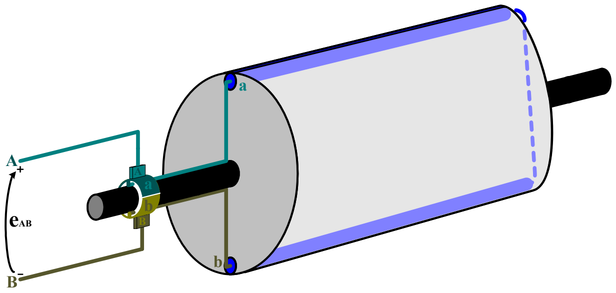
The two slip rings are replaced by a single “split” ring, the halves of which are electrically isolated from each other.

The brushes are mounted such that they are directly opposing each other in order to provide a contact connection to both sides of the ring.



Mechanical Rectifier

The top half of the ring is then connected to the front end of the top conductor and the bottom half of the ring is connected to the front end of the bottom conductor.

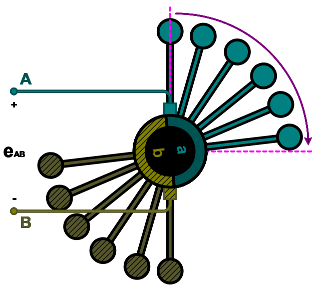


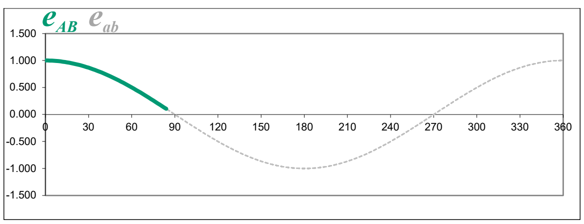
Mechanical Rectifier

Starting at the 0° position, *brush A* will be pressed against *ring-half a* and *brush B* will be pressed against *ring-half b*.

$(e_{AB} = e_{ab})$

This will remain true for almost the 1st 90° of revolution.

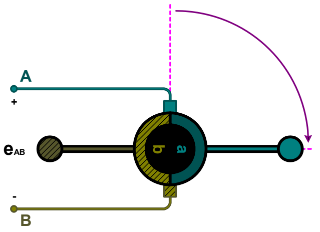


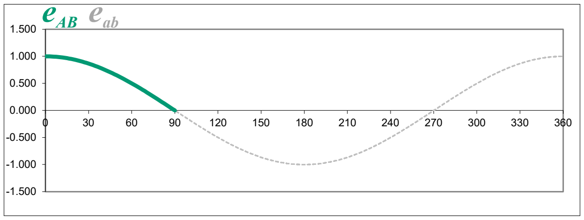


Mechanical Rectifier

As the rotor reaches the 90° position, the brushes will short across the gaps that separate the *ring-halves a* and *b*.

This is not a problem because the potential difference between the ring-halves is 0-volts at this position.

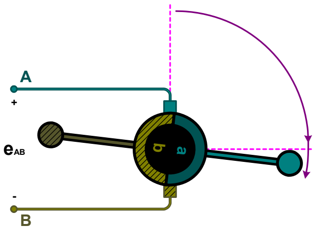


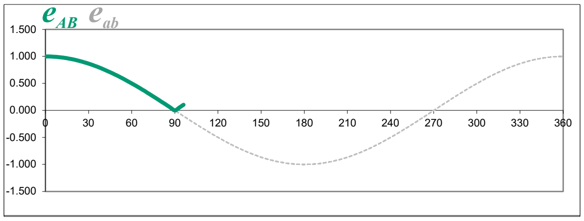


Mechanical Rectifier

Slightly past the 90° position, the *ring-halves a* and *b* will have rotated under *brushes B* and *A* respectively, effectively negating the polarity of the brush terminals at the same time that the rotor voltage becomes negative.

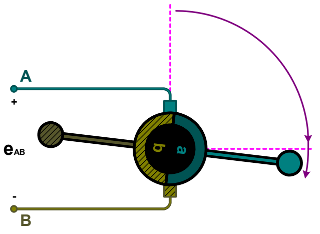
$$(e_{AB} = -e_{ab})$$

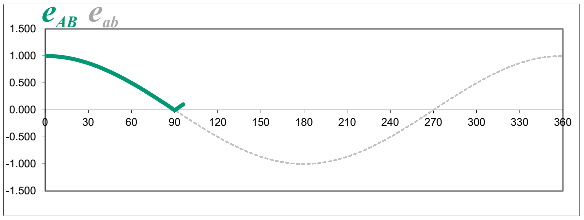




Mechanical Rectifier

By negating the polarity of the brush terminals at the same time that the rotor voltage becomes negative, the sinusoidal rotor voltage is effectively rectified such that the voltage seen at the brush terminals will be the same as that seen at the output terminals for a full-wave rectifier.

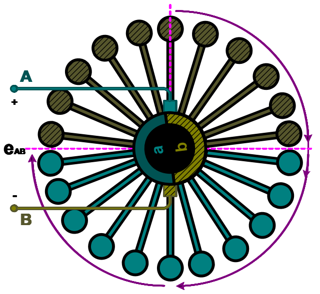


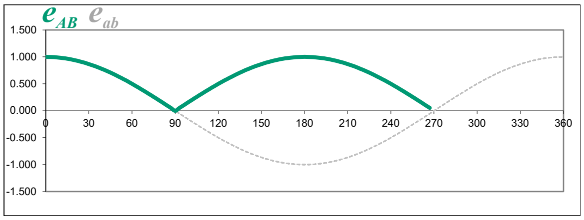


Mechanical Rectifier

Thus, for almost the next 180° of rotation, the *ring-half a* will remain under *brush B* and *ring-half b* will remain under *brush A*, maintaining an overall positive voltage at the stationary brush terminals.

$$(e_{AB} = -e_{ab})$$

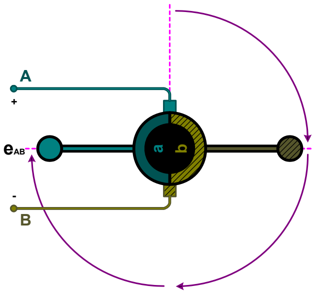


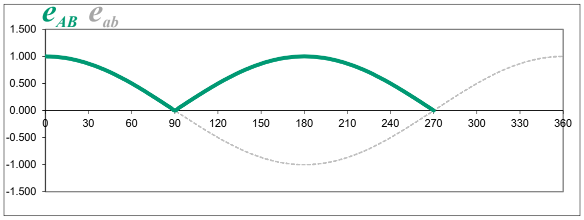


Mechanical Rectifier

When the rotor rotates a total of 270°, the brushes will once again short across the gaps that separate *ring-halves a* and *b*.

As before, this is not a problem because a 0-volt potential difference exists between the ring-halves at this position.

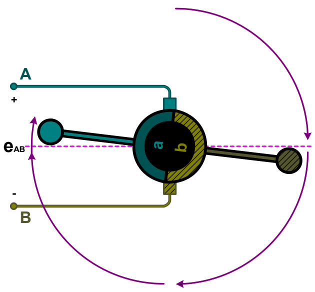


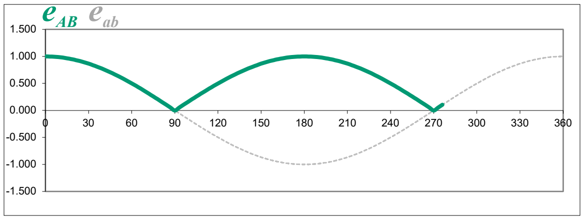


Mechanical Rectifier

When the rotor rotates past the 270° position, the ring-halves will reconnect to the original brushes at the same time that the rotor voltage becomes positive, again reverting back to a positive polarity and maintaining a positive terminal voltage.

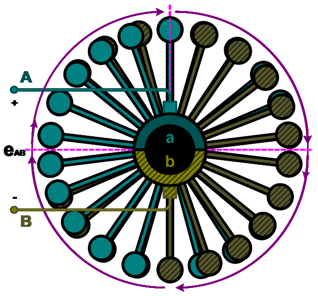
$(e_{AB} = e_{ab})$

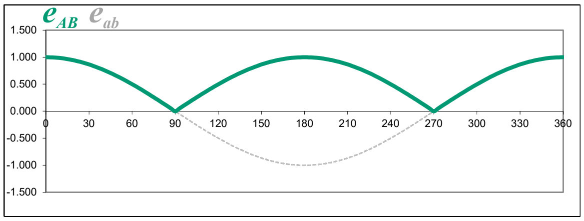




Mechanical Rectifier

The trend will continue as the rotor rotates for a total of 360°, (i.e. – back to the original 0° position) at which point the process repeats so long as the rotor continues rotating.

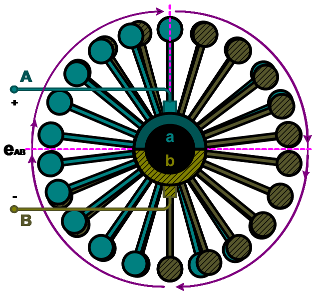


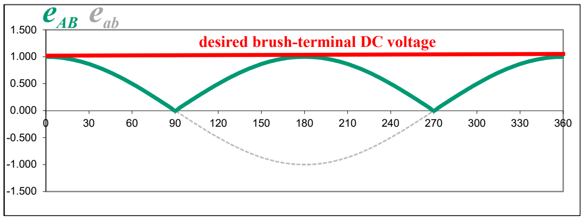


Mechanical Rectifier

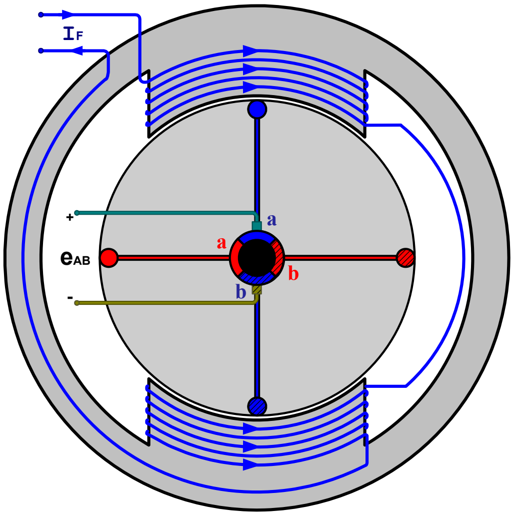
Unlike the original sinusoidal waveform, the rectified sinusoid does have a constant or average (DC) component.

Despite this fact, the voltage seen at the brush terminals is still far from the desired DC output voltage.



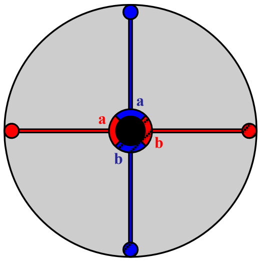


Mechanical Rectifier – 2 Conductor Pairs



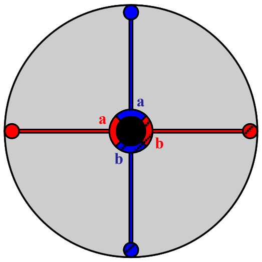
Two pair of conductors w/ four-section ring.

Mechanical Rectifier – 2 Conductor Pairs

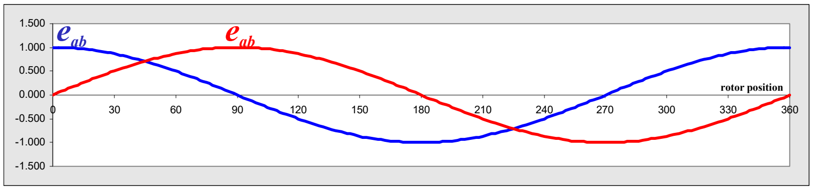


Consider the addition of a second pair of conductors that are embedded within the outer surface of the rotor, the rear-ends of which are shorted together and the front-ends of which are connected to the appropriate sections of the shaft-mounted ring that is now split into four sections.

Mechanical Rectifier – 2 Conductor Pairs

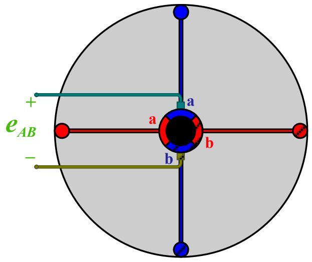


The voltage across the second pair of conductors will be the same as the voltage across the original pair of conductors, but delayed with respect to rotor position by ¼ revolution of the rotor.

