Transformers

(Part A)

Ideal Transformers

ECET 3500 – *Survey of Electric Machines*

Introduction to Ideal Transformers

An **<u>ideal transformer</u>** is a two-port device that receives an AC voltage, $\tilde{E}_p$, at one magnitude and transforms it into a new AC voltage, $\tilde{E}_s$, at a different magnitude.

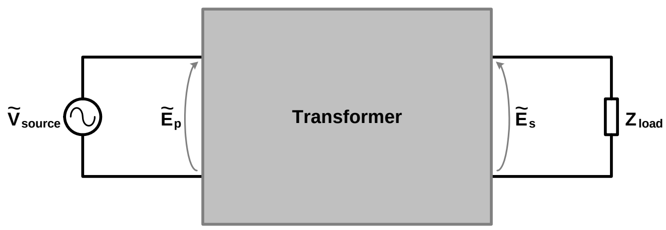
The port (side) to which the voltage is applied is called the "**primary**" side and the port (side) at which the transformed voltage appears is called the "**secondary**" side.

Ratio of Voltages

The **turns-ratio** of the transformer, ***a***, defines the ratio of the two voltage magnitudes:

$$a = \frac{E_p}{E_s}$$

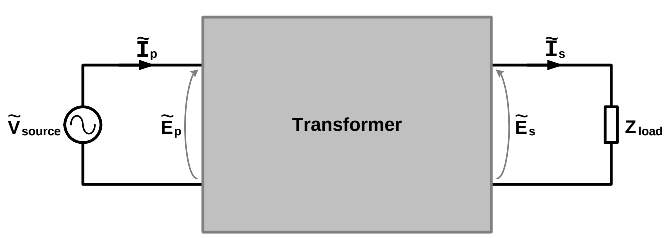
Note that an **ideal transformer** <u>does not change the phase angle</u> of the voltage; <u>it only transforms the magnitude</u>.



Ratio of Currents

Since P_{in} must equal P_{out} for an ideal (lossless) transformer, the magnitudes of the **primary** and **secondary currents** flowing into and out of the transformer must have an <u>inverse ratio</u> compared to the voltages:

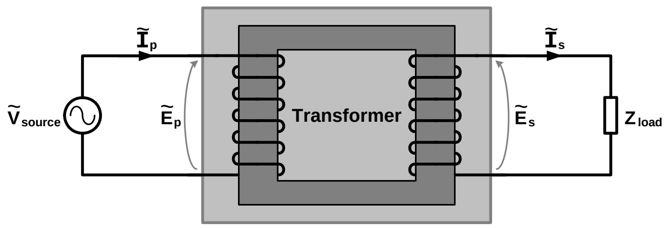
$$\frac{E_p}{E_s} = a \qquad \frac{I_p}{I_s} = \frac{1}{a}$$



Underlying Theory

An **ideal transformer's** theoretical operation is based on the electro-magnetic interactions that occur between two coils that are coupled together by a magnetic core.

Thus, the analysis of its theoretical operation will begin with the analysis of a simple, AC-supplied magnetic circuit.

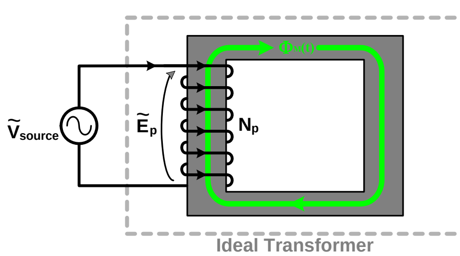


Simple AC-Supplied Magnetic Circuit

If an **AC source** is connected to a coil, a **magnetic flux**, Φ_M , will be created by that coil, the value of which may be determined from the following relationship :

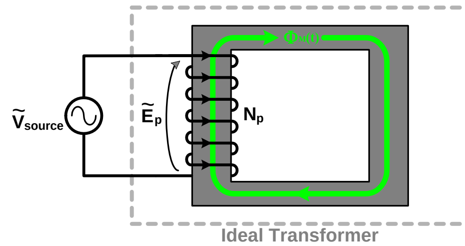
$$v(t) = \tilde{E}_P = N_p \cdot \frac{d\Phi_M(t)}{dt} \quad \text{(Faraday's Law)}$$

where N_p is the number of turns of the primary (sourced) coil.



