ECET 3500

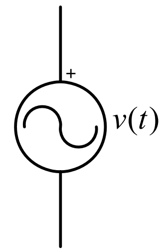
Electric Machines

(Balanced)

Three-Phase Systems

Single-Phase AC Voltage Sources

A (single-phase) **AC voltage source** is a source whose voltage varies sinusoidally, as defined by the function:

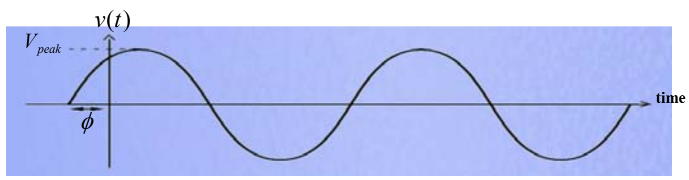


$$v(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

where: V_{peak} is the peak value of the voltage,

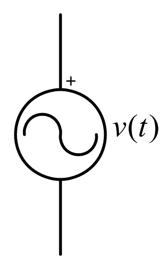
ω is the angular frequency ($2\pi f$) of the waveform, and

ϕ is the phase angle of the waveform.



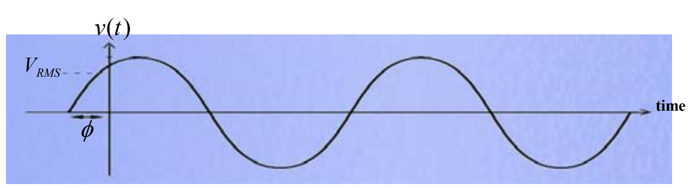
Single-Phase AC Voltage Sources

The voltage waveform may also be expressed in terms of its ***RMS voltage magnitude***:



$$v(t) = \sqrt{2} \cdot V_{RMS} \cdot \sin(\omega \cdot t + \phi)$$

where: $V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$ is the **RMS** or "***effective***" voltage magnitude of the AC waveform.

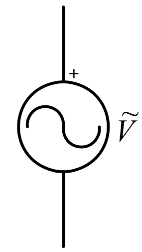


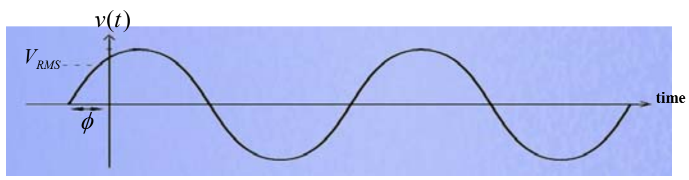
Single-Phase AC Voltage Sources

When performing a steady-state AC analysis, voltages (and currents) are often expressed in "***phasor***" form:

$$\tilde{V} = Ve^{j\phi} = V\angle\phi$$

where: V is the RMS magnitude of the voltage, and
ϕ is the phase angle of the waveform.

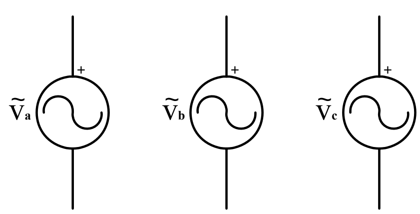




Three-Phase AC Voltage Sources

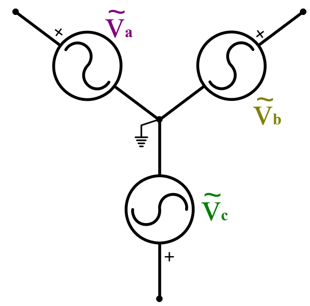
A ***three-phase*** (3Φ) ***AC voltage*** source is a composite source that can be modeled using three single-phase AC voltage sources that are connected together to function as one complete unit.

Note that the three single-phase AC voltage sources must be connected together in a symmetrical fashion.



Wye-connected Three-Phase Source

The three sources are typically connected together in a "***Wye***" (**Y**) format such that the reference terminals of the three supplies are tied to a common node.



The common point of connection is referred to as the "***neutral point***".

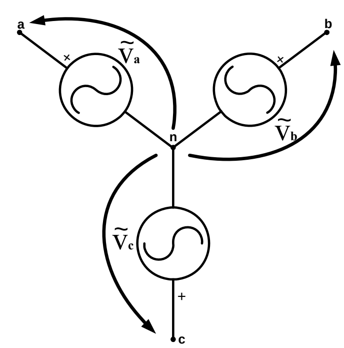
(node **n** in the figure)

Note that the neutral point is often grounded in order to provide a zero-volt reference for the source.

Wye-connected Three-Phase Source

If the remaining nodes are labeled **a**, **b**, and **c**, then:

- Then the voltage $\tilde{V}_a$ can be defined as the voltage-rise from the neutral point **n** to node **a**.
- Similarly, voltages $\tilde{V}_b$ and $\tilde{V}_c$ can be defined as the rises from node **n** to **b** and node **n** to **c** respectively.



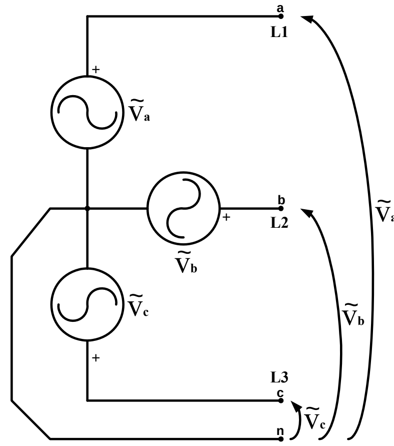
Wye-connected Three-Phase Source

The primary source terminals (connection points to the source) are nodes **a**, **b**, and **c**.

Nodes **a**, **b**, and **c** are sometimes defined as ***line terminals*** **L1**, **L2**, and **L3**.

The neutral-point **n** is typically utilized as a forth (***neutral***) terminal.

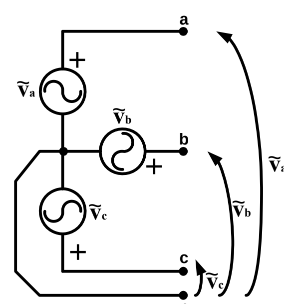
The three voltages are shown in the figure, but this time with respect to the four source terminals **a**, **b**, **c**, and **n**.



Phase Voltages

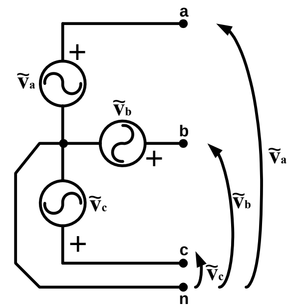
The voltages $\tilde{V}_a$, $\tilde{V}_b$, and $\tilde{V}_c$ are referred to as "***phase voltages***" because they correspond to the voltage across each individual phase of the wye-connected source.

The phase voltages are sometimes referred to as "***line-to-neutral voltages***", and as such may be expressed as $\tilde{V}_{an}$, $\tilde{V}_{bn}$, and $\tilde{V}_{cn}$.



Balanced Three-Phase Voltages

A "***balanced***" 3Φ source is a source whose phase voltages have <u>equal magnitudes</u> and whose <u>phase angles are separated by 120º</u>.



Note that most practical 3Φ sources are assumed to be balanced despite the slight magnitude differences between the individual phases.

Phase Voltages

If expressed as phasors, the phase voltages can be defined by the following expressions:

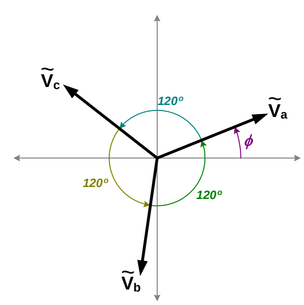
$$\tilde{V}_a = V\angle\phi$$

$$\tilde{V}_b = V\angle\phi - 120°$$

$$\tilde{V}_c = V\angle\phi - 240°$$

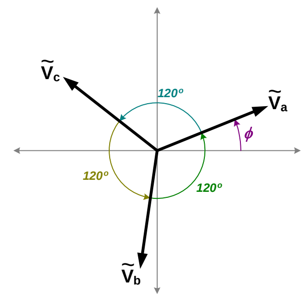
where: V is the RMS magnitude of the voltages, and

ϕ is the phase angle of the phase **a** source.



Phase Voltages

The figure below is a plot of the three phase voltages:

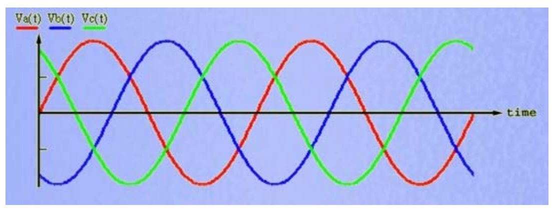


$$\tilde{V}_a = V\angle\phi$$

$$\tilde{V}_b = V\angle\phi - 120°$$

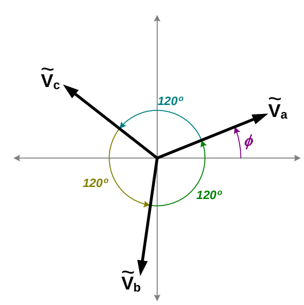
$$\tilde{V}_c = V\angle\phi - 240°$$

as a function of time with $\phi = 0°$ as shown.