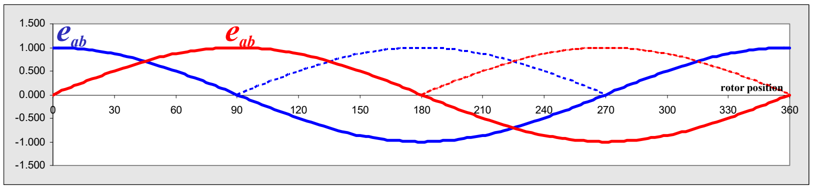


Two pair of conductors w/ four-section ring.

Mechanical Rectifier – 2 Conductor Pairs

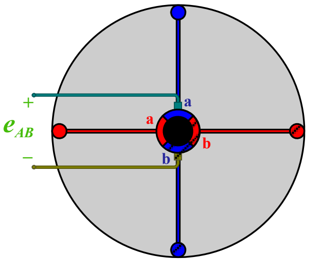


If the brushes are placed correctly, then only the peak portions of the rectified conductor voltages from both pairs will be present across the brush conductors as the rotor rotates beneath the brushes.

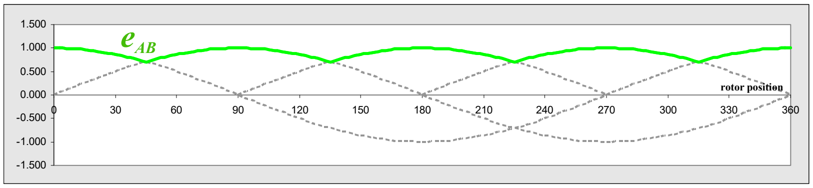


Two pair of conductors w/ four-section ring.

Mechanical Rectifier – 2 Conductor Pairs



If the brushes are placed correctly, then only the peak portions of the rectified conductor voltages from both pairs will be present across the brush conductors as the rotor rotates beneath the brushes.



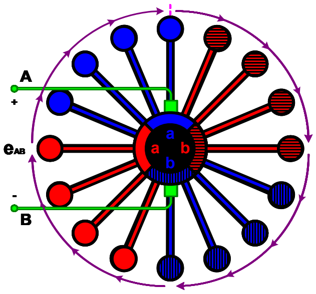
Two pair of conductors w/ four-section ring.

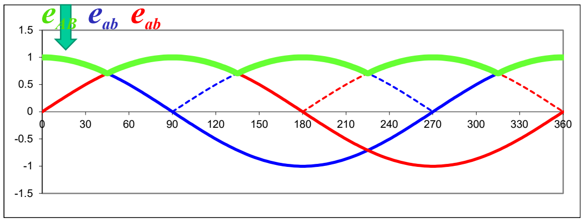
Mechanical Rectifier – 2 Conductor Pairs

Starting at the 0° position, *brush A* will be pressed against *blue-a* and *brush B* will be pressed against *blue-b*.

$$(e_{AB} = e_{ab})$$

This will remain true for almost the 1st 45° of revolution.



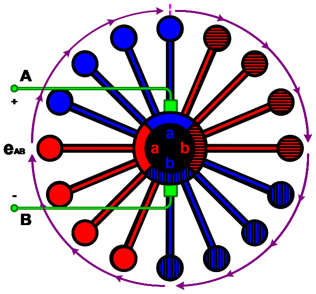


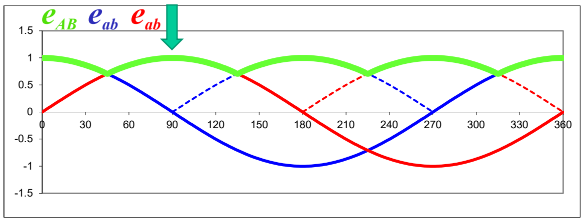
Mechanical Rectifier – 2 Conductor Pairs

But, when the rotor reaches the 45° position, *brush A* will transition to *red-a* and *brush B* will transition to *red-b*.

($e_{AB} = e_{ab}$)

This will remain true for the next 90° of revolution (135° total).



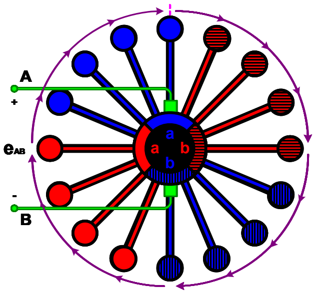


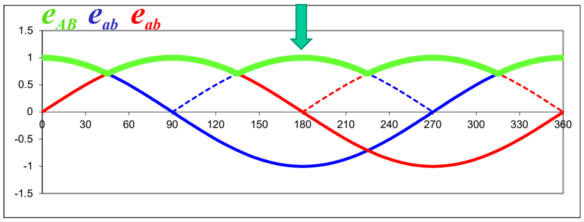
Mechanical Rectifier – 2 Conductor Pairs

When the rotor reaches the 225° position, *brush A* will then transition to *blue-b* and *brush B* will transition to *blue-a*.

($e_{AB} = -\, e_{ab}$)

This will remain true for the next 90° of revolution (225° total).



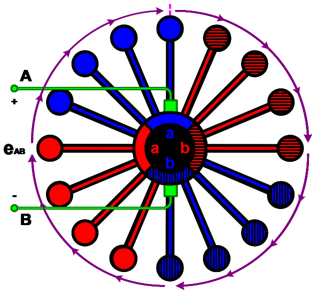


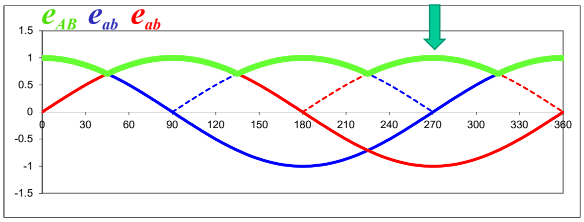
Mechanical Rectifier – 2 Conductor Pairs

Then, when the rotor reaches the 135° position, *brush A* will transition to *red-b* and *brush B* will transition to *red-a*.

$$(e_{AB} = -e_{ab})$$

This will remain true for the next 90° of revolution (315° total).



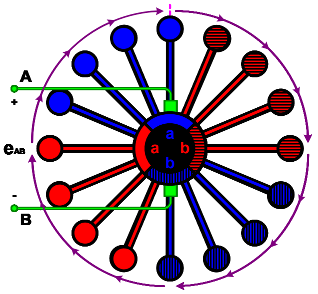


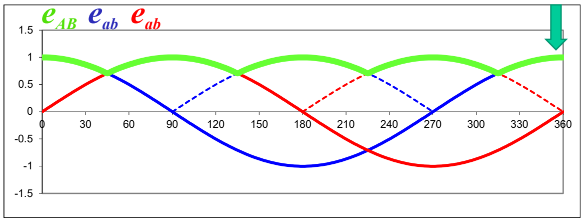
Mechanical Rectifier – 2 Conductor Pairs

And finally, when the rotor reaches the 315° position, *brush A* will transition back to *blue-a* and *brush B* back to *blue-b*.

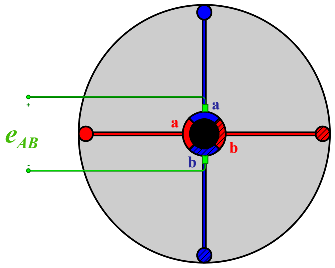
$$(e_{AB} = e_{ab})$$

This will remain true for the next 90° of revolution, and the rotation will repeat. $(e_{AB} = e_{ab} \rightarrow e_{ab} \rightarrow -e_{ab} \rightarrow -e_{ab} \rightarrow \ldots)$

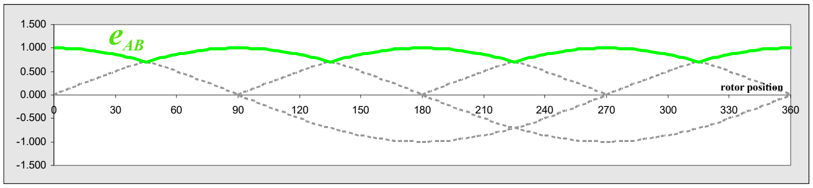




Mechanical Rectifier – 2 Conductor Pairs

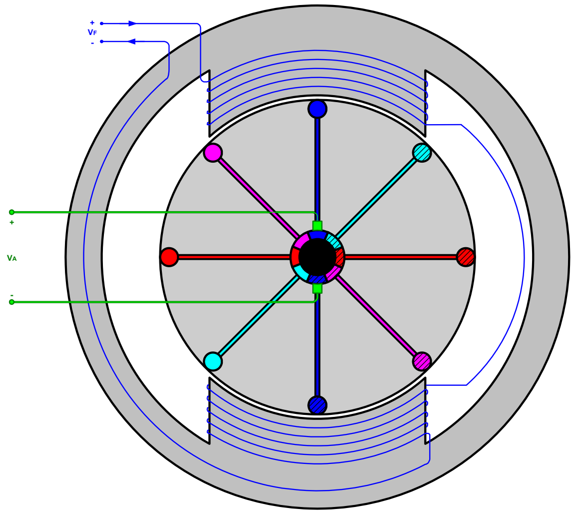


Thus, brush conductors effectively sample the conductor voltages in repeating 90° sections during which the voltages vary from 70% to 100% of their peak values.



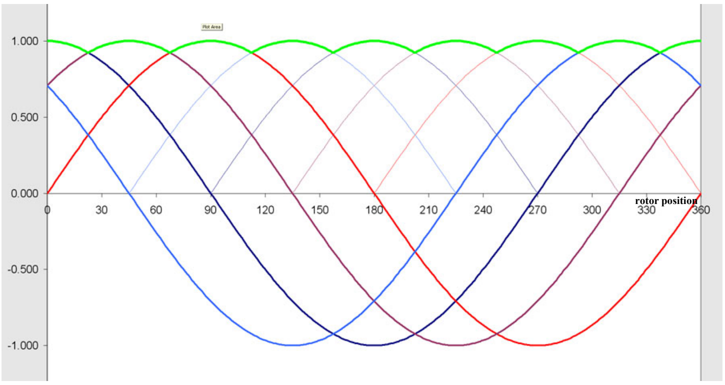
Two pairs of conductors w/ four-section ring.

Mechanical Rectifier – 4 Conductor Pairs



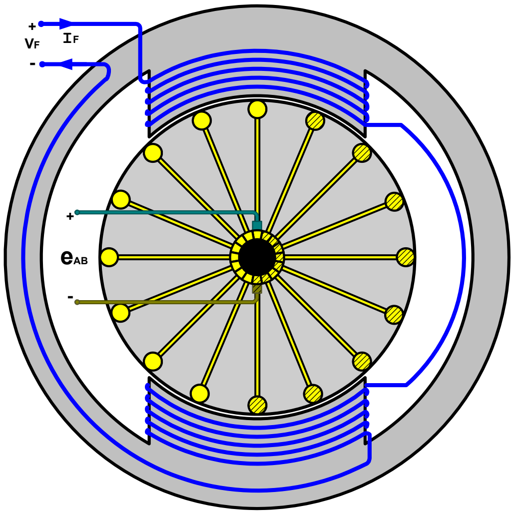
Four pairs of conductors w/ eight-section ring.

Mechanical Rectifier – 4 Conductor Pairs



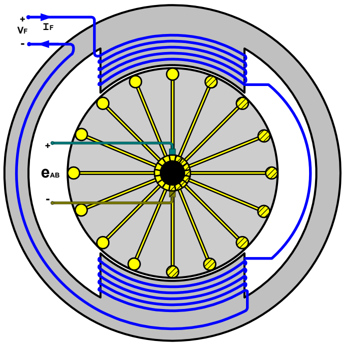
Four pairs of conductors w/ eight-section ring.

Mechanical Rectifier – 8 Conductor Pairs



Eight pairs of conductors w/ sixteen-section ring.

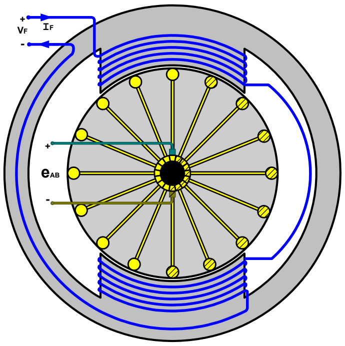
DC Machines



Modeling the DC Machine

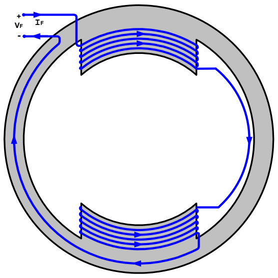
Generator Operation

Modeling the DC Machine



As with the squirrel-cage induction machine, it can be useful to derive a **circuit model** for the DC machine in order to predict or analyze its operational characteristics.