Simple AC-Supplied Magnetic Circuit

Thus, given the voltage:



$$v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$$

the **magnetic flux** can be solved as:

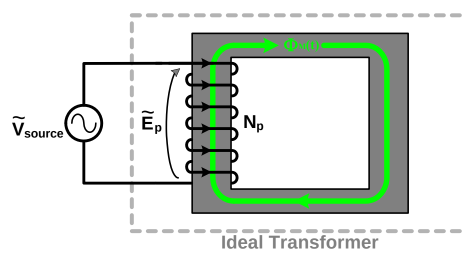
$$\Phi_M(t) = \frac{1}{N_p} \cdot \int v(t)dt$$

$$= \frac{1}{N_p} \cdot \int \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)dt$$

$$= \sqrt{2} \cdot \frac{V}{\omega \cdot N_p} \cdot \sin(\omega \cdot t) = \sqrt{2} \cdot \Phi \cdot \sin(\omega \cdot t)$$

Simple AC-Supplied Magnetic Circuit

The resultant **flux**:



$$\Phi_M(t) = \sqrt{2} \cdot \Phi \cdot \sin(\omega \cdot t)$$

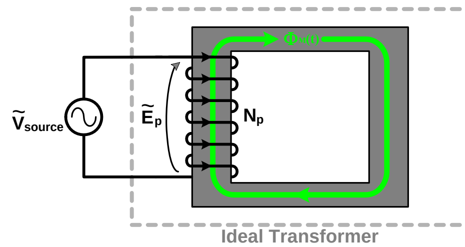
may be rewritten as:

$$\Phi_M(t) = \sqrt{2} \cdot \Phi \cdot \cos(\omega \cdot t - 90°)$$

since sine is equivalent to cosine that is phase-shifted by –90°.

Simple AC-Supplied Magnetic Circuit

Thus, the sinusoidally-supplied coil will induce a sinusoidally-varying **flux** that out of phase from the voltage by –90°:



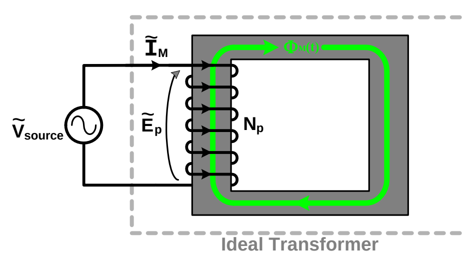
$$v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$$

$$\Downarrow$$

$$\Phi_M(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot N_p} \cdot \cos(\omega \cdot t - 90°)$$

Simple AC-Supplied Magnetic Circuit

If the core has <u>finite permeability</u>, μ, then a **magnetization current**, i_m, will be drawn into the coil from the source, as defined by the relationship:



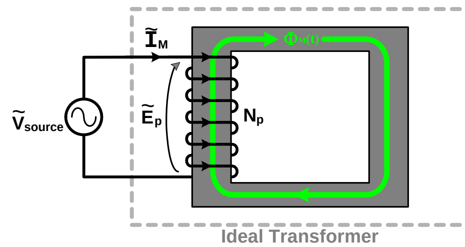
$$N_p \cdot i_m(t) = \Phi_M(t) \cdot \Re$$

where $\Re$ is the **reluctance** of the field path (core).