Phase Sequence

Note that the voltage relationships:

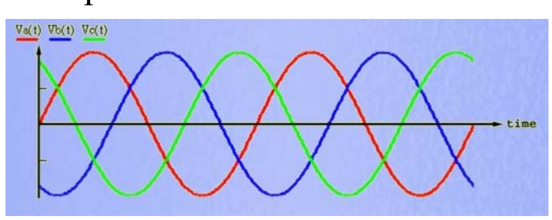


$$\tilde{V}_a = V\angle\phi$$

$$\tilde{V}_b = V\angle\phi - 120°$$

$$\tilde{V}_c = V\angle\phi - 240°$$

define a "***positive-sequence***" (***a-b-c***) source since phase **a** leads **b** and phase **b** leads **c**.

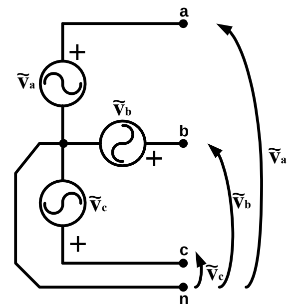


Phase Voltage Example

Given the phase voltage:

$$\tilde{V}_a = 120\angle 0°$$

determine the other phase voltages $\tilde{V}_b$ and $\tilde{V}_c$.



Since $V = 120$ volts and $\phi = 0°$:

$$\tilde{V}_b = V\angle\phi - 120° = 120\angle -120°$$

$$\tilde{V}_c = V\angle\phi - 240° = 120\angle -240°$$

→

$$\tilde{V}_a = 120\angle 0°$$

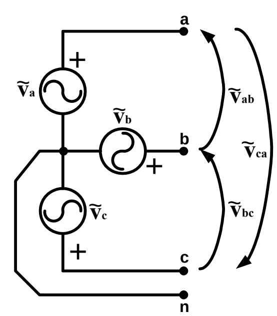
$$\tilde{V}_b = 120\angle -120°$$

$$\tilde{V}_c = 120\angle -240°$$

Line Voltages

A second set of voltages can also be defined for the 3Φ source in terms of the voltage rise between each pair of terminals:

a-b, **b-c**, and **c-a**.



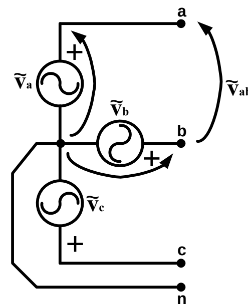
The voltages $\tilde{V}_{ab}$, $\tilde{V}_{bc}$ and $\tilde{V}_{ca}$ are referred to as "***line voltages***".

Line Voltages

The ***line voltages*** for a balanced 3Φ source are closely related to the source's phase voltages.

For example, the line voltage defines the $\tilde{V}_{ab}$ voltage rise from terminal **b** to terminal **a**, and can be expressed in terms of the phase voltages by the KVL equation:

$$\tilde{V}_{ab} = -\tilde{V}_b + \tilde{V}_a = \tilde{V}_a - \tilde{V}_b$$

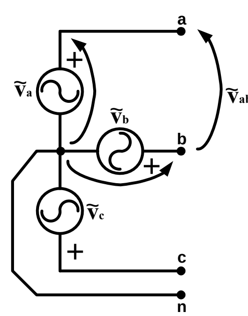


Line Voltages

The ***line voltages*** for a balanced 3Φ source are closely related to the source's phase voltages.

The same logic can be used to express all three line voltages in terms of their respective phase voltages:

$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b$$
$$\tilde{V}_{bc} = \tilde{V}_b - \tilde{V}_c$$
$$\tilde{V}_{ca} = \tilde{V}_c - \tilde{V}_a$$



Line Voltage Example

Given a balanced 3Φ source that has the phase voltage:

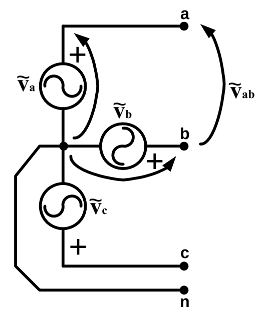
$$\tilde{V}_a = 120\angle 0°$$

the line voltage $\tilde{V}_{ab}$ for that source can be determined as follows:

$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b$$

$$= 120\angle 0° - 120\angle -120°$$

$$= 208\angle +30°$$



Line Voltage Example

A complete analysis of the 3Φ source with phase voltages:

$$\tilde{V}_a = 120\angle 0°$$

$$\tilde{V}_b = 120\angle -120°$$

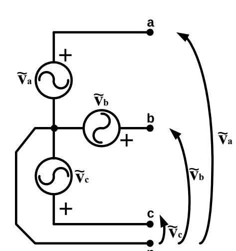
$$\tilde{V}_c = 120\angle -240°$$

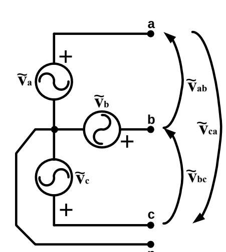
will provide the following line voltages:

$$\tilde{V}_{ab} = 208\angle 30°$$

$$\tilde{V}_{bc} = 208\angle -90°$$

$$\tilde{V}_{ca} = 208\angle -210°$$





Phase ↔ Line Voltage Relationship

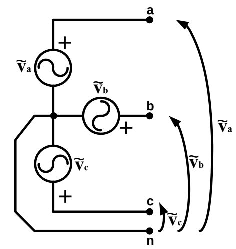
A comparison of the phase and line voltages:

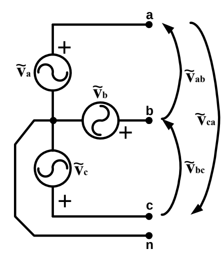
$$\tilde{V}_a = 120\angle 0^\circ \quad \tilde{V}_{ab} = 208\angle 30^\circ$$

reveals that the line voltage is:

- $\sqrt{3}$x greater in magnitude, and
- 30° greater in phase angle

compared to the phase voltage.





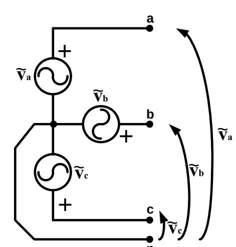
Phase ↔ Line Voltage Relationship

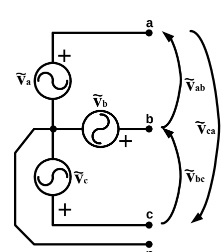
Thus, the relationship between the phase and line voltages:

$$\tilde{V}_a = 120\angle 0^\circ \quad \tilde{V}_{ab} = 208\angle 30^\circ$$

can be expressed as:

$$\tilde{V}_{ab} = (\sqrt{3}\angle 30^\circ)\cdot \tilde{V}_a$$





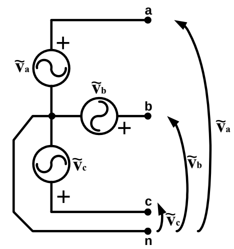
Phase ↔ Line Voltage Relationship

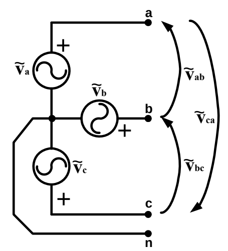
Similarly, given the 3Φ source:

$$\tilde{V}_a = 120\angle 0° \qquad \tilde{V}_{ab} = 208\angle 30°$$

$$\tilde{V}_b = 120\angle -120° \qquad \tilde{V}_{bc} = 208\angle -90°$$

$$\tilde{V}_c = 120\angle -240° \qquad \tilde{V}_{ca} = 208\angle -210°$$





the complete set of phase-to-line voltage relationships are:

$$\tilde{V}_{ab} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_a$$

$$\tilde{V}_{bc} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_b$$

$$\tilde{V}_{ca} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_c$$

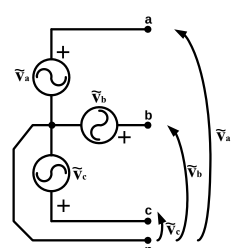
Phase ↔ Line Voltage Relationship

It turns out that the relationships:

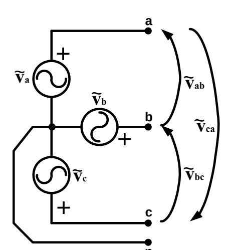
$$\tilde{V}_{ab} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_a$$

$$\tilde{V}_{bc} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_b$$

$$\tilde{V}_{ca} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_c$$



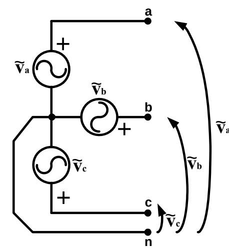
hold true for all balanced 3Φ sources.

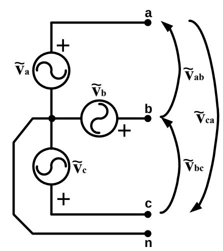


Thus, given any phase or line voltage for a specific source, all of the other voltages can be determined by applying the above relationships.