DC Machine – Stator (Field) Circuit Model

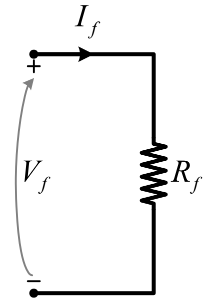


Stator of a DC Machine w/ Field Coil

When modeling the **stator** of the DC machine for steady-state operation, only the resistance of the field coil needs to be considered.

Note that in the case of a permanent magnet DC machine, the stator flux is provided by the permanent magnets and thus there is <u>no</u> "field circuit" associated with the machine.

DC Machine – Stator (Field) Circuit Model

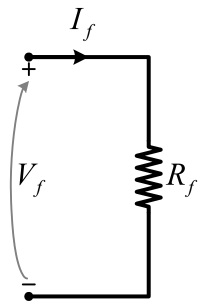


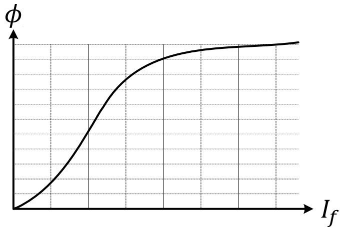
Field Circuit

Thus, a single resistor can be used to model the **field circuit**, where $\boldsymbol{V_f}$ is the voltage applied to the field coil and $\boldsymbol{I_f}$ is the resultant field current that creates the stator flux, $\boldsymbol{\phi}$, within the rotor region.

$$\phi = \mathrm{f}(I_f)$$

DC Machine – Stator (Field) Circuit Model

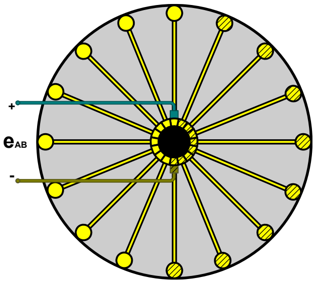




Field Circuit

Note that, since the magnetic properties (***B*** *vs.* ***H***) of the materials used to form the stator and rotor of a typical DC machine are **non-linear**, the relationship between the **stator flux** (***ϕ***) and the **field current** (I_f) will often resemble a **saturation curve**.

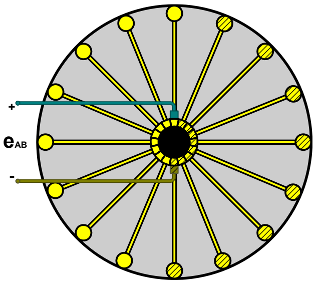
DC Machine – Armature Circuit Model



Armature of the DC Machine

The **armature** (**rotor**) portion of the DC machine consists of both the rotor conductors and the brush-commutator connection.

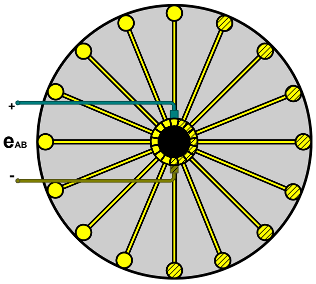
DC Machine – Armature Circuit Model



Armature of the DC Machine

The circuit model for the **armature** must account for the voltages that are induced across the rotor conductors as they rotate through the stator flux, but from the perspective of the armature (brush) terminals.

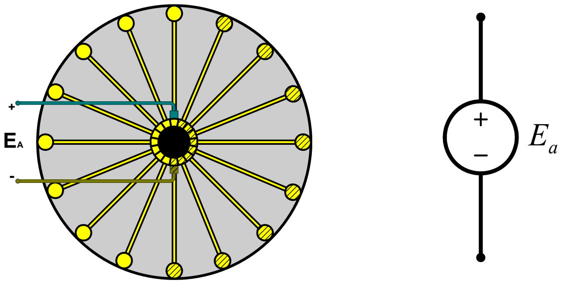
DC Machine – Armature Circuit Model



Armature of the DC Machine

Since the **brush-commutator connection** provides a mechanism that rectifies and samples the peaks of the rotor conductor voltages, the brush terminal voltage approaches DC as the number of conductor pairs increases.

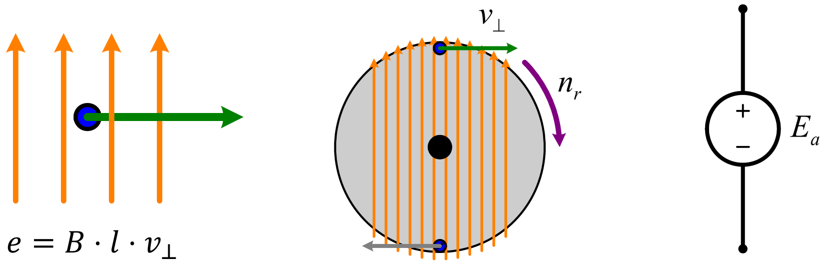
DC Machine – Armature Circuit Model



Armature of the DC Machine

For this reason, a DC source with magnitude $\boldsymbol{E_a}$ can be used to model the induced **armature circuit voltage**.

DC Machine – Armature Circuit Model



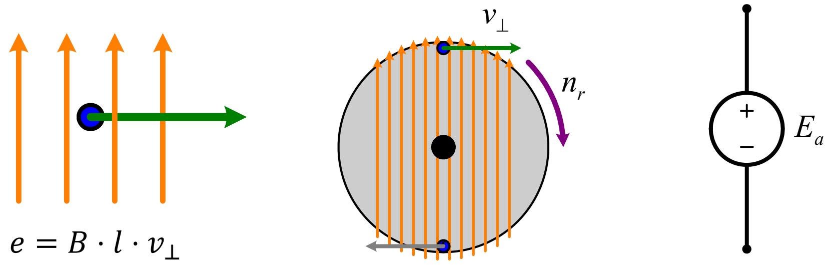
$e = B \cdot l \cdot v_\perp$

Deriving the Induced Armature Terminal Voltage Relationship

The **magnitude** of the **voltage** $\boldsymbol{E_a}$ is a function of the peak value of the rotor conductor voltages that were previously characterized based on the Faraday's Law expression:

$$e = B \cdot l \cdot v_\perp$$

DC Machine – Armature Circuit Model



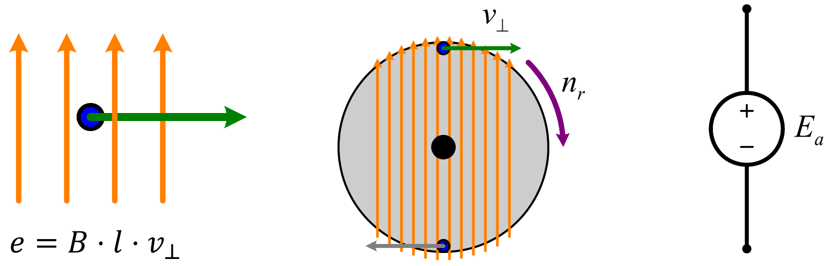
$$e = B \cdot l \cdot v_\perp$$

Deriving the Induced Armature Terminal Voltage Relationship

Since the opposing rotor conductors are connected in series, the **voltage** induced **across the commutator bars** is 2x the voltage (***e***) induced across the individual conductors.

$$E_a \equiv 2e = 2 \cdot B \cdot l \cdot v_\perp$$

DC Machine – Armature Circuit Model

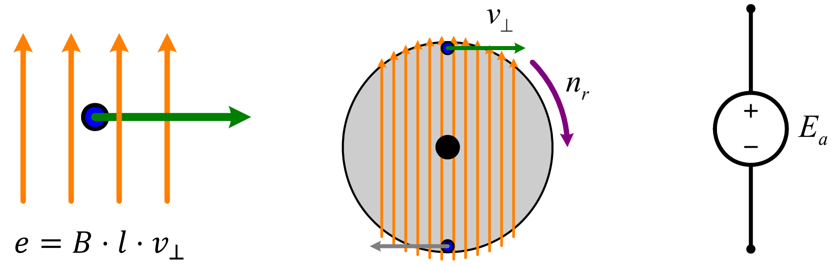


$$e = B \cdot l \cdot v_\perp$$

Deriving the Induced Armature Terminal Voltage Relationship

Furthermore, if the brushes are placed such that they always connect to the rotor conductors that are traveling orthogonal to the stator field, then the brush-commutator connection results in a **DC armature terminal voltage** that is equal to the peak value of voltages developed across the commutator bars.

DC Machine – Armature Circuit Model



$e = B \cdot l \cdot v_\perp$

Deriving the Induced Armature Terminal Voltage Relationship

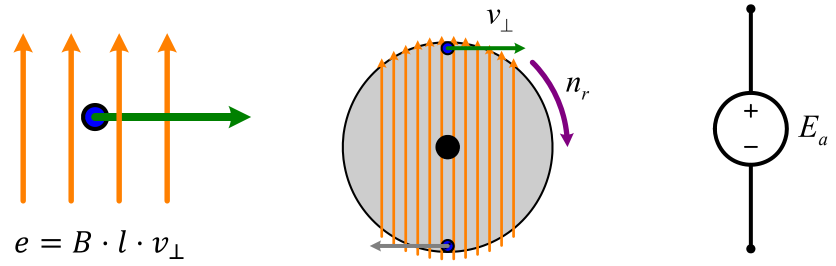
When traveling orthogonal to the field, the **linear speed** of the **conductors** is:

$$v_\perp = \frac{n_r}{60} \cdot 2\pi \cdot r$$

thus:

$$E_a = 2 \cdot B \cdot l \cdot \frac{n_r}{60} \cdot 2\pi \cdot r$$

DC Machine – Armature Circuit Model

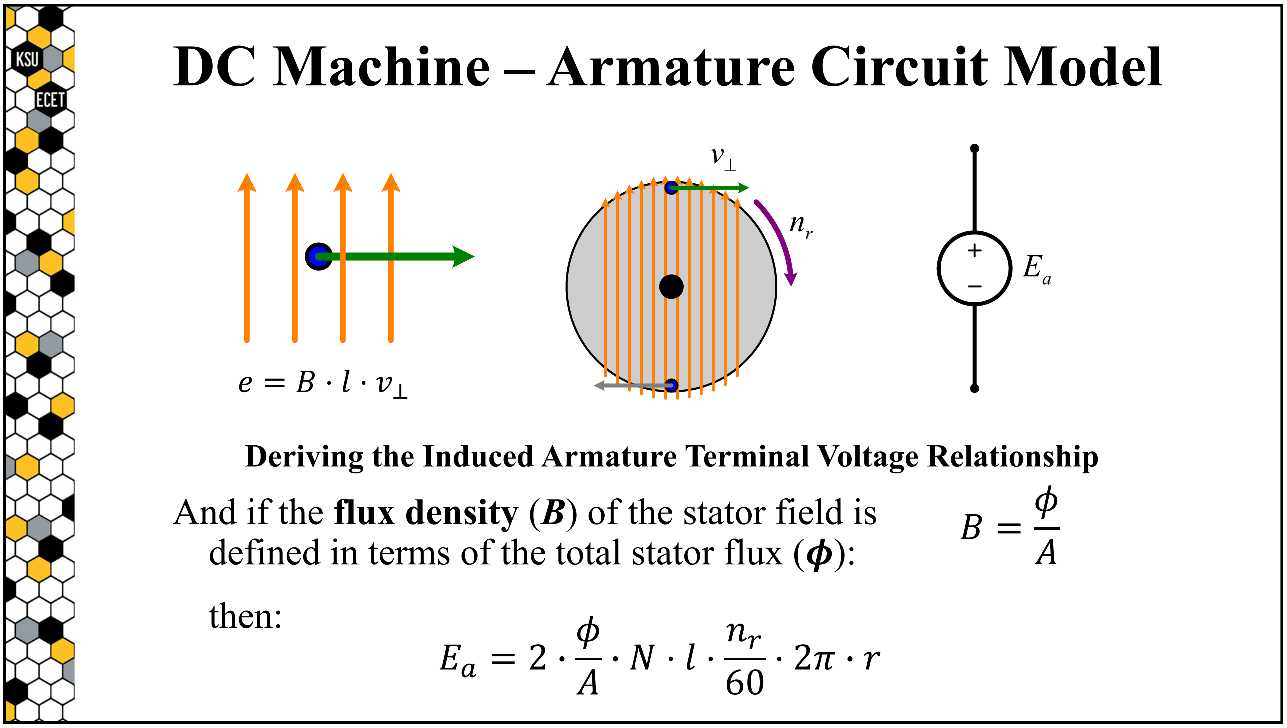


$e = B \cdot l \cdot v_\perp$

Deriving the Induced Armature Terminal Voltage Relationship

But in a practical machine, a coil of wire would be used instead of a single pair of conductors, resulting in an **induced voltage** that is also proportional to the number of turns (***N***) of the coil:

$$E_a = 2 \cdot B \cdot N \cdot l \cdot \frac{n_r}{60} \cdot 2\pi \cdot r$$



DC Machine – Armature Circuit Model

$$e = B \cdot l \cdot v_\perp$$

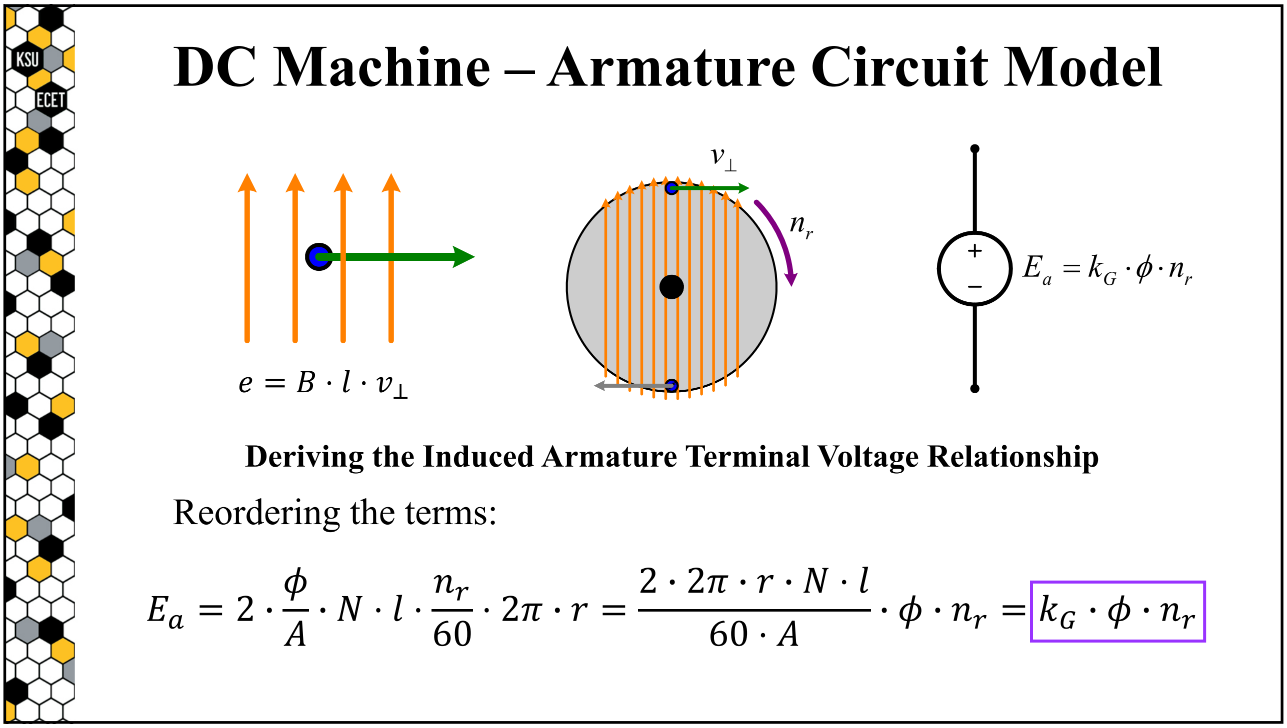
Deriving the Induced Armature Terminal Voltage Relationship

And if the **flux density** (***B***) of the stator field is defined in terms of the total stator flux (***ϕ***):

$$B = \frac{\phi}{A}$$

then:

$$E_a = 2 \cdot \frac{\phi}{A} \cdot N \cdot l \cdot \frac{n_r}{60} \cdot 2\pi \cdot r$$



DC Machine – Armature Circuit Model

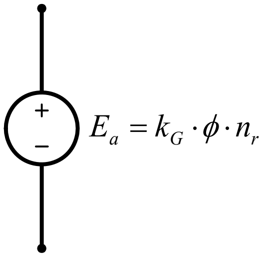
$$e = B \cdot l \cdot v_\perp$$

Deriving the Induced Armature Terminal Voltage Relationship

Reordering the terms:

$$E_a = 2 \cdot \frac{\phi}{A} \cdot N \cdot l \cdot \frac{n_r}{60} \cdot 2\pi \cdot r = \frac{2 \cdot 2\pi \cdot r \cdot N \cdot l}{60 \cdot A} \cdot \phi \cdot n_r = \boxed{k_G \cdot \phi \cdot n_r}$$

DC Machine – Armature Circuit Model

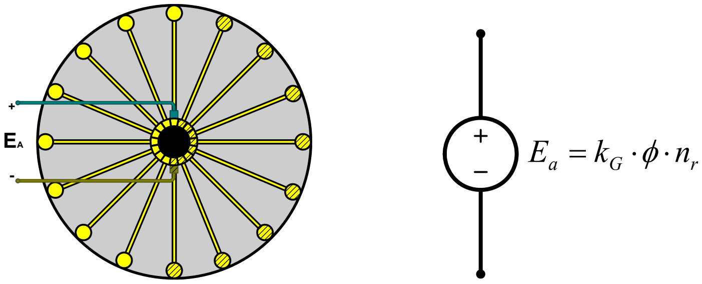


Induced Armature Voltage

$$E_a = k_G \cdot \phi \cdot n_r$$

where: $\boldsymbol{\phi}$ is the **flux** created by the stator field winding (or permanent magnets),
$\boldsymbol{n_r}$ is the **rotational speed** of the **rotor** (in rpm), and
$\boldsymbol{k_G}$ is a **machine constant** the depends on the physical parameters of the machine along with some unit-conversion factors.

DC Machine – Armature Circuit Model

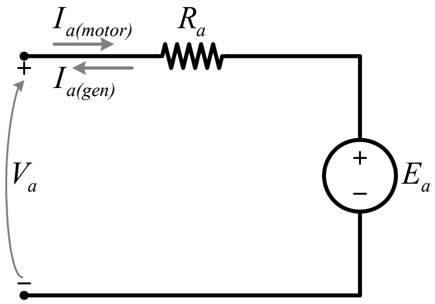


Armature of the DC Machine

In addition to the induced armature voltage, if current is flowing in the rotor conductors, then the armature circuit model must also account for the overall **resistance** of the **armature circuit**.

The armature circuit resistance may include both the resistance of the rotor conductors as well as a factor to account for the non-ideal connections between the brushes and the commutator.

DC Machine – Armature Circuit Model

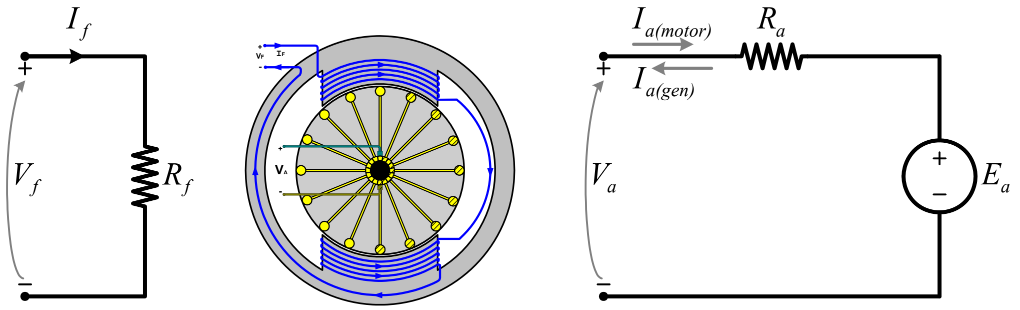


Armature of the DC Machine

If a series resistance is added to account for the overall armature circuit resistance, then the **voltage**, V_a, can be defined at the **armature** (brush) **terminals** such that:

$$V_a = E_a \pm I_a \cdot R_a$$

DC Machine – Overall Circuit Model

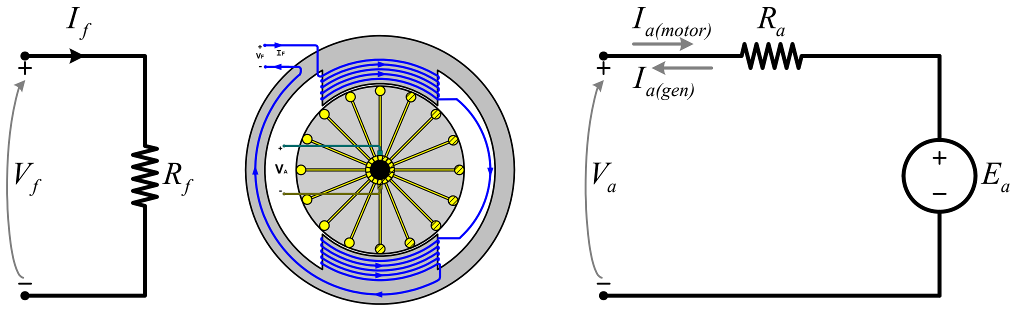


Field Circuit

Armature Circuit

Thus, the complete model for the DC machine consists of two parts; a **field circuit** and an **armature circuit**.

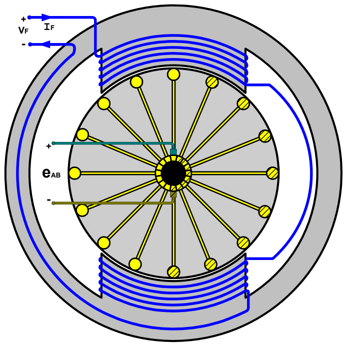
DC Machine – Overall Circuit Model



Field Circuit **Armature Circuit**

Although the field and armature circuits may be electrically isolated from each other, their operation is linked due to the relationship between **armature voltage E_a** and **stator flux ϕ**, which in-turn is a function of the **field current I_f**.

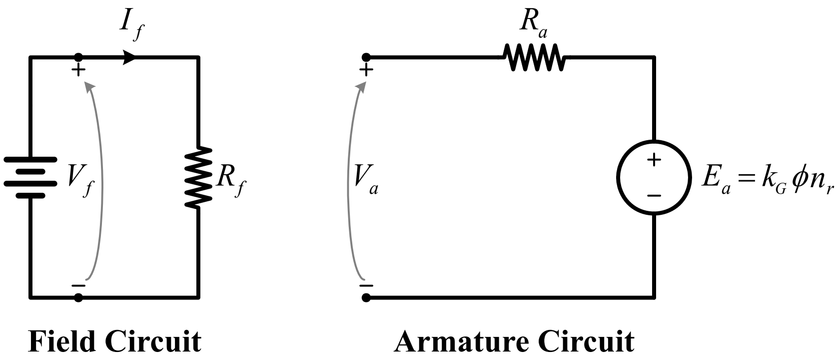
DC Machines



Separately-Excited DC Generators

Operational Characteristics

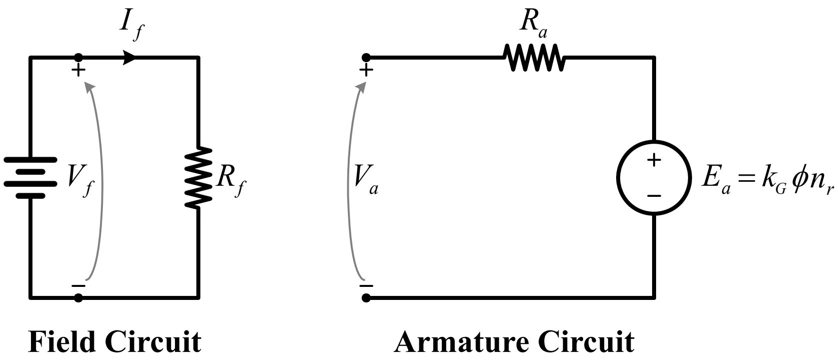
Separately-Excited DC Generator



When operating as a **separately-excited DC generator**:

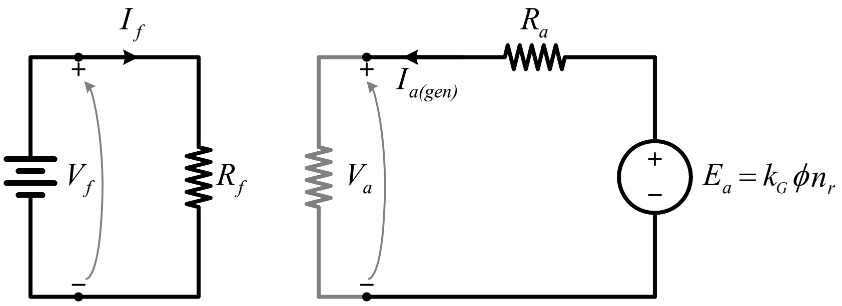
A **voltage** is applied to the **field winding** in order to create the required stator flux while a **mechanical force** is applied to the **shaft** of the machine in order to rotate it at a constant speed, causing the machine to develop an armature voltage ***E_a***.