Reluctance of simple, uniform magnetic core → $\Re = \dfrac{l}{\mu \cdot A}$

Simple AC-Supplied Magnetic Circuit

Additionally, if the core material is **linear** (**μ_r = constant**), then the **magnetization current** will be <u>proportional to the flux</u>:

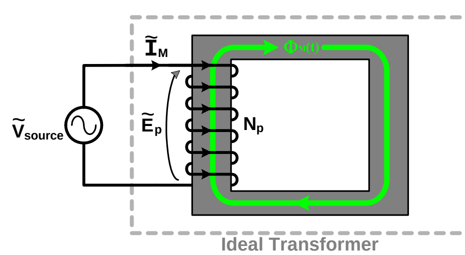


$$i_m(t) = \Phi_M(t) \cdot \frac{\Re}{N_p}$$

Note – $i_m(t)$ is **<u>not the primary current</u>** that was defined within the initial introduction to "ideal transformers". Instead, $i_m(t)$ is a non-ideal (loss) component that occurs within a transformer due to a finite core permeability.

Simple AC-Supplied Magnetic Circuit

If the solution for **flux**, Φ_M, is substituted into the current equation, then the **magnetization current**, i_m, can be expressed in terms of the original source voltage's parameters:



$$i_m(t) = \sqrt{2} \cdot \frac{V \cdot \Re}{\omega \cdot N_p^2} \cdot \cos(\omega \cdot t - 90°)$$

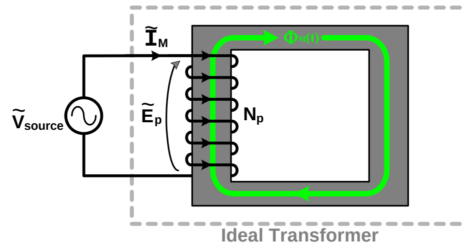
Simple AC-Supplied Magnetic Circuit

The resultant **magnetization current**:

$$i_m(t) = \sqrt{2} \cdot \frac{V \cdot \Re}{\omega \cdot N_p^{\,2}} \cdot \cos(\omega \cdot t - 90°)$$

can be rewritten as:

$$i_m(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot \frac{N_p^{\,2}}{\Re}} \cdot \cos(\omega \cdot t - 90°)$$



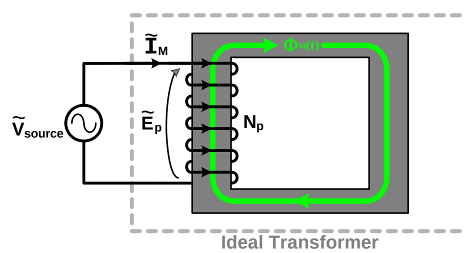
Simple AC-Supplied Magnetic Circuit

If we define a new variable, ***L***, such that:

$$L = \frac{N_p^{\,2}}{\Re}$$

we can substitute ***L*** into the expression for **current**:

$$i_m(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot L} \cdot \cos(\omega \cdot t - 90°)$$

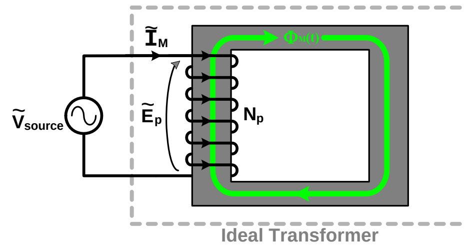


Simple AC-Supplied Magnetic Circuit

Looking at the overall response of an AC-supplied coil wrapped around a finite-permeability core;

$$i_m(t) = \sqrt{2} \cdot I_M \cdot \cos(\omega \cdot t - 90°)$$

the resultant **current** is out of phase with the supply voltage by –90°.