Phase ↔ Line Voltage Relationship

Given a balanced 3Φ source, the following phase-to-line voltage relationships can be used to specify the complete set of phase and line voltages:

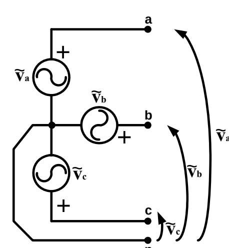


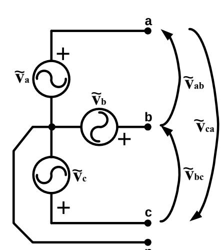


Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

Phase ↔ Line Voltage Relationship

Note that the line voltages also have equal magnitudes and a 120º phase separation between each pair; thus they maintain the same balanced relationship as the phase voltages:



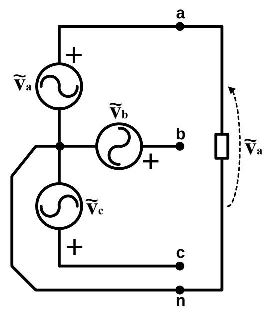


Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

1Φ Voltages Available from 3Φ Source

A single-phase load may be supplied from a single-phase source if the load is connected across two of the source's terminals.

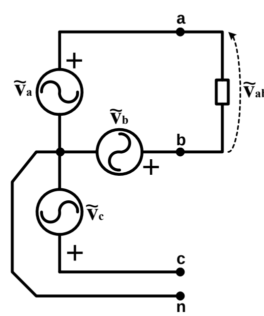
If the load is connected between a <u>line terminal</u> and the <u>neutral terminal</u>, then a ***phase voltage*** will appear across the load.



1Φ Voltages Available from 3Φ Source

A single-phase load may be supplied from a single-phase source if the load is connected across two of the source's terminals.

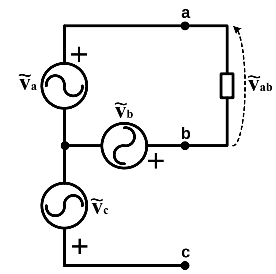
If the load is connected between two of the <u>line terminals</u>, then a ***line voltage*** will appear across the load.



1Φ Voltages Available from 3Φ Source

A single-phase load may be supplied from a single-phase source if the load is connected across two of the source's terminals.

If the load is connected between two of the line terminals, then a ***line voltage*** will appear across the load.



Note – if the neutral terminal is not available, then only ***line voltages*** can be received from the supply, but not phase voltages.

Balanced Three-Phase Loads

A ***three-phase load*** consists of three individual loads that are connected together to form a symmetrical, composite load that can be supplied by connecting it to the terminals of a 3Φ source.

A ***balanced*** 3Φ load is constructed using three loads that all have the same impedance value.

When a balanced 3Φ load is connected to a balanced 3Φ source, the resultant currents will also maintain a balanced relationship similar to that of the phase or line voltages.

Three-Phase Load Configurations

There are two different load configurations that can be utilized in order to connect the three individual loads together in a symmetrical manner:

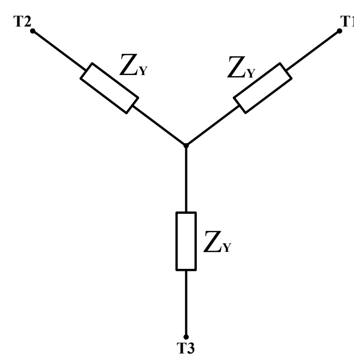
- Wye (Y)
- Delta (Δ)

Wye-connected Three-Phase Loads

A ***wye-connected***, three-phase load is constructed by connecting one end of the three individual loads to form a common (neutral) node.

The other end of the three individual loads provide the terminals for connection into a 3Φ system.

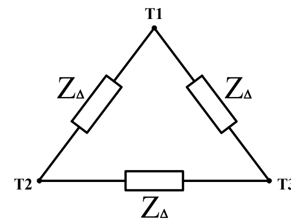
These terminals are sometimes defined as terminals **T1**, **T2**, and **T3**.



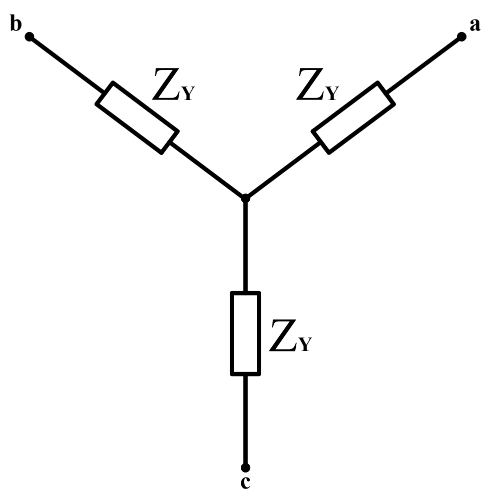
Delta-connected Three-Phase Loads

A ***delta-connected***, three-phase load is constructed by connecting one end of each individual load to only one of the other loads.

The three nodes that connect each pair of impedances provide the terminals for connection into a 3Φ system.



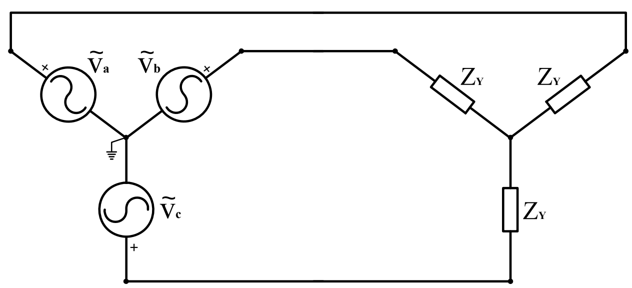
Wye-connected Three-Phase Loads



Wye-connected Loads in 3Φ Systems

The simple 3Φ system shown below consists of a ***wye-connected source*** and a ***wye-connected load***.

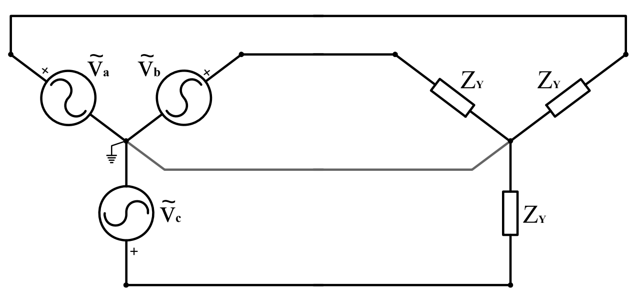
The neutral-point of the source is grounded to provide a zero-volt reference for the system.



Wye-connected Loads in 3Φ Systems

Three wires or "***lines***" are used to connect the source terminals to the terminals of the Y-connected load.

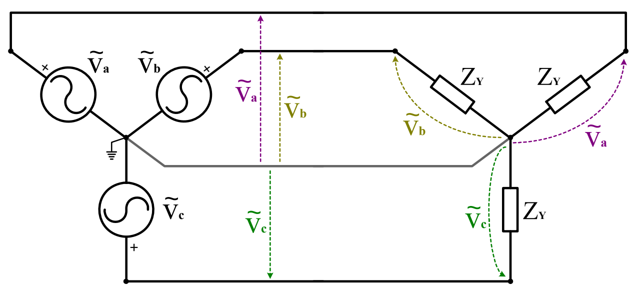
A "***neutral wire***" can be added to connect the grounded neutral-point of the source to the center-point of the load, holding both neutral points at a zero-volt potential.



Wye-connected Load Voltages

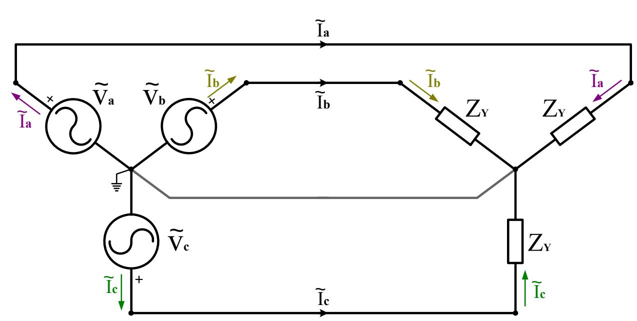
Note that the voltage potential present on each line (w.r.t. the neutral wire) is equal to the ***phase voltage*** of the source's phase to which the line is connected.

Thus, the four-wire connection results in the presence of a ***phase voltage*** across each phase of the load.



Wye-connected Load Currents

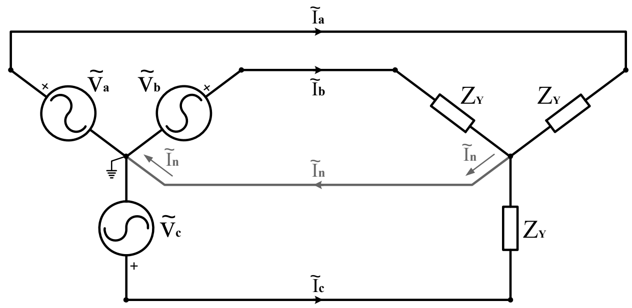
A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) can be defined that flow from each phase of the source, down the lines and into the individual phases of the load.