Separately-Excited DC Generator



Under **open-circuit conditions**, the armature terminal voltage ***V_a*** will equal the induced armature voltage ***E_a*** since there is no current flowing in the armature circuit.

Separately-Excited DC Generator

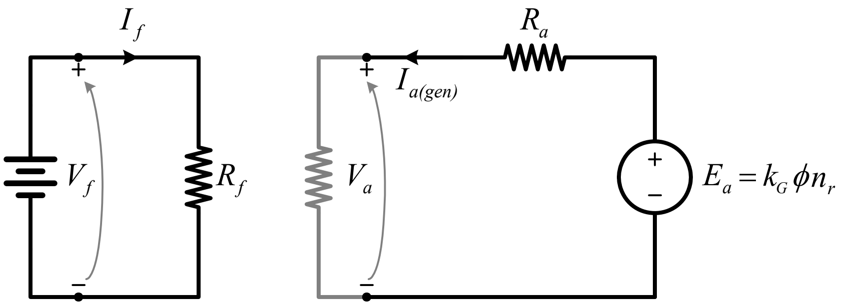


Field Circuit **Armature Circuit**

But if a **load** is connected to the armature terminals, then the induced armature voltage $\boldsymbol{E_a}$ will cause a current to flow out of the positive terminal of the armature circuit, such that:

$$V_a = E_a - I_a \cdot R_a$$

Separately-Excited DC Generator

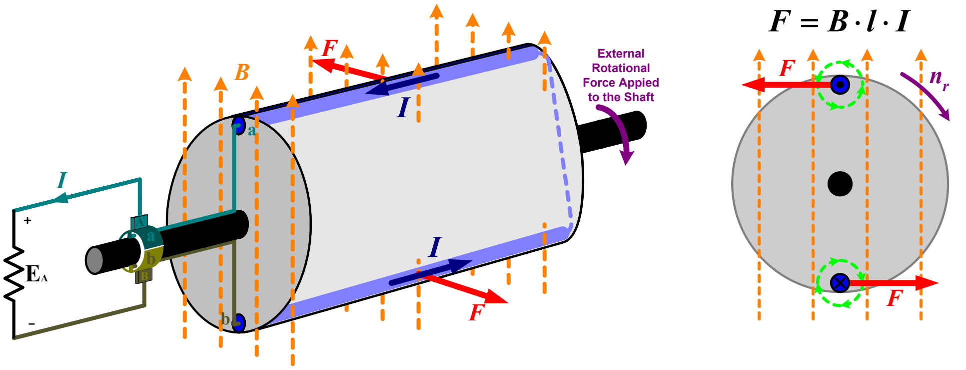


Field Circuit **Armature Circuit**

Note that, when operating as a **generator**, to electric power produced by the source $\boldsymbol{E_a}$ must equal to the total mechanical power converted to an electric form, thus:

$$P_{mech} = E_a \cdot I_a$$

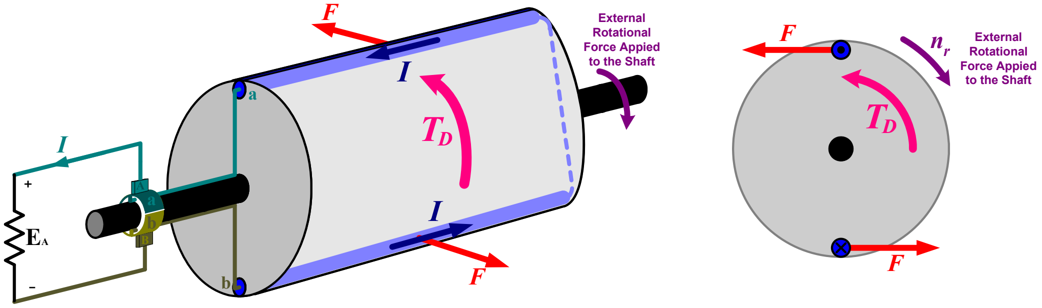
Separately-Excited DC Generator



Also note that, due to the orientation of the rotor conductors within the stator field, opposing **forces** (***F***) will be created upon the conductors when a current (I_a) is flowing in the conductors.

$$F = B \cdot l \cdot I$$

Separately-Excited DC Generator

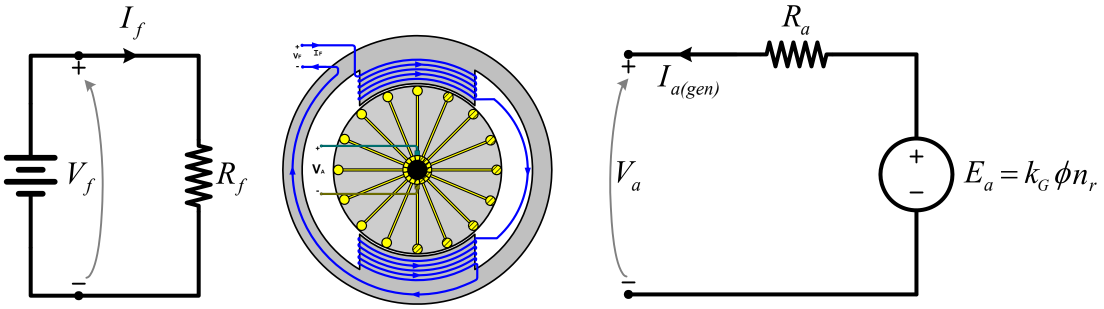


The opposing forces cause a **torque** (T_D) to be developed upon the rotor that opposes its rotation, such that:

$$T_D = k_m \cdot \phi \cdot I_a$$

k_m is another machine constant, similar to k_G, that also depends on the physical parameters of the machine along with some unit-conversion factors, the exact details of which will not be presented at this time.

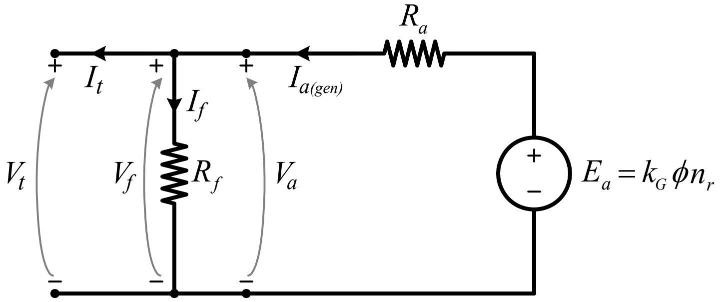
Separately-Excited DC Generator



Field Circuit **Armature Circuit**

When operating as a **separately-excited DC generator**, a voltage source must be connected to the field circuit in order to supply the field current (***I_f***) required to create the stator flux (***ϕ***) and, in-turn, the armature voltage ***E_a***.

Self-Excited DC Generator

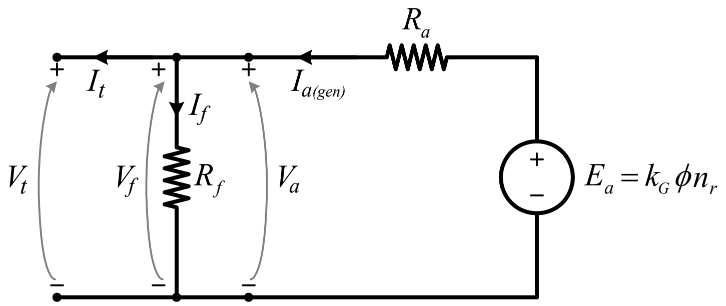


Shunt-Connected Field & Armature Circuits

If an external source is not available to supply a field current, the machine can be configured as a **self-excited DC generator**, in which the field and armature circuits are connected in parallel.

shunt-connected ≡ parallel-connected

Self-Excited DC Generator



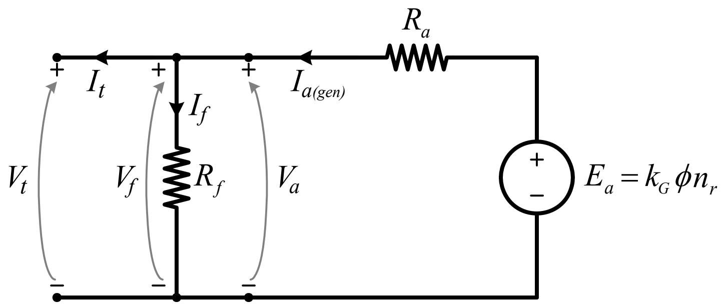
Shunt-Connected Field & Armature Circuits

In this configuration:

$$V_t = V_a = V_f$$

$$I_t = I_a - I_f$$

Self-Excited DC Generator

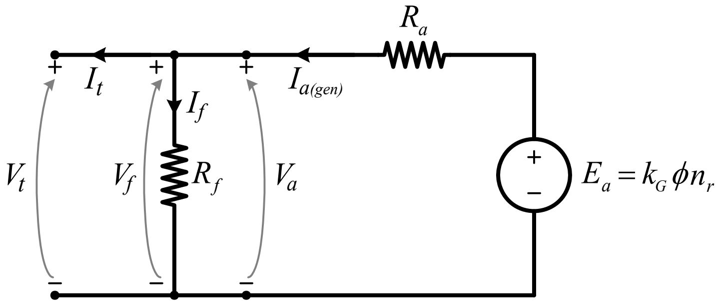


Shunt-Connected Field & Armature Circuits

Although no initial source of field current exists in this configuration, a **residual magnetization** (**ϕ**) often exists in the machine's core.

Exposure to an externally-created flux will cause most ferro-magnetic materials to become magnetized, at least to some small extent, resulting in a residual magnetism to remain within the materials even after the externally-created flux is removed.

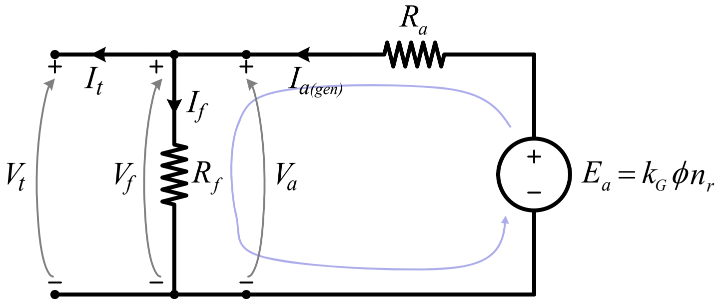
Self-Excited DC Generator



Shunt-Connected Field & Armature Circuits

Thus, when an external force is applied to the shaft of the machine to rotate it at some speed, the **residual stator flux** (***ϕ***) will cause a small voltage (***E_a***) to be induced within the armature circuit.

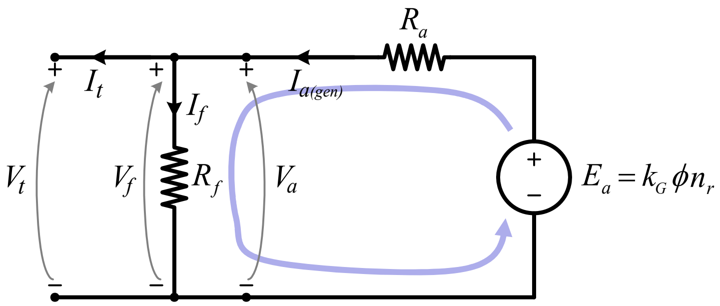
Self-Excited DC Generator



Shunt-Connected Field & Armature Circuits

Provided the field resistance isn't too large, a small **current** will begin to flow through the armature and field circuits due to the small armature voltage (***E_a***) that resulted from the residual field, in-turn causing the field coil to develop a slightly larger flux (***ϕ***).

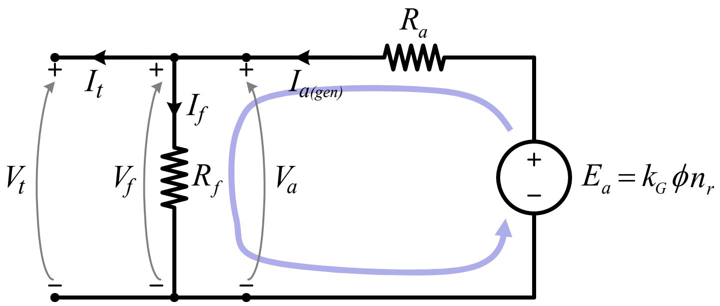
Self-Excited DC Generator



Shunt-Connected Field & Armature Circuits

The increasing **flux ($\boldsymbol{\phi}$)** causes a further increase in the induced armature voltage (**$\boldsymbol{E_a}$**) and, in-turn, the field current, which results in an even larger flux to be created by the field winding.