Additionally, the voltage and current magnitudes have the proportionality relationship:

$$I_M = \frac{V}{\omega \cdot L}$$

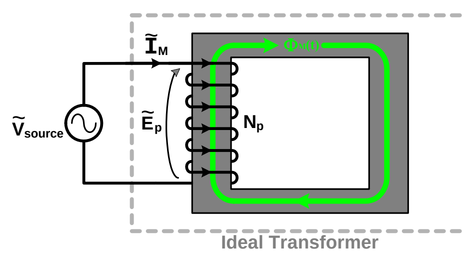
Simple AC-Supplied Magnetic Circuit

If the **magnetization current**:

$$i_m(t) = \frac{\sqrt{2} \cdot V}{\omega \cdot L} \cdot \cos(\omega \cdot t - 90°)$$

is compared to the **flux** formed by the coil:

$$\Phi_M(t) = \frac{\sqrt{2} \cdot V}{\omega \cdot N_p} \cdot \cos(\omega \cdot t - 90°)$$



Simple AC-Supplied Magnetic Circuit

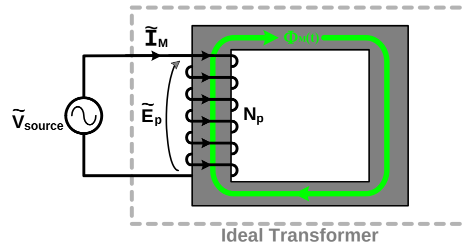
It can be determined that:

$$\frac{d\Phi_M(t)}{dt} = \frac{L}{N_p} \cdot \frac{di_m(t)}{dt}$$

This result may be substituted into the **Faraday's Law** equation to provide the relationship:

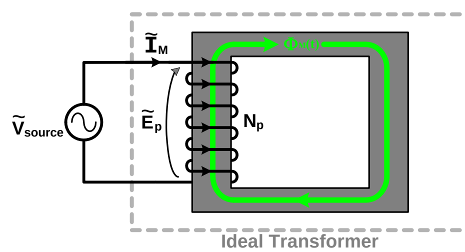
$$v(t) = N_p \cdot \frac{d\Phi_M(t)}{dt}$$

$$= L \cdot \frac{di_M(t)}{dt}$$



Simple AC-Supplied Magnetic Circuit

Thus, an AC-supplied coil that is wrapped around a finite-permeability magnetic core will function as an **inductor**, whose voltage-current relationship is defined by:



$$v(t) = L \cdot \frac{di_M(t)}{dt}$$

and whose **inductance** is a function of the number of turns in the coil, N_p, and the reluctance of the field-path, $\Re$:

$$L = \frac{N_p^{\,2}}{\Re}$$

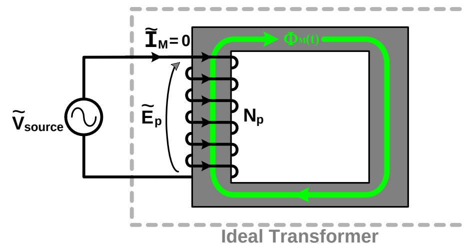
Simple AC-Supplied Magnetic Circuit

Note – if the core is assumed to be **infinitely-permeable** ($\boldsymbol{\mu \to \infty}$),

then the **reluctance** of the core will be zero ($\boldsymbol{\Re \to 0}$), $\Re = \frac{l}{\mu \cdot A}$

and the **inductance** will be infinite ($\boldsymbol{L \to \infty}$). $L = \frac{N_p^{\,2}}{\Re}$

Thus, the coil will draw zero **magnetizing current** ($\boldsymbol{i_m \to 0}$). $I_M = \frac{V}{\omega \cdot L}$

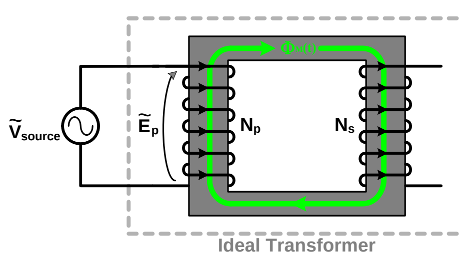


No **magnetization current** is required to create a flux within an infinitely-permeable magnetic core.

Mutually Linked Coils

Given the previously defined magnetic circuit…

What would happen if a **second coil** is coupled to (wrapped around) the magnetic core?