Wye-connected Load Currents

A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) can be defined that flow from each phase of the source, down the lines and into the individual phases of the load.

A ***neutral current*** ($\tilde{I}_n$) can also be defined that flows in the neutral wire from the load back to the source.

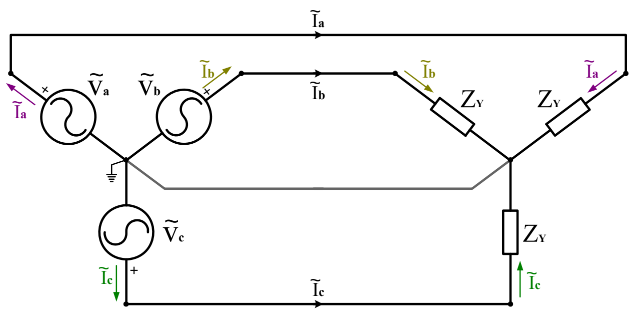


Wye-connected Load Currents

Note that the ***line currents*** may also be referred to as

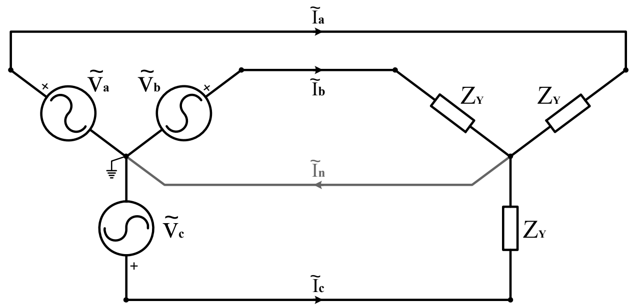
- ***phase currents of the source***, or
- ***phase currents of the load***

because they flow through the individual phases of both the source and the load.



Wye-connected Load Currents

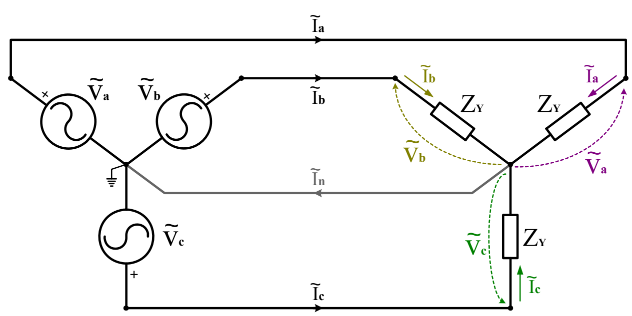
If the source voltages and load impedances are all known, then the ***line currents*** and the ***neutral current*** can all be determined using basic circuit theory.



Wye-connected Load Currents

Since the phase voltages of the load and source are equal, the line currents can each be solved independently by applying Ohm's Law at each load.

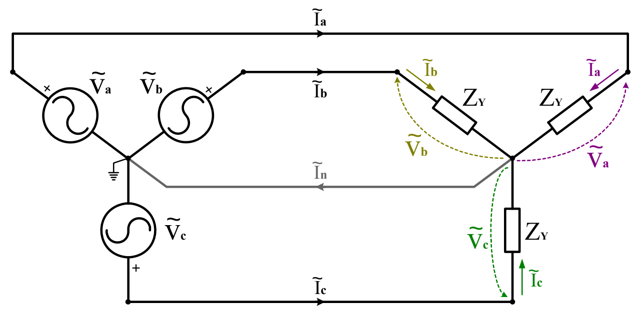
$$\tilde{I}_a = \frac{\tilde{V}_a}{Z_Y} \qquad \tilde{I}_b = \frac{\tilde{V}_b}{Z_Y} \qquad \tilde{I}_c = \frac{\tilde{V}_c}{Z_Y}$$



Wye-connected Load Currents

Furthermore, if the source voltages are balanced and the load impedances are all equal, then the line currents will also be balanced.

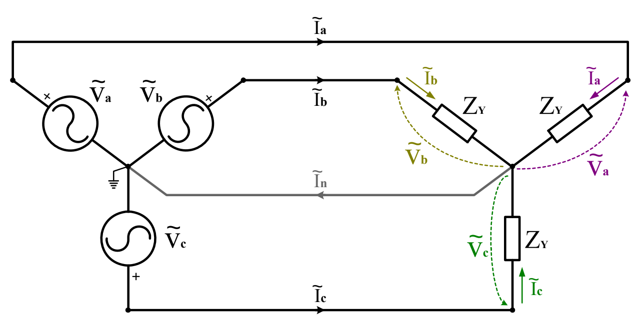
$$\tilde{I}_a = I\angle\delta \qquad \tilde{I}_b = I\angle\delta - 120° \qquad \tilde{I}_c = I\angle\delta - 240°$$



Complex Power in 3Φ Systems

The total complex power produced or consumed by a 3Φ source or load is equal to the sum of the complex powers produced or consumed by each of the source's or load's three individual phases.

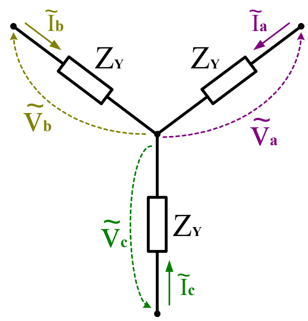
$$S_{3\Phi} = S_a + S_b + S_c$$



Complex Power in Y-connected Loads

In the case of a 3Φ, Y-connected load, the complex powers consumed by each of the load's three individual phases are:

$$S_a = \tilde{V}_a \cdot \tilde{I}_a^* \qquad S_b = \tilde{V}_b \cdot \tilde{I}_b^* \qquad S_c = \tilde{V}_c \cdot \tilde{I}_c^*$$



Complex Power in Y-connected Loads

If the system is balanced, with voltages and currents:

$$\tilde{V}_a = V\angle\phi \qquad \tilde{I}_a = I\angle\delta$$

$$\tilde{V}_b = V\angle\phi - 120° \qquad \tilde{I}_b = I\angle\delta - 120°$$

$$\tilde{V}_c = V\angle\phi - 240° \qquad \tilde{I}_c = I\angle\delta - 240°$$

then:

$$S_a = \tilde{V}_a \cdot \tilde{I}_a^* = [V\angle\phi]\cdot[I\angle-(\delta)] = V\cdot I\angle\phi-\delta$$

$$S_b = \tilde{V}_b \cdot \tilde{I}_b^* = [V\angle\phi-120°]\cdot[I\angle-(\delta-120°)] = V\cdot I\angle\phi-\delta$$

$$S_c = \tilde{V}_c \cdot \tilde{I}_c^* = [V\angle\phi-240°]\cdot[I\angle-(\delta-240°)] = V\cdot I\angle\phi-\delta$$

Complex Power in Y-connected Loads

If the system is balanced, with voltages and currents:

$$\tilde{V}_a = V\angle\phi \qquad \tilde{I}_a = I\angle\delta$$
$$\tilde{V}_b = V\angle\phi - 120° \qquad \tilde{I}_b = I\angle\delta - 120°$$
$$\tilde{V}_c = V\angle\phi - 240° \qquad \tilde{I}_c = I\angle\delta - 240°$$

then:

$$S_a = \tilde{V}_a \cdot \tilde{I}_a^* = V \cdot I\angle\phi - \delta$$
$$S_b = \tilde{V}_b \cdot \tilde{I}_b^* = V \cdot I\angle\phi - \delta$$
$$S_c = \tilde{V}_c \cdot \tilde{I}_c^* = V \cdot I\angle\phi - \delta$$

all three phases will consume equal complex powers.

Complex Power in Y-connected Loads

Thus, the total complex power consumed by a balanced, 3Φ, Y-connected load will be equal to **3x** the power consumed by any individual phase:

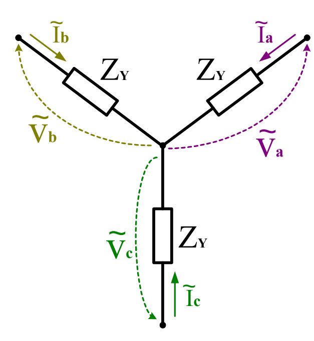
$$S_{3\Phi} = S_a + S_b + S_c = 3 \cdot S_{1\Phi}$$

allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_a \cdot \tilde{I}_a^* = 3 \cdot V \cdot I\angle\phi - \delta$$

where:

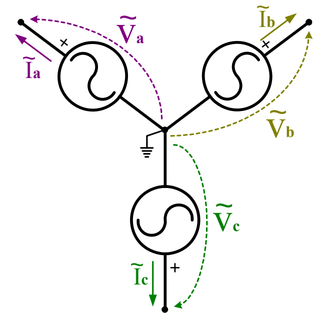
$$\tilde{V}_a = V\angle\phi$$
$$\tilde{I}_a = I\angle\delta$$



Complex Power in Y-connected Sources

Additionally, the total complex power produced by a balanced, 3Φ, Y-connected source will be equal to 3x the power produced by any individual phase:

$$S_{3\Phi} = S_a + S_b + S_c = 3 \cdot S_{1\Phi}$$



allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_a \cdot \tilde{I}_a^* = 3 \cdot V \cdot I \angle \phi - \delta$$

where:

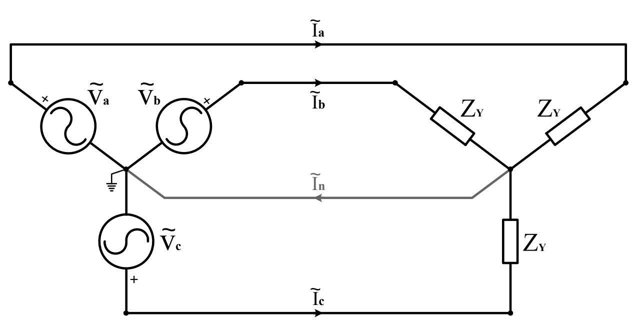
$$\tilde{V}_a = V \angle \phi$$

$$\tilde{I}_a = I \angle \delta$$

Neutral Current in 3Φ Systems

The neutral current $\tilde{I}_n$ can be determined by solving the node equation:

$$\tilde{I}_n = \tilde{I}_a + \tilde{I}_b + \tilde{I}_c$$

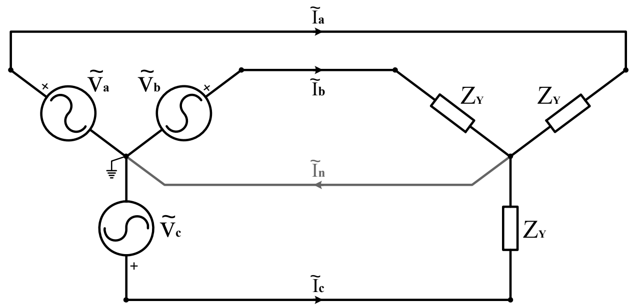


Neutral Current in 3Φ Systems

In a balanced system, the neutral current will be:

$$\tilde{I}_n = \tilde{I}_a + \tilde{I}_b + \tilde{I}_c = I\angle\delta + I\angle(\delta - 120°) + I\angle(\delta - 240°) = 0$$

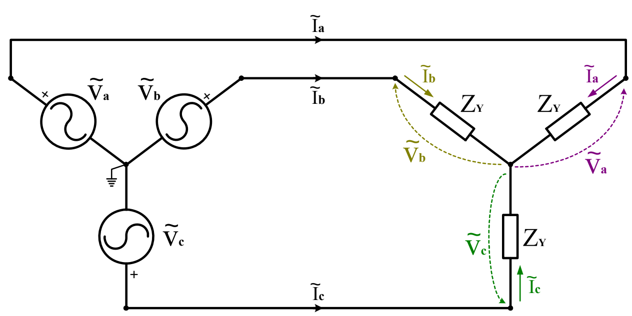
If the line currents are <u>balanced</u>, then they will sum to zero
→ **<u>no current will flow in the neutral wire</u>**.



Neutral Wires in 3Φ Systems

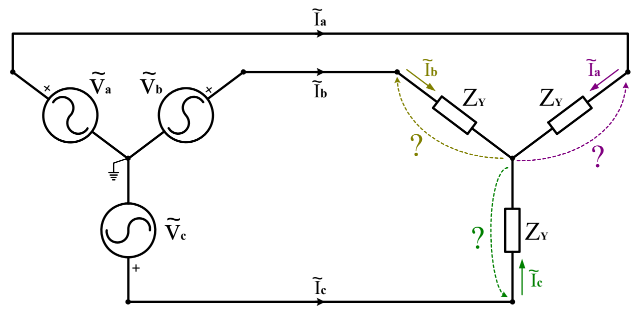
If the system is balanced such that the neutral current is zero,

Then removal of the neutral wire will theoretically have no effect on the operation of the system.



Neutral Wires in 3Φ Systems

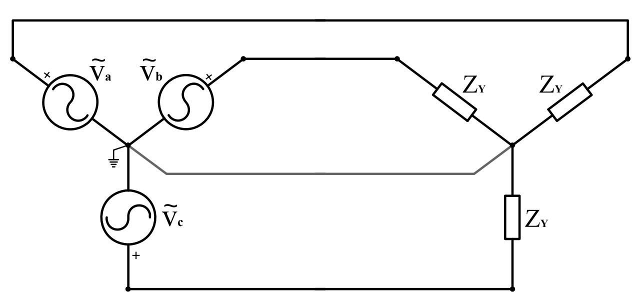
Although removal of the neutral wire will not affect the operation of a balanced system, <u>the importance of the neutral wire comes into play during times of unbalanced operation</u> during which it's existence can greatly affect the system.



3Φ Wye-connected Load Example

Given a 480V, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Y-connected, balanced load with individual per-phase impedances:

$$Z_Y = 80 + j60\ \Omega,$$



3Φ Wye-connected Load Example

Given a 480V, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Y-connected, balanced load with individual per-phase impedances:

$$Z_Y = 80 + j60\ \Omega,$$

Determine:

a) all of the phase and line voltages in the system,

b) all of the line currents in the system, and

c) the total complex power provided by the source to the Y- load.

Note – choose the angle of the phase voltage $\tilde{V}_a$ to be the 0° reference angle.

3Φ Wye-connected Load Example

Since the source is a Y-connected, positive-sequence, balanced source, the phase and line voltages will adhere to the following relationships:

Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

The values of $\boldsymbol{V}$ and $\boldsymbol{\phi}$ can be determined from the information provided in the problem statement.

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

Standard: if a single voltage magnitude is specified for a 3Φ source, then the value specified is the source's line-voltage magnitude.

Thus, given a balanced 480V source, the line and phase voltage magnitudes can all be specified as:

$$V_{line} = \sqrt{3}\cdot V = 480\,volts \quad \rightarrow \quad V_{phase} = V = \frac{480}{\sqrt{3}} = 277\,volts$$

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle\phi$	$\tilde{V}_{ab} = 480\angle\phi + 30°$
$\tilde{V}_b = 277\angle\phi - 120°$	$\tilde{V}_{bc} = 480\angle\phi - 90°$
$\tilde{V}_c = 277\angle\phi - 240°$	$\tilde{V}_{ca} = 480\angle\phi - 210°$

Standard: if a single voltage magnitude is specified for a 3Φ source, then the value specified is the source's line-voltage magnitude.

Note – if the source is Y-connected with an accessible neutral point, then the line and phase voltage magnitudes are often specified for convenience.

I.e. – 480/277V

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle\phi$	$\tilde{V}_{ab} = 480\angle\phi + 30°$
$\tilde{V}_b = 277\angle\phi - 120°$	$\tilde{V}_{bc} = 480\angle\phi - 90°$
$\tilde{V}_c = 277\angle\phi - 240°$	$\tilde{V}_{ca} = 480\angle\phi - 210°$

As with any steady-state AC circuit solution, the first phase angle in a 3Φ circuit may be chosen arbitrarily, after which all other phase angles (voltage and current) must be calculated based to the initial choice.

For convenience, the first angle is often chosen to be 0°.

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle\phi$	$\tilde{V}_{ab} = 480\angle\phi + 30°$
$\tilde{V}_b = 277\angle\phi - 120°$	$\tilde{V}_{bc} = 480\angle\phi - 90°$
$\tilde{V}_c = 277\angle\phi - 240°$	$\tilde{V}_{ca} = 480\angle\phi - 210°$

In this example, the problem statement instructed that an initial angle of 0° was to be chosen for the phase voltage $\tilde{V}_a$.

Thus, as defined in the relationships shown above:

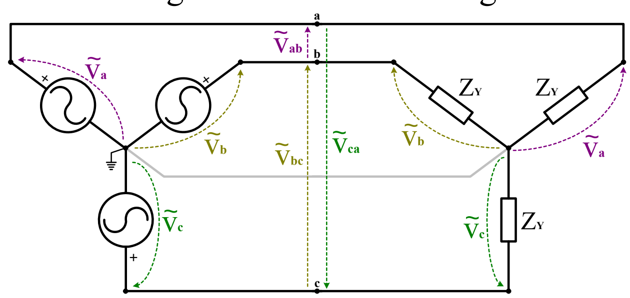
$$\phi = 0°,$$

to which all of the other angles can be referenced.

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

The phase and line voltages are shown in the figure below:



3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Now that all of the voltages have been specified in the system, the next step is to solve for all of the line currents that will flow in the 3Φ system from the source to the load.

3Φ Wye-connected Load Example

<u>Phase Voltages</u>	<u>Line Voltages</u>
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

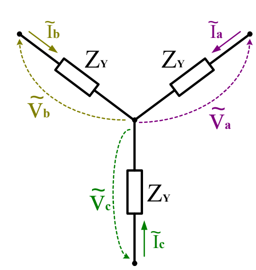
Since both the source and the load are both balanced, the resultant line currents will also be balanced.

Because of this, the complete set of line currents may be determined by first solving for one of the currents and then utilizing the balanced relationship in order to specify the remaining currents.