Self-Excited DC Generator

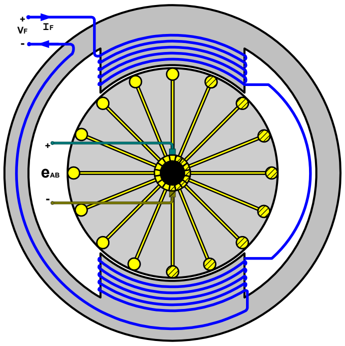


Shunt-Connected Field & Armature Circuits

This **buildup of voltage** will continue until a steady-state operating point is reached based on the electrical characteristics of the machine and the magnetic properties of the core material.

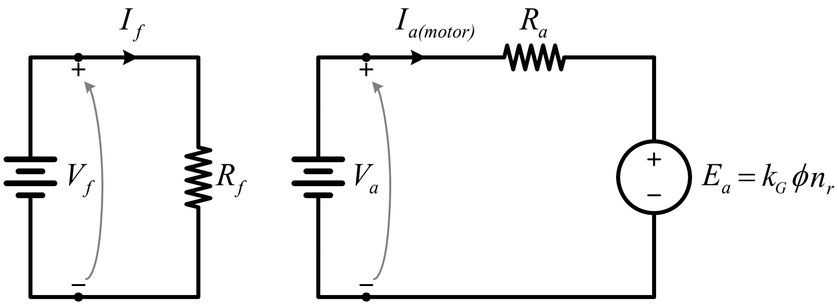
The exact mechanism that results in steady-state operation of a Shunt-Excited DC Generator will not be discussed in detail at this time.

DC Machines



Separately-Excited DC Motors
Operational Characteristics

Separately-Excited DC Motor

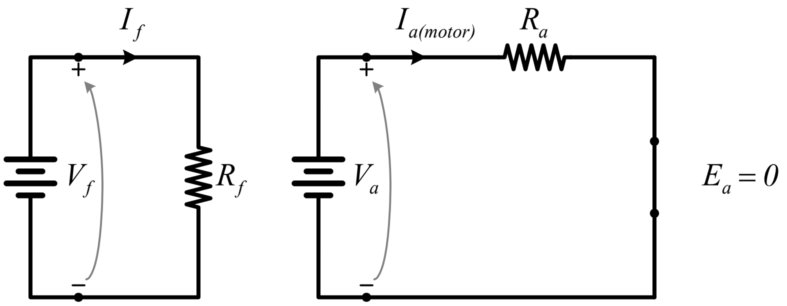


Field Circuit **Armature Circuit**

When operating as a **separately-excited DC motor**:

A **voltage** is applied to both the **field winding** (in order to create the required stator flux) and the **armature terminals**.

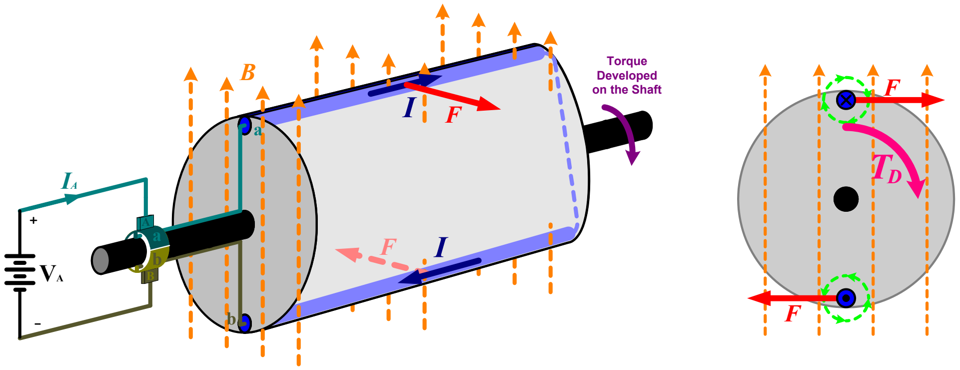
Separately-Excited DC Motor



At **start-up** (***$n_r = 0$***), the induced armature voltage ***E_a*** will be **zero**, resulting in a large armature current and in-turn, a large developed torque:

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_a}{R_a} \qquad T_D = k_m \cdot \phi \cdot I_a$$

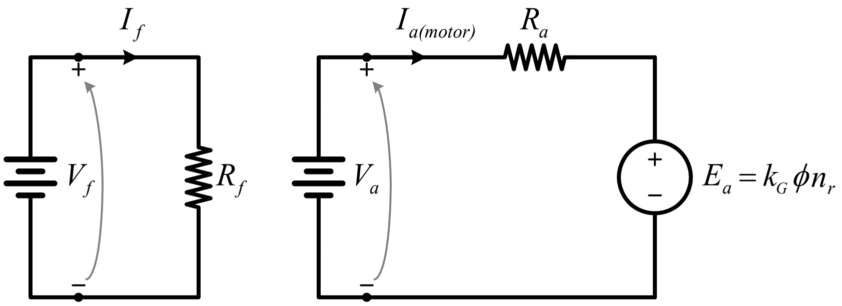
Separately-Excited DC Motor



And when a current flows into the armature circuit, a field will be induced around the armature conductors that will interact with the stator field, resulting in a **developed torque** that will try to accelerate the rotor.

$$T_D = k_m \cdot \phi \cdot I_a$$

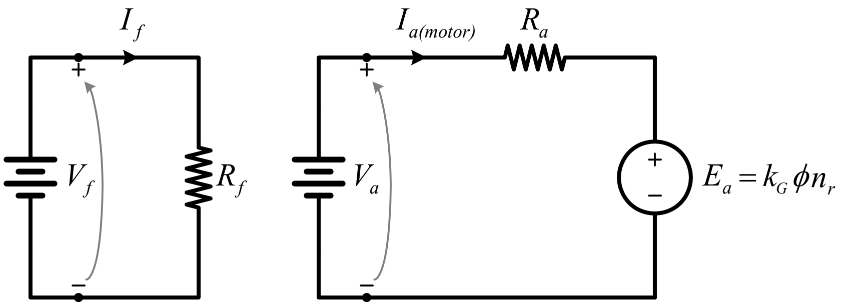
Separately-Excited DC Motor



But as the motor begins to accelerate, the **armature voltage E_a** will increase, in-turn causing a decrease in both the armature current I_a and the developed torque T_D :

$$I_a = \frac{V_a - E_a}{R_a} \qquad T_D = k_m \cdot \phi \cdot I_a$$

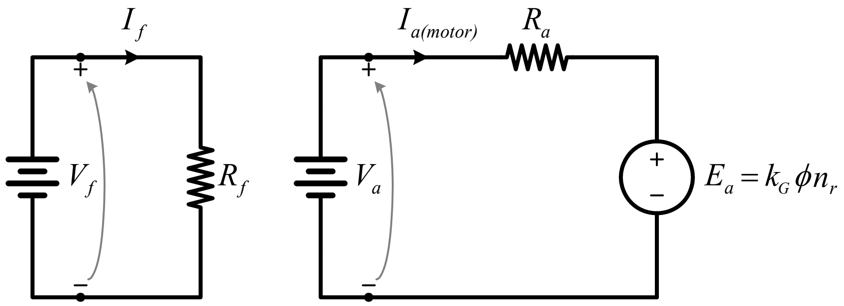
Separately-Excited DC Motor



Under **"no-load" conditions**, the motor will continue to accelerate until the **developed torque $T_D = 0$**, which occurs when $E_a = V_a$, resulting in an armature current $I_a = 0$:

$$I_a = \frac{V_a - E_a}{R_a} = 0 \quad when \quad V_a = E_a$$

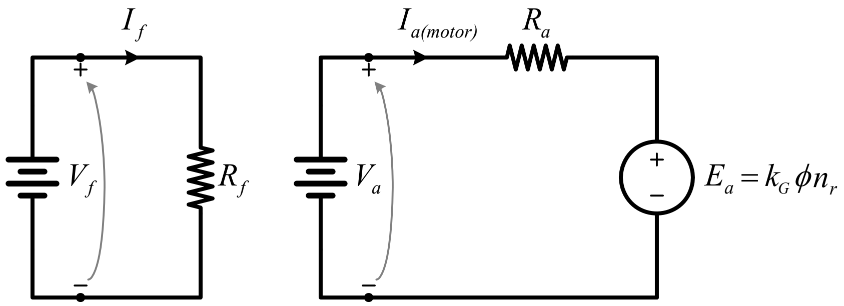
Separately-Excited DC Motor



If a **mechanical load** is applied to the shaft of the motor, then the motor will **slow down**, resulting in a **decrease** in the **armature voltage *E_a*** and an associated **increase** in the **armature current *I_a***.

$$I_a = \frac{V_a - E_a}{R_a}$$

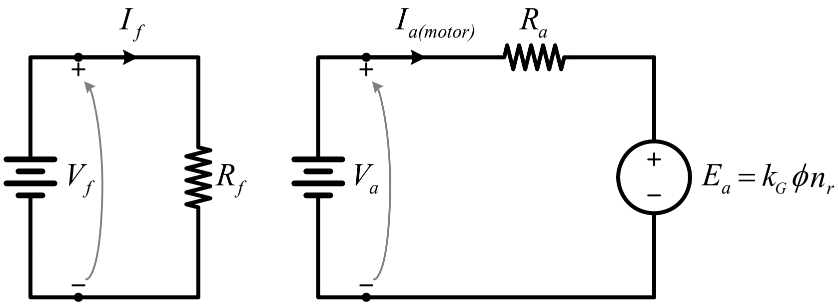
Separately-Excited DC Motor



The "loaded" motor will slow down to a **steady-state operational speed** at which the associated armature current results in a developed torque **$T_D = T_{load}$**.

$$I_a = \frac{V_a - E_a}{R_a} \qquad T_D = k_m \cdot \phi \cdot I_a$$

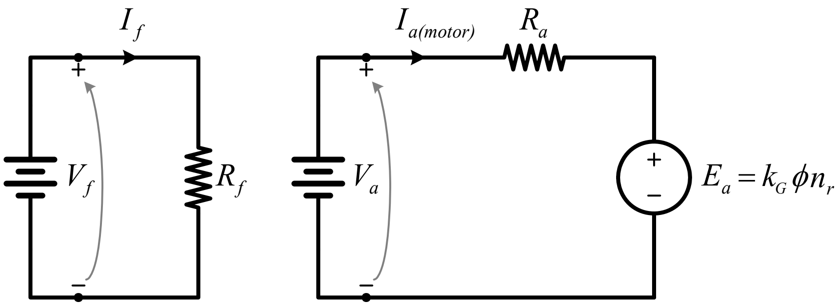
Separately-Excited DC Motor



Note that for a DC **motor**, the power "consumed" by the source E_a in the model is the amount of electrical power converted to a mechanical form:

$$P_{mech} = E_a \cdot I_a$$

Separately-Excited DC Motor



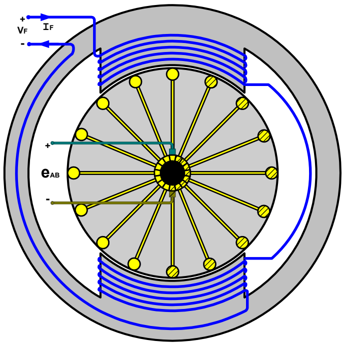
Summary of relationships for a **separately-excited DC motor**:

$$V_f = I_f \cdot R_f$$
$$P_{elec} = V_a \cdot I_a + V_f \cdot I_f$$
$$P_{mech} = E_a \cdot I_a$$

$$E_a = V_a - I_a \cdot R_a$$
$$E_a = k_G \cdot \phi \cdot n_r$$
$$T_D = k_m \cdot \phi \cdot I_a$$

$$I_a = \frac{V_a - E_a}{R_a} \qquad 1\,hp = 746\,W$$
$$T_{D(lb\cdot ft)} = \frac{P_{mech(hp)} \cdot 5252}{n_{r(rpm)}}$$

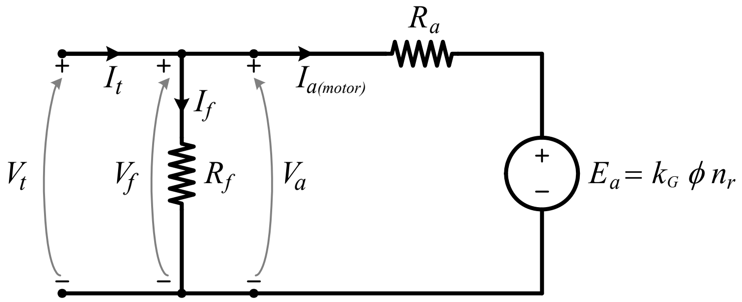
DC Machines



Shunt-Excited & Series-Excited DC Motors

If only a **single source** is available to supply a DC motor, then the **field and armature circuits** can be connected together in either a parallel or a series format.