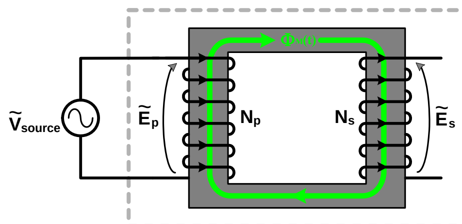


Mutually Linked Coils

If the magnetic core is assumed to be **ideal**, then the total flux created by the sourced coil will pass through the second coil.

Since a time-varying flux passes through the second coil, a **voltage** will be induced across that coil, also defined by:

$$\tilde{E}_s = N_s \cdot \frac{d\Phi_M(t)}{dt}$$



Ideal Transformer

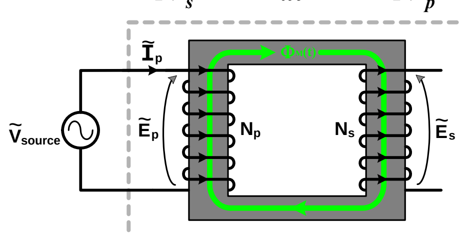
An ideal magnetic core is both lossless and infinitely-permeable.

Mutually Linked Coils

If the **total flux** passes through both coils, then the rate of change of the flux, $\frac{d\Phi(t)}{dt}$, passing through the coils must be the same.

The following relationship may be derived by solving for $\frac{d\Phi(t)}{dt}$ in both coils and equating the results:

$$\frac{\tilde{E}_s}{N_s} = \frac{d\Phi_M(t)}{dt} = \frac{\tilde{E}_p}{N_p}$$



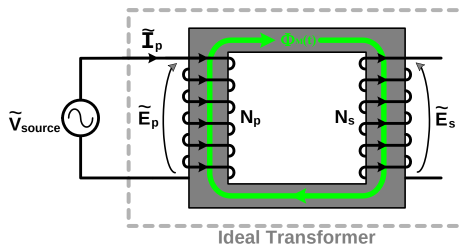
Ideal Transformer

Voltage Relationship

The relationship between the two coil voltages is typically expressed as a **ratio of the voltages**, which equals to the ratio of their respective number of turns.

(I.e. – the "**turns ratio**" of the transformer).

$$\frac{\tilde{E}_p}{N_p} = \frac{\tilde{E}_s}{N_s} \Rightarrow \boxed{\frac{\tilde{E}_p}{\tilde{E}_s} = \frac{N_p}{N_s}}$$

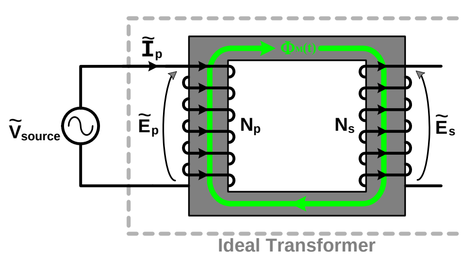


Turns-Ratio

The ratio relationship, referred to as the **<u>turns ratio</u>** (***a***):

$$a = \frac{\tilde{E}_p}{\tilde{E}_s} = \frac{N_p}{N_s}$$

defines the basic operation of an ideal transformer in terms of the primary and secondary voltages.

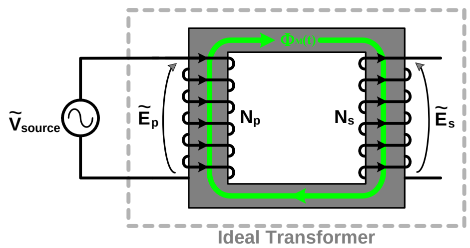


Polarity Relationship & Lenz's Law

Note – the **polarity** of the voltage induced across the second coil is based upon both the direction of the flux within the core and the direction that the coil is wrapped around the core.

The correct polarity relationship can be determined by applying **Lenz's Law**, which states:

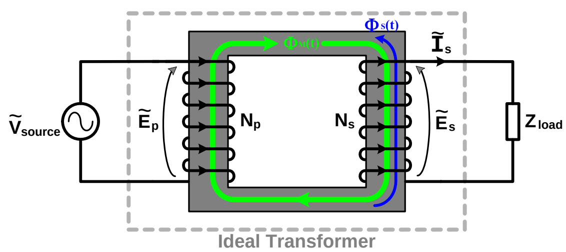
"Any induced effect will always oppose its source."