3Φ Wye-connected Load Example

<u>Phase Voltages</u>	<u>Line Voltages</u>
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Applying Ohm's Law to "***phase a***" of the load results the line current:



$$\tilde{I}_a = \frac{\tilde{V}_a}{Z_Y} = \frac{277\angle 0°}{80 + j60} = 2.77\angle -36.9°$$

from which the remaining line currents can be solved.

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Given: $\tilde{I}_a = 2.77\angle -36.9°$ → $I = 2.77$ $\delta = -36.9°$

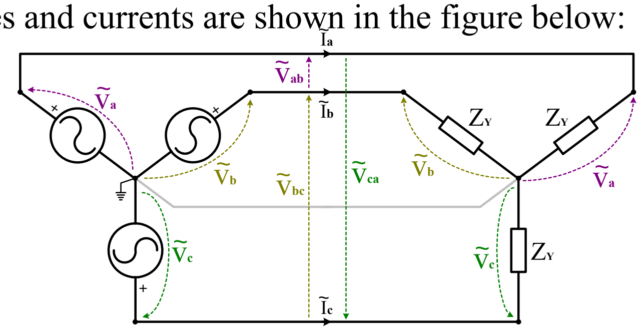
The remaining line currents can be determined from:

Balanced Relationships		Line Currents
$\tilde{I}_a = I\angle\delta$		$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{I}_b = I\angle\delta - 120°$	→	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{I}_c = I\angle\delta - 240°$		$\tilde{I}_c = 2.77\angle -276.9°$

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

The voltages and currents are shown in the figure below:



3Φ Wye-connected Load Example

Phase Voltages	Line Voltages	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

Now that all of the voltages and currents have been specified in the system, the next step is to solve for the total complex power that will be provided from the 3Φ source to the 3Φ load.

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

Since the total complex power produced/consumed in a balanced, 3Φ system is equal to 3x the complex power produced/consumed in a any individual phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_a \cdot \tilde{I}_a^* = 3 \cdot [277\angle 0°] \cdot [2.77\angle -(-36.9°)]$$
$$= 3 \cdot [614.4 + j460.8] = 1843.2 + j1382.4$$

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

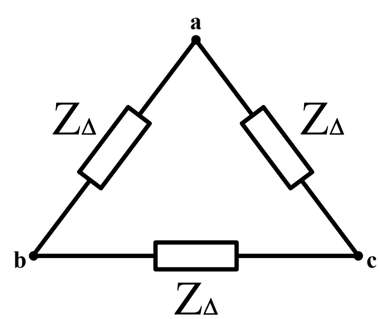
If desired, the complex power result:

$$S_{3\Phi} = 1843.2 + j1382.4$$

can be broken down into its real and reactive power components:

$$P_{3\Phi} = 1843.2\,Watts \qquad Q_{3\Phi} = 1382.4\,Vars$$

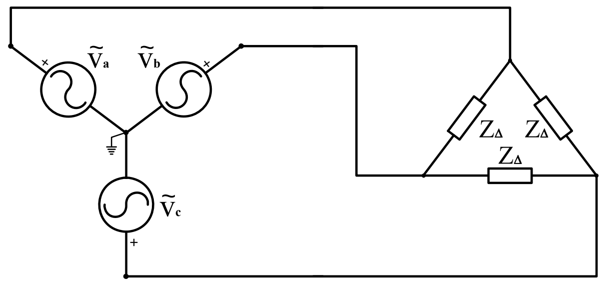
Delta-connected Three-Phase Loads



Delta-connected Loads in 3Φ Systems

The simple 3Φ system shown below consists of a ***wye-connected source*** and a ***delta-connected load***.

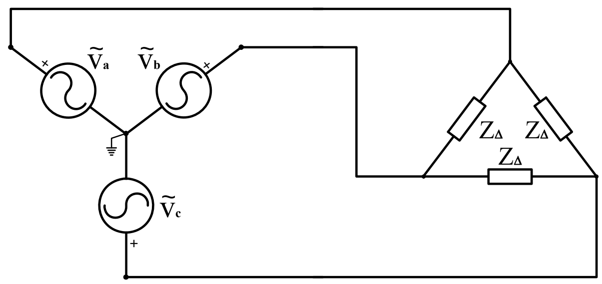
The neutral-point of the source is still grounded to provide a zero-volt reference for the system.



Delta-connected Loads in 3Φ Systems

Three "***lines***" are also used to connect the source terminals to the terminals of the Δ-connected load.

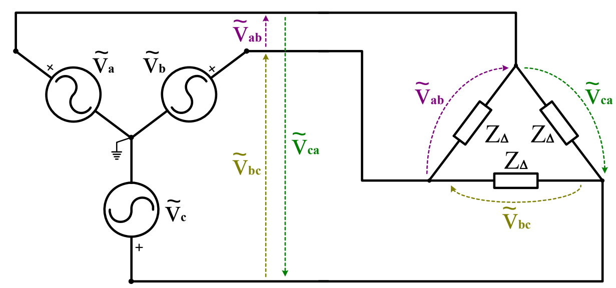
Note that <u>no neutral wire can be connected to the load</u> because the Δ-connected load has no central node to which the wire can be symmetrically connected.



Delta-connected Load Voltages

The voltage potential between each pair of lines is equal to the ***line voltage*** of the source.

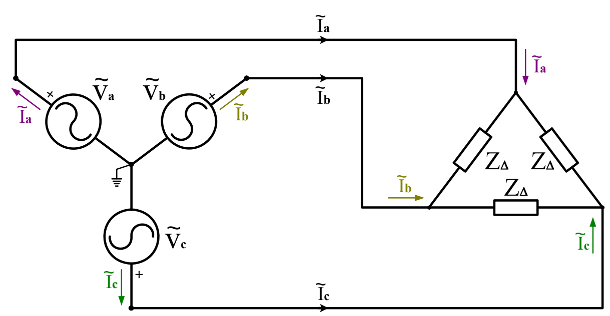
Since each phase of the Δ-connected load connects across a pair of lines, the three-wire connection provides a ***line voltage*** across each phase of the load.



Delta-connected Load Currents

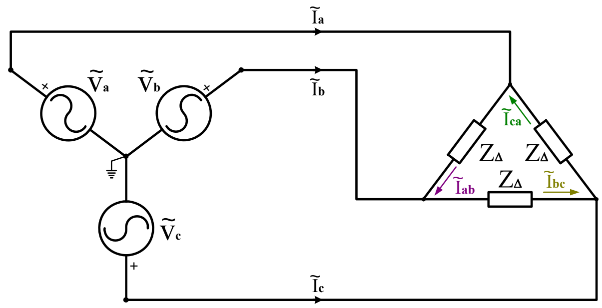
A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) was defined to flow in the lines from the source to the load.

Although the line currents flow through each phase of the source, they do **not** flow through the individual phases of the Δ-load.



Delta-connected Load Currents

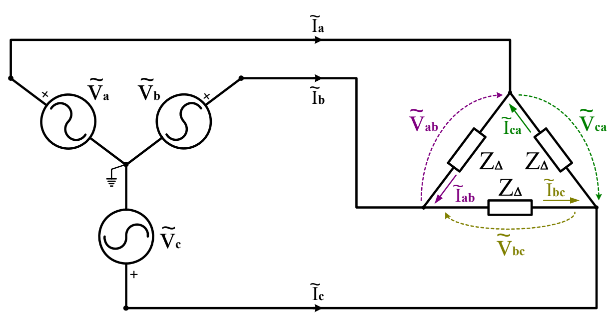
In order to fully characterize the Δ-connected load's operation, a set of ***phase currents*** ($\tilde{I}_{ab}$, $\tilde{I}_{bc}$ and $\tilde{I}_{ca}$) that flow through the individual phases of the load must also be defined.



Delta-connected Load Currents

If the line voltages and load impedances are known, then the ***phase currents of the load*** can each be solved independently by applying Ohm's Law for each phase:

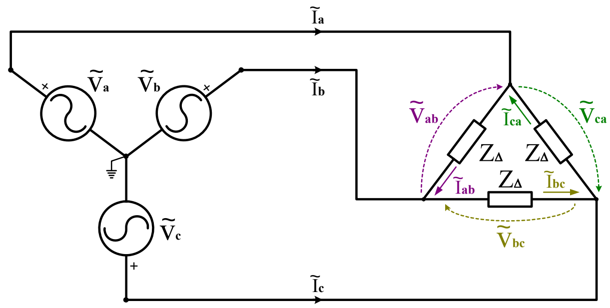
$$\tilde{I}_{ab} = \frac{\tilde{V}_{ab}}{Z_\Delta} \qquad \tilde{I}_{bc} = \frac{\tilde{V}_{bc}}{Z_\Delta} \qquad \tilde{I}_{ca} = \frac{\tilde{V}_{ca}}{Z_\Delta}$$



Delta-connected Load Currents

Furthermore, if the source voltages are balanced and the load impedances are all equal, then the phase currents of the load will also be balanced.

$$\tilde{I}_{ab} = I_{\Delta}\angle\beta \qquad \tilde{I}_{bc} = I_{\Delta}\angle\beta - 120° \qquad \tilde{I}_{ca} = I_{\Delta}\angle\beta - 240°$$



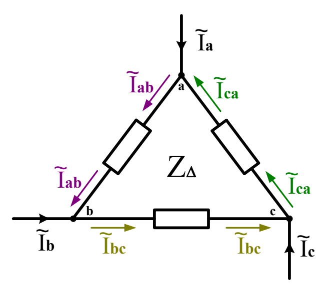
Delta-connected Load Currents

Once the phase currents of the load have been determined, the ***line currents*** flowing into the load may also be determined by solving a node equation for each of the three connection points to the load.