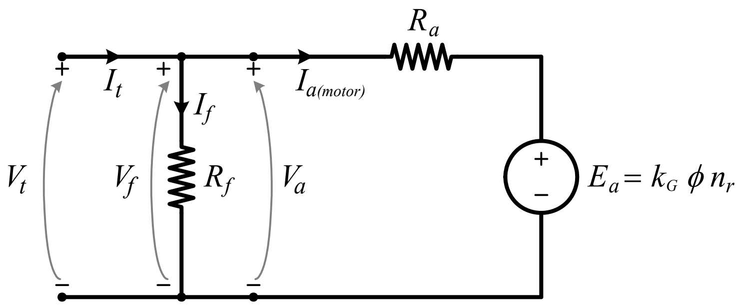
Shunt Excited DC Motor



Parallel-connected Field and Armature Circuits

If the **field and armature circuits** are connected in **parallel** to a single source, then the machine is being operated as a:
“**Shunt Excited**” **DC Motor**.

Shunt Excited DC Motor



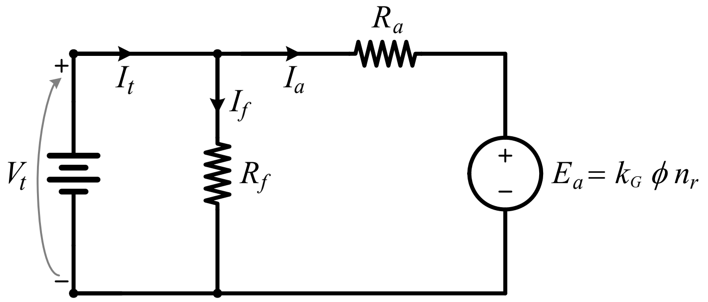
Due to the parallel connection of the field and armature circuits in a **shunt-excited motor**:

$$V_t = V_a = V_f$$

$$I_t = I_a + I_f$$

Since a shunt-excited motor only has one pair of terminals to which an external source can be connected, V_t is used to define the terminal voltage and I_t is used to define the terminal current.

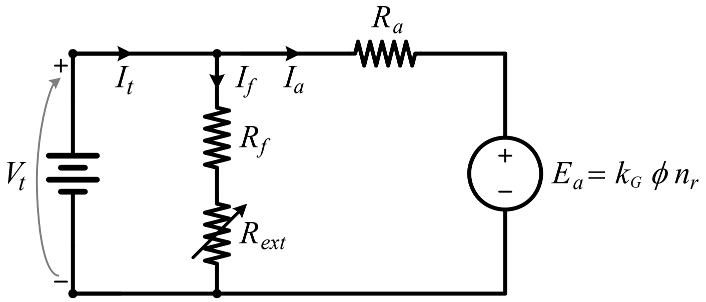
Shunt Excited DC Motor



If a **constant** voltage source V_t is connected across the motor's terminals, then the **field current I_f** (and the stator flux ϕ) will remain **constant** during the motor's operation since:

$$I_f = \frac{V_t}{R_f}$$

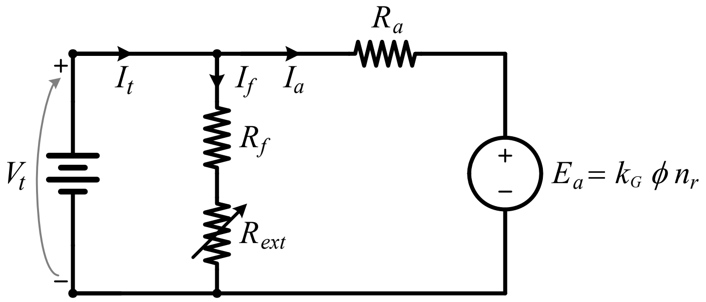
Shunt Excited DC Motor



In order to provide a method for **controlling the speed** of a DC shunt-excited motor, an **external** (variable) **resistance** can be connected in-series with the field circuit:

$$\boldsymbol{I_f = \frac{V_t}{R_f + R_{ext}}}$$

Shunt Excited DC Motor



During normal operation, **increasing** the **field circuit resistance**, in-turn decreasing the field current I_f (and the stator flux $\boldsymbol{\phi}$), will cause the **motor to speed-up** to a new, higher, steady-state operating speed.

Shunt Excited DC Motor

Summary of equations for a "**Shunt Excited**" **DC Motor**.

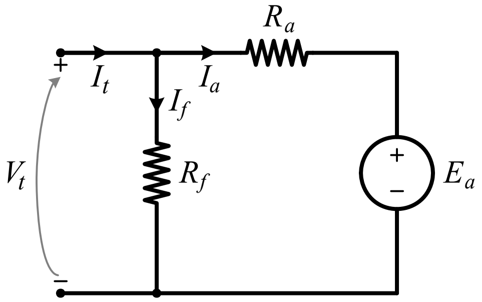
$$V_t = V_a = V_f \qquad E_a = k_G \cdot \phi \cdot n_r$$

$$I_t = I_a + I_f \qquad T_D = k_m \cdot \phi \cdot I_a$$

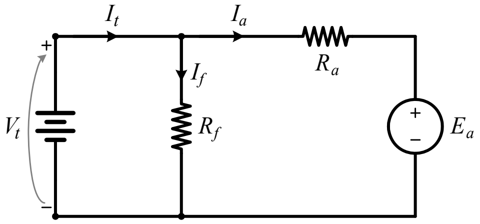
$$E_a = V_a - I_a \cdot R_a \qquad I_f = \frac{V_t}{R_f}$$

$$P_{elec} = V_t \cdot I_t \qquad P_{mech} = E_a \cdot I_a$$

$$1\,hp = 746\,W \qquad T_{D(lb\cdot ft)} = \frac{P_{mech(hp)} \cdot 5252}{n_{r(rpm)}}$$



Shunt DC Motor – Example Problem



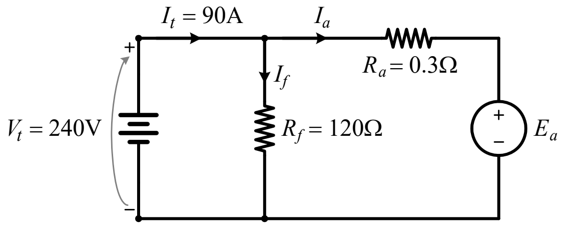
Given a **240V** shunt-excited DC motor having field and armature resistances **R_f = 120Ω** and **R_a = 0.3Ω,**

If the motor draws **90A** and rotates at **900rpm** when supplied with rated voltage, determine:

P_{elec} – the **total electrical power** supplied to the motor by its source,
P_{mech} – the **total mechanical power** produced by the motor, and
T_D – the **total torque developed** by the motor.

Shunt DC Motor – Example Problem

P_{elec} = 21,600W

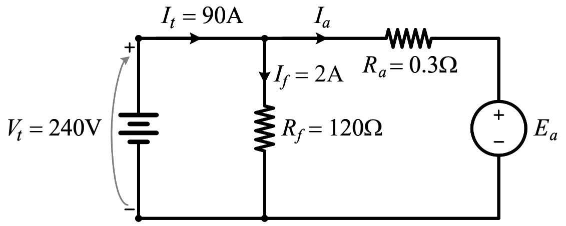


Since a shunt-excited DC motor is supplied by a single voltage source, the **total electrical power**, ***P_{elec}***, supplied to the motor is:

$$P_{elec} = V_t \cdot I_t = 240\text{V} \cdot 90\text{A} = 21{,}600 \text{ W}$$

Shunt DC Motor – Example Problem

P_{elec} = 21,600W



Since a shunt-excited motor's field and armature circuits are connected in parallel:

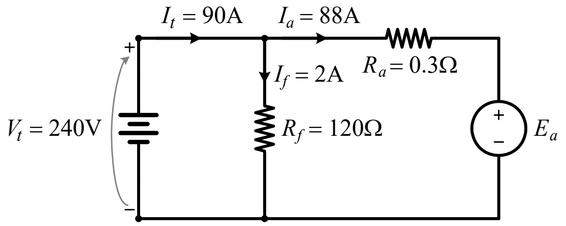
$$V_t = V_a = V_f = 240 \text{ V}$$

and thus, the **field current *I_f*** is:

$$I_f = \frac{V_t}{R_f} = \frac{240\text{V}}{120\Omega} = 2 \text{ A}$$

Shunt DC Motor – Example Problem

$P_{elec} = 21{,}600\text{W}$

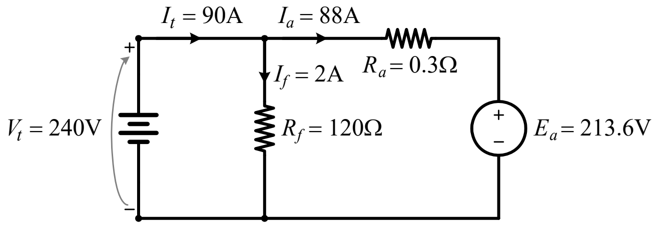


Now that both the terminal current $\boldsymbol{I_t}$ and the field current $\boldsymbol{I_f}$ are known, the **armature current** $\boldsymbol{I_a}$ can be determined based on the KCL equation:

$$I_a = I_t - I_f = 90 - 2 = 88 \text{ A}$$

Shunt DC Motor – Example Problem

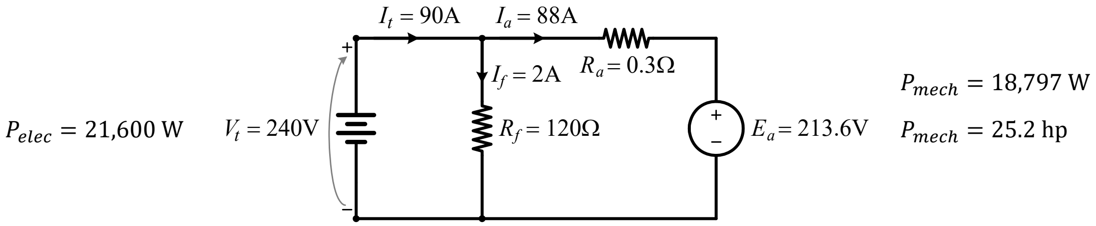
$P_{elec} = 21{,}600\text{W}$



The **induced armature voltage** $\boldsymbol{E_a}$ can then be determined based on the KVL equation:

$$E_a = V_a - I_a \cdot R_a = 240\text{V} - (88\text{A})(0.3\Omega) = 213.6 \text{ V}$$

Shunt DC Motor – Example Problem



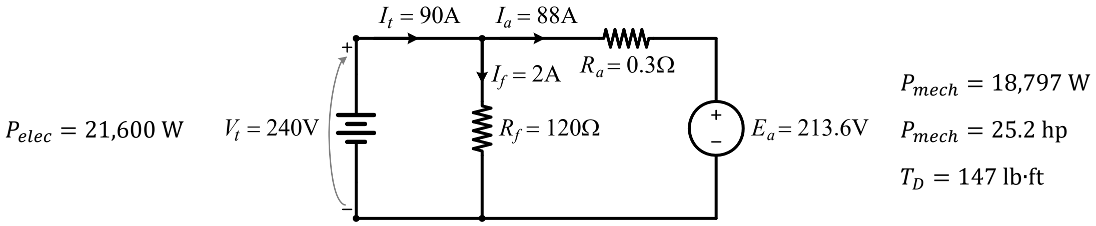
And now that both the induced armature voltage E_a and the armature current I_a are both known, the **total mechanical power P_{mech}** produced by the motor is:

$$P_{mech} = E_a \cdot I_a = (213.6\text{V})(88\text{A}) = 18{,}797\text{ W}$$

which can also be expressed in terms of horsepower:

$$P_{mech} = 18{,}797\text{W} \cdot \frac{1\text{hp}}{746\text{W}} = 25.2\text{ hp}$$

Shunt DC Motor – Example Problem

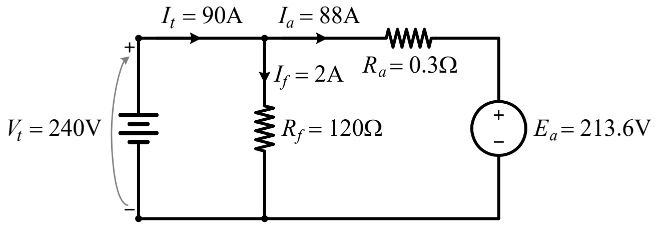


The **total torque T_D developed** by the motor can then be determined based on the total mechanical power P_{mech} and rotational speed n_r of the motor:

$$T_D = \frac{P_{mech(hp)} \cdot 5252}{n_{r(rpm)}} = \frac{25.2 \cdot 5252}{900} = 147\text{ lb·ft}$$

Shunt DC Motor – Example Problem

$P_{elec} = 21{,}600$ W



$P_{mech} = 18{,}797$ W

$P_{mech} = 25.2$ hp

$T_D = 147$ lb·ft

Although it was not requested, the **operational efficiency *η*** of the motor can be determined based on the previous results:

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{mech}}{P_{elec}} \cdot 100\% = \frac{18{,}797}{21{,}600} \cdot 100\% = 87.0\ \%$$

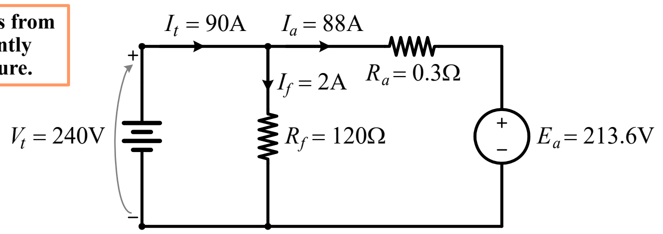
Note that the motor's mechanical losses were not considered in this problem. If mechanical losses are included, then the output power of the motor would be the shaft power P_{shaft}, which is equal to:

$$P_{shaft} = P_{mech} - P_{mechlosses}$$

Shunt DC Motor – Example Problem pt. II

The original values from part I are currently shown in the figure.