Determining the Polarity Relationship

If a load is connected to the second winding, then a **secondary current** $\tilde{I}_s$ will flow out of the secondary winding and through the load due to the induced voltage.

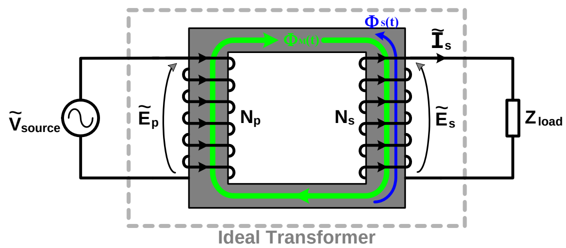
Based on Lenz's Law, the **polarity of the voltage** must be such that <u>the resultant coil-current will create a counter-flux</u> in the core that opposes the original flux.



Secondary Current Effects

But, the existence of a counter-flux produced by the current that is flowing in the second coil would tend to decrease the overall flux within the magnetic core, in-turn decreasing the total flux passing through the primary coil.

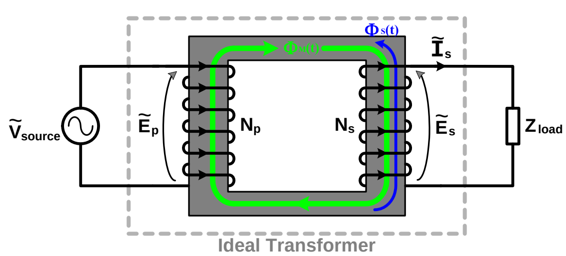
$$\Phi_{Net} = \Phi_M - \Phi_S$$



Secondary Current Effects

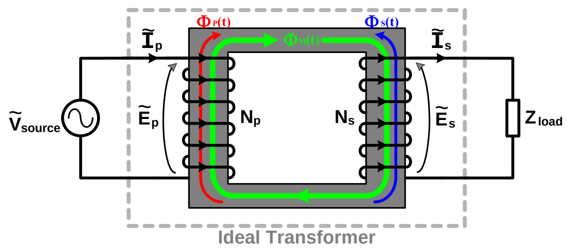
Assuming that the source is ideal, this presents a problem because Faraday's Law does not allow for a change in the flux passing through the primary coil unless the supply voltage itself changes accordingly.

$$\tilde{E}_P = N_P \cdot \frac{d\Phi_{Net}(t)}{dt}$$



Secondary Current Effects

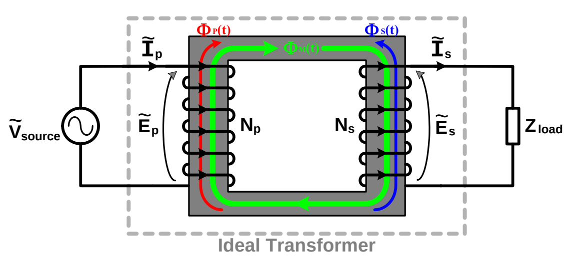
Thus, the existence of the secondary current's counter-flux, $\mathbf{\Phi_s}$, requires that a **primary current**, $\tilde{I}_p$, be drawn into the primary winding that will create an additional flux component, $\mathbf{\Phi_p}$, within the core that is equal in magnitude but opposite in direction compared to the secondary flux $\mathbf{\Phi_s}$.



Primary Current

Since the primary and secondary fluxes are equal in magnitude but opposite in direction, they will cancel, leaving the net flux in the core the same as defined by Faraday's Law applied to the primary winding:

$$\Phi_{Net} = \Phi_M - \Phi_S + \Phi_P = \Phi_M$$



Primary/Secondary Current Ratio