Node a: $\tilde{I}_a = \tilde{I}_{ab} - \tilde{I}_{ca}$

Node b: $\tilde{I}_b = \tilde{I}_{bc} - \tilde{I}_{ab}$

Node c: $\tilde{I}_c = \tilde{I}_{ca} - \tilde{I}_{bc}$



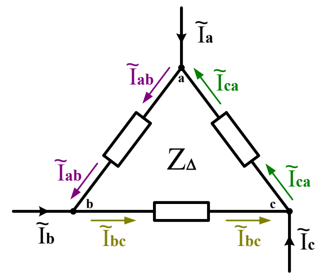
Delta-connected Load Currents

Given the balanced set of phase currents:

$$\tilde{I}_{ab} = I_\Delta \angle \beta \qquad \tilde{I}_{bc} = I_\Delta \angle \beta - 120° \qquad \tilde{I}_{ca} = I_\Delta \angle \beta - 240°$$

the line current $\tilde{I}_a$ can be determined as follows:

$$\begin{aligned}\tilde{I}_a &= \tilde{I}_{ab} - \tilde{I}_{ca} \\ &= I_\Delta \angle \beta - I_\Delta \angle (\beta - 240°) \\ &= \sqrt{3} \cdot I_\Delta \angle (\beta - 30°) \\ &= (\sqrt{3} \angle -30°) \cdot \tilde{I}_{ab}\end{aligned}$$



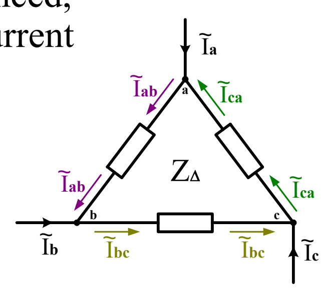
Delta-connected Load Currents

Since the phase currents are balanced:

$$\tilde{I}_{ab} = I_\Delta \angle \beta \qquad \tilde{I}_{bc} = I_\Delta \angle \beta - 120° \qquad \tilde{I}_{ca} = I_\Delta \angle \beta - 240°$$

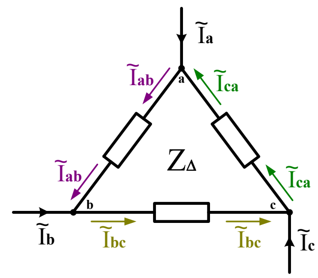
the resultant line currents will also be balanced, allowing a complete set of phase-to-line current relationships to be defined:

$$\begin{aligned}\tilde{I}_a &= (\sqrt{3} \angle -30°) \cdot \tilde{I}_{ab} \\ \tilde{I}_b &= (\sqrt{3} \angle -30°) \cdot \tilde{I}_{bc} \\ \tilde{I}_c &= (\sqrt{3} \angle -30°) \cdot \tilde{I}_{ca}\end{aligned}$$



Phase ↔ Line Current Relationship

The phase-to-line current relationships can then be used to specify a complete set of currents for a balanced, Δ-connected load as follows:

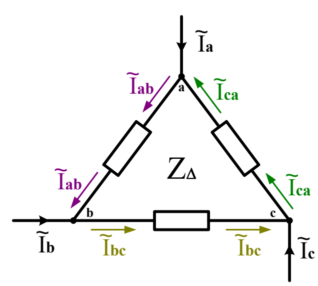


Phase Currents	**Line Currents**
$\tilde{I}_{ab} = I_\Delta \angle \beta$	$\tilde{I}_a = \sqrt{3} \cdot I_\Delta \angle \beta - 30°$
$\tilde{I}_{bc} = I_\Delta \angle \beta - 120°$	$\tilde{I}_b = \sqrt{3} \cdot I_\Delta \angle \beta - 150°$
$\tilde{I}_{ca} = I_\Delta \angle \beta - 240°$	$\tilde{I}_c = \sqrt{3} \cdot I_\Delta \angle \beta - 270°$

Phase ↔ Line Current Relationship

Note – to correspond with the line-currents defined for the Y-connected load, the phase and line current expressions can be rewritten such that:

$$I = \sqrt{3} \cdot I_\Delta \qquad \delta = \beta - 30°$$

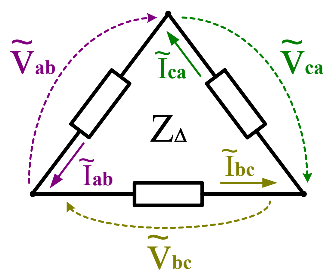


Line Currents	**Phase Currents**
$\tilde{I}_a = I \angle \delta$	$\tilde{I}_{ab} = \frac{I}{\sqrt{3}} \angle \delta + 30°$
$\tilde{I}_b = I \angle \delta - 120°$	$\tilde{I}_{bc} = \frac{I}{\sqrt{3}} \angle \delta - 90°$
$\tilde{I}_c = I \angle \delta - 240°$	$\tilde{I}_{ca} = \frac{I}{\sqrt{3}} \angle \delta - 210°$

Complex Power in Δ-connected Loads

In the case of a 3Φ, Δ-connected load, the complex power consumed by each of the load's three individual phases are:

$$S_{ab} = \tilde{V}_{ab} \cdot \tilde{I}_{ab}^* \qquad S_{bc} = \tilde{V}_{bc} \cdot \tilde{I}_{bc}^* \qquad S_{ca} = \tilde{V}_{ca} \cdot \tilde{I}_{ca}^*$$



Complex Power in Δ-connected Loads

If the system is balanced, with voltages and currents defined as:

$$\tilde{V}_{ab} = \sqrt{3} \cdot V\angle\phi + 30° \qquad \tilde{I}_{ab} = \frac{I}{\sqrt{3}}\angle\delta + 30°$$

$$\tilde{V}_{bc} = \sqrt{3} \cdot V\angle\phi - 90° \qquad \tilde{I}_{bc} = \frac{I}{\sqrt{3}}\angle\delta - 90°$$

$$\tilde{V}_{ca} = \sqrt{3} \cdot V\angle\phi - 210° \qquad \tilde{I}_{ca} = \frac{I}{\sqrt{3}}\angle\delta - 210°$$

then:

$$S_{ab} = \tilde{V}_{ab} \cdot \tilde{I}_{ab}^* = \left[\sqrt{3} \cdot V\angle\phi + 30°\right] \cdot \left[\frac{I}{\sqrt{3}}\angle -(\delta + 30°)\right] = V \cdot I\angle\phi - \delta$$

$$S_{bc} = \tilde{V}_{bc} \cdot \tilde{I}_{bc}^* = \left[\sqrt{3} \cdot V\angle\phi - 90°\right] \cdot \left[\frac{I}{\sqrt{3}}\angle -(\delta - 90°)\right] = V \cdot I\angle\phi - \delta$$

$$S_{ca} = \tilde{V}_{ca} \cdot \tilde{I}_{ca}^* = \left[\sqrt{3} \cdot V\angle\phi - 210°\right] \cdot \left[\frac{I}{\sqrt{3}}\angle -(\delta - 210°)\right] = V \cdot I\angle\phi - \delta$$

Complex Power in Δ-connected Loads

If the system is balanced, with voltages and currents defined as:

$$\tilde{V}_{ab} = \sqrt{3} \cdot V \angle \phi + 30° \qquad \tilde{I}_{ab} = \frac{I}{\sqrt{3}} \angle \delta + 30°$$

$$\tilde{V}_{bc} = \sqrt{3} \cdot V \angle \phi - 90° \qquad \tilde{I}_{bc} = \frac{I}{\sqrt{3}} \angle \delta - 90°$$

$$\tilde{V}_{ca} = \sqrt{3} \cdot V \angle \phi - 210° \qquad \tilde{I}_{ca} = \frac{I}{\sqrt{3}} \angle \delta - 210°$$

then:

$$S_{ab} = \tilde{V}_{ab} \cdot \tilde{I}_{ab}^* = V \cdot I \angle \phi - \delta$$

$$S_{bc} = \tilde{V}_{bc} \cdot \tilde{I}_{bc}^* = V \cdot I \angle \phi - \delta$$

$$S_{ca} = \tilde{V}_{ca} \cdot \tilde{I}_{ca}^* = V \cdot I \angle \phi - \delta$$

all three phases will consume equal complex power.