$P_{elec} = 21{,}600$ W



$P_{mech} = 18{,}797$ W

$P_{mech} = 25.2$ hp

$T_D = 147$ lb·ft

Given the results of the previous problem, and assuming that the motor is driving a **constant torque load** and that the terminal voltage remains constant,

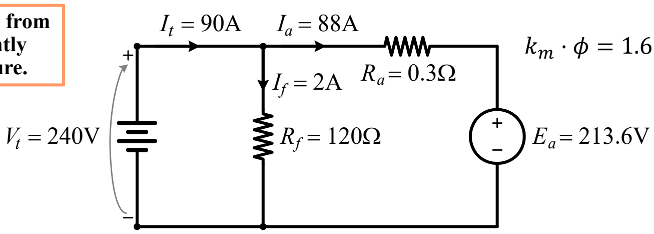
If a resistance is added in series with the field circuit such that the increased resistance causes the stator flux ϕ to decrease by 20%, determine:

the **new operational speed $n_{r(new)}$** for the motor.

Shunt DC Motor – Example Problem pt. II

The original values from part I are currently shown in the figure.

$P_{elec} = 21{,}600$ W



$k_m \cdot \phi = 1.6705\,\frac{\text{lb·ft}}{\text{A}}$

$P_{mech} = 18{,}797$ W

$P_{mech} = 25.2$ hp

$T_D = 147$ lb·ft

Based on the previous results and the expression for developed torque $\boldsymbol{T_D}$:

$$T_D = k_m \cdot \phi \cdot I_a$$

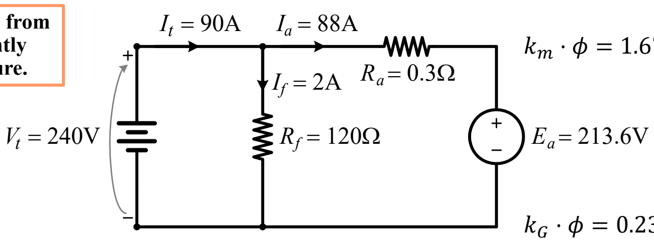
the original value for $\boldsymbol{k_m \cdot \phi}$ can be determined:

$$k_m \cdot \phi = \frac{T_D}{I_a} = \frac{147}{88} = 1.6705\,\frac{\text{lb·ft}}{\text{A}}$$

Shunt DC Motor – Example Problem pt. II

The original values from part I are currently shown in the figure.

$P_{elec} = 21{,}600$ W



$k_m \cdot \phi = 1.6705\,\frac{\text{lb·ft}}{\text{A}}$

$P_{mech} = 18{,}797$ W

$P_{mech} = 25.2$ hp

$T_D = 147$ lb·ft

$k_G \cdot \phi = 0.23733\,\frac{\text{V}}{\text{rpm}}$

Based on the previous results and the expression for armature voltage $\boldsymbol{E_a}$:

$$E_a = k_G \cdot \phi \cdot n_r$$

the original value for $\boldsymbol{k_G \cdot \phi}$ can be determined:

$$k_G \cdot \phi = \frac{E_a}{n_r} = \frac{213.6}{900} = 0.23733\,\frac{\text{V}}{\text{rpm}}$$

Shunt DC Motor – Example Problem pt. II

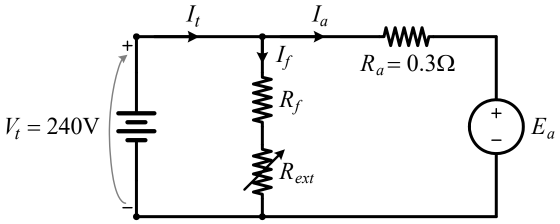
Original values from part I that are needed for part II:

$k_m \cdot \phi = 1.6705 \, \frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi = 0.23733 \, \frac{\text{V}}{\text{rpm}}$

$I_a = 88 \text{ A}$

$E_a = 213.6 \text{ V}$



$k_m \cdot \phi_{(new)} = 1.3364 \, \frac{\text{lb·ft}}{\text{A}}$

Since $\boldsymbol{k_m}$ is a constant, if the stator flux $\boldsymbol{\phi}$ decreases by 20%, then the value of $\boldsymbol{k_m \cdot \phi}$ must also decrease by 20%, thus the new value for $\boldsymbol{k_m \cdot \phi_{(new)}}$ will be:

$$k_m \cdot \phi_{(new)} = k_m \cdot \phi_{(orig)} \cdot 0.8 = 1.6705 \cdot 0.8 = 1.3364 \, \frac{\text{lb·ft}}{\text{A}}$$

Shunt DC Motor – Example Problem pt. II

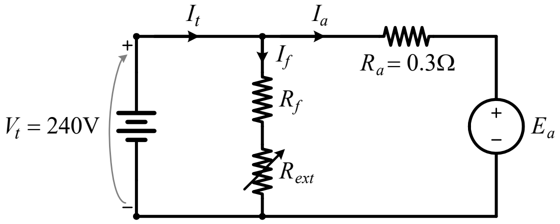
Original values from part I that are needed for part II:

$k_m \cdot \phi = 1.6705 \, \frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi = 0.23733 \, \frac{\text{V}}{\text{rpm}}$

$I_a = 88 \text{ A}$

$E_a = 213.6 \text{ V}$



$k_m \cdot \phi_{(new)} = 1.3364 \, \frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi_{(new)} = 0.18986 \, \frac{\text{V}}{\text{rpm}}$

Similarly, since $\boldsymbol{k_G}$ is a constant, if the stator flux $\boldsymbol{\phi}$ decreases by 20%, then the value of $\boldsymbol{k_G \cdot \phi}$ must also decrease by 20%, thus the new value for $\boldsymbol{k_G \cdot \phi_{(new)}}$ will be:

$$k_G \cdot \phi_{(new)} = k_G \cdot \phi_{(orig)} \cdot 0.8 = 0.23733 \cdot 0.8 = 0.18986 \, \frac{\text{V}}{\text{rpm}}$$

Shunt DC Motor – Example Problem pt. II

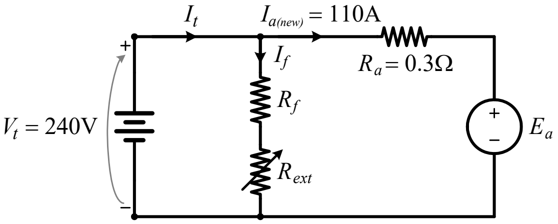
Original values from part I that are needed for part II:

$k_m \cdot \phi = 1.6705\,\frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi = 0.23733\,\frac{\text{V}}{\text{rpm}}$

$I_a = 88\ \text{A}$

$E_a = 213.6\ \text{V}$



$k_m \cdot \phi_{(new)} = 1.3364\,\frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi_{(new)} = 0.18986\,\frac{\text{V}}{\text{rpm}}$

But based on the relationship:

$$T_D = k_m \cdot \phi \cdot I_a$$

if the motor is driving a constant torque load, then a new value for the **armature current $I_{a(new)}$** can be determined:

$$I_{a(new)} = \frac{T_D}{k_m \cdot \phi_{(new)}} = \frac{147}{1.3364} = 110\ A$$

Shunt DC Motor – Example Problem pt. II

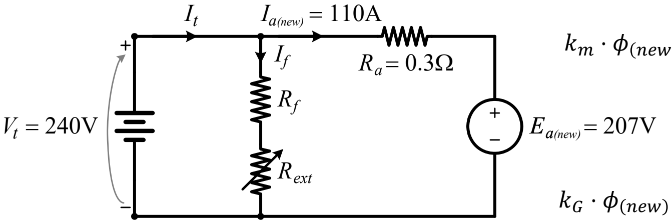
Original values from part I that are needed for part II:

$k_m \cdot \phi = 1.6705\,\frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi = 0.23733\,\frac{\text{V}}{\text{rpm}}$

$I_a = 88\ \text{A}$

$E_a = 213.6\ \text{V}$



$k_m \cdot \phi_{(new)} = 1.3364\,\frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi_{(new)} = 0.18986\,\frac{\text{V}}{\text{rpm}}$

And given the new armature current $I_{a(new)}$, a new value for the **induced armature voltage $E_{a(new)}$** can be determined:

$$E_{a(new)} = V_a - I_{a(new)} \cdot R_a = 240\text{V} - (110\text{A})(0.3\Omega) = 207\ \text{V}$$

Shunt DC Motor – Example Problem pt. II

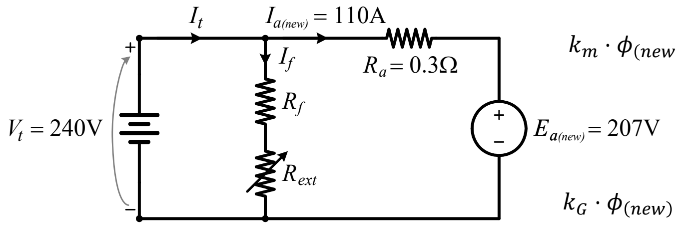
Original values from part I that are needed for part II:

$k_m \cdot \phi = 1.6705\ \frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi = 0.23733\ \frac{\text{V}}{\text{rpm}}$

$I_a = 88\ \text{A}$

$E_a = 213.6\ \text{V}$



$k_m \cdot \phi_{(new)} = 1.3364\ \frac{\text{lb·ft}}{\text{A}}$

$k_G \cdot \phi_{(new)} = 0.18986\ \frac{\text{V}}{\text{rpm}}$

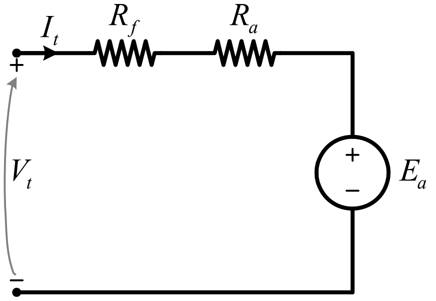
Now that new values for both $\boldsymbol{k_G \cdot \phi_{(new)}}$ and armature voltage $\boldsymbol{E_{a(new)}}$ have been determined, a new operational speed $\boldsymbol{n_{r(new)}}$ can be determined as follows:

$$n_{r(new)} = \frac{E_{a(new)}}{k_G \cdot \phi_{(new)}} = \frac{207}{0.18986} = 1090\ \text{rpm}$$

Series Excited DC Motor

$$V_t = V_a + V_f$$

$$I_t = I_a = I_f$$



Series Field and Armature Circuits

If the **field and armature circuits** are connected in **series** with a single source, then the machine is being operated as a:
"<u>Series Excited</u>" DC Motor.

Series Excited DC Motor

Summary of equations for a “**Series Excited**” **DC Motor**.

$$V_t = V_a + V_f \qquad E_a = k_G \cdot \phi \cdot n_r$$

$$I_t = I_a = I_f \qquad T_D = k_m \cdot \phi \cdot I_a$$

$$E_a = V_t - I_a \cdot (R_a + R_f)$$

$$P_{elec} = V_t \cdot I_t \qquad P_{mech} = E_a \cdot I_a$$

$$1\,hp = 746\,W \qquad T_{D(lb\cdot ft)} = \frac{P_{mech(hp)} \cdot 5252}{n_{r(rpm)}}$$