Complex Power in Δ-connected Loads

Thus, the total complex power consumed by a balanced, 3Φ, Y-connected load will be equal to **3x** the power consumed by any individual phase:

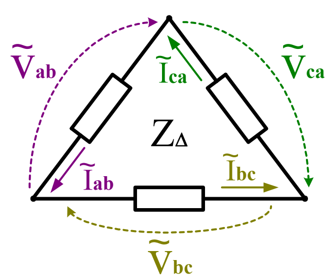
$$S_{3\Phi} = S_{ab} + S_{bc} + S_{ca} = 3 \cdot S_{1\Phi}$$

allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_{ab} \cdot \tilde{I}_{ab}^* = 3 \cdot V \cdot I \angle \phi - \delta$$

where: $\tilde{V}_{ab} = \sqrt{3} \cdot V \angle \phi + 30°$

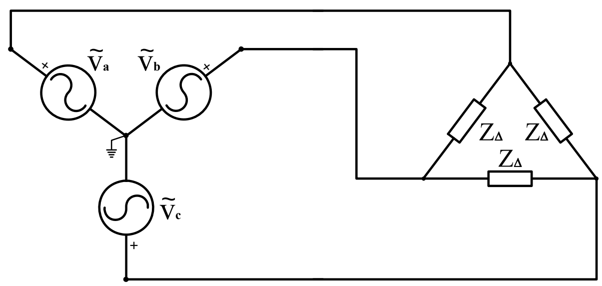
$\tilde{I}_{ab} = \frac{I}{\sqrt{3}} \angle \delta + 30°$



3Φ Delta-connected Load Example

Given a 480V, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Δ-connected, balanced load with individual phase impedances

$$Z_\Delta = 80 + j60\ \Omega,$$



3Φ Delta-connected Load Example

Given a 480V, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Δ-connected, balanced load with individual phase impedances

$$Z_\Delta = 80 + j60\ \Omega,$$

Determine:

a) all of the phase and line voltages in the system,

b) all of the phase and line currents in the system, and

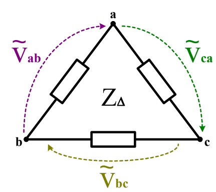
c) the total complex power provided by the source to the Δ-connected load.

Note – choose the angle of the phase voltage $\tilde{V}_a$ to be the 0° reference angle for the system.

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0^\circ$	$\tilde{V}_{ab} = 480\angle +30^\circ$
$\tilde{V}_b = 277\angle -120^\circ$	$\tilde{V}_{bc} = 480\angle -90^\circ$
$\tilde{V}_c = 277\angle -240^\circ$	$\tilde{V}_{ca} = 480\angle -210^\circ$

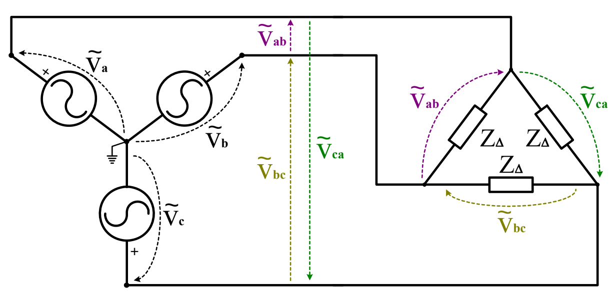
Since the source defined in this example is the same as that in the Y-connected load example, the phase and line voltages shown above are provided without the logic required to obtain those values.



3Φ Delta-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0^\circ$	$\tilde{V}_{ab} = 480\angle +30^\circ$
$\tilde{V}_b = 277\angle -120^\circ$	$\tilde{V}_{bc} = 480\angle -90^\circ$
$\tilde{V}_c = 277\angle -240^\circ$	$\tilde{V}_{ca} = 480\angle -210^\circ$

The phase and line voltages are shown in the figure below:



3Φ Delta-connected Load Example

Line Voltages

$$\tilde{V}_{ab} = 480\angle +30^\circ$$

$$\tilde{V}_{bc} = 480\angle -90^\circ$$

$$\tilde{V}_{ca} = 480\angle -210^\circ$$

Note that although the phase and line voltages both exist at the Y-connected source, only the line voltages appear at the Δ-connected load due to the absence of a neutral point.

3Φ Delta-connected Load Example

Line Voltages

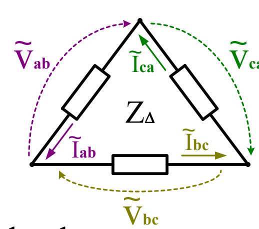
$$\tilde{V}_{ab} = 480\angle +30^\circ$$

$$\tilde{V}_{bc} = 480\angle -90^\circ$$

$$\tilde{V}_{ca} = 480\angle -210^\circ$$

By applying Ohm's Law to the load connected across nodes **a** and **b**, the phase current can be determined:

$$\tilde{I}_{ab} = \frac{\tilde{V}_{ab}}{Z_\Delta} = \frac{480\angle 30^\circ}{80 + j60} = 4.8\angle -6.9^\circ$$



from which the remaining phase currents can then be solved.

3Φ Delta-connected Load Example

Line Voltages

$$\tilde{V}_{ab} = 480\angle +30°$$
$$\tilde{V}_{bc} = 480\angle -90°$$
$$\tilde{V}_{ca} = 480\angle -210°$$

Given: $\tilde{I}_{ab} = 4.8\angle -6.9°$ → $\frac{I}{\sqrt{3}} = 4.8 \quad \delta + 30° = -6.9°$

The remaining phase currents can be determined from:

Balanced Relationships

$$\tilde{I}_{ab} = \frac{I}{\sqrt{3}}\angle \delta + 30°$$
$$\tilde{I}_{bc} = \frac{I}{\sqrt{3}}\angle \delta - 90°$$
$$\tilde{I}_{ca} = \frac{I}{\sqrt{3}}\angle \delta - 210°$$

→

Phase Currents

$$\tilde{I}_{ab} = 4.8\angle -6.9°$$
$$\tilde{I}_{bc} = 4.8\angle -126.9°$$
$$\tilde{I}_{ca} = 4.8\angle -246.9°$$

3Φ Delta-connected Load Example

Line Voltages

$$\tilde{V}_{ab} = 480\angle +30°$$
$$\tilde{V}_{bc} = 480\angle -90°$$
$$\tilde{V}_{ca} = 480\angle -210°$$

Phase Currents

$$\tilde{I}_{ab} = 4.8\angle -6.9°$$
$$\tilde{I}_{bc} = 4.8\angle -126.9°$$
$$\tilde{I}_{ca} = 4.8\angle -246.9°$$

Additionally: $\frac{I}{\sqrt{3}} = 4.8 \quad \delta + 30° = -6.9°$ → $I = 8.31 \quad \delta = -36.9°$

The line currents can be determined from:

Balanced Relationships

$$\tilde{I}_{a} = I\angle \delta$$
$$\tilde{I}_{b} = I\angle \delta - 120°$$
$$\tilde{I}_{c} = I\angle \delta - 240°$$

→

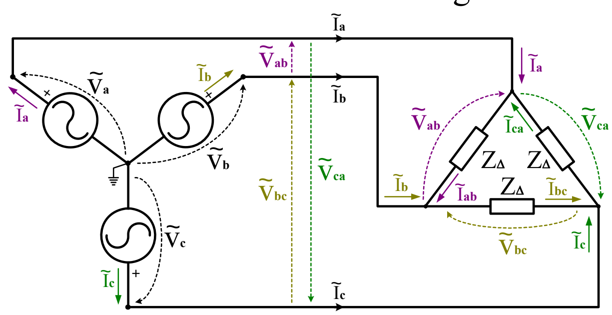
Line Currents

$$\tilde{I}_{a} = 8.31\angle -36.9°$$
$$\tilde{I}_{b} = 8.31\angle -156.9°$$
$$\tilde{I}_{c} = 8.31\angle -276.9°$$

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

The voltages and currents are shown in the figure below:



3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

Now that all of the voltages and currents have been specified in the system, the next step is to solve for the total complex power that will be provided by the 3Φ source to the 3Φ load.

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

Since the total complex power consumed by a balanced Δ-connected load is equal to 3x the complex power consumed by each individual phase of the load:

$$S_{3\Phi} = 3\cdot\tilde{V}_{ab}\cdot\tilde{I}_{ab}^{*} = 3\cdot[480\angle 30°]\cdot[4.8\angle -(-6.9°)]$$
$$= 3\cdot[1843.2 + j1382.4] = 5529.6 + j4147.2$$

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

If desired, the complex power result:

$$S_{3\Phi} = 5529.6 + j4147.2$$

can be broken down into its real and reactive power components:

$$P_{3\Phi} = 5529.6\,Watts \qquad Q_{3\Phi} = 4147.2\,Vars$$

Y ↔ Δ Load Comparison

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Based on the results of the previous examples:

If a balanced 3Φ source is supplying both a Y-connected load and a Δ-connected load, each having the same per-phase impedances:

$$Z_\Delta = Z_Y$$

then the Δ-connected load will consume **3x** more power than the Y-connected load.

Y ↔ Δ Load Comparison

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

It can also be proven that:

If a balanced 3Φ source is supplying both a Y-connected load and a Δ-connected load, but the per-phase Δ-impedances are **3x** larger than the per-phase Y-impedances:

$$Z_\Delta = 3 \cdot Z_Y$$

then the Δ-connected load and the Y-connected load will consume the **same** amount of power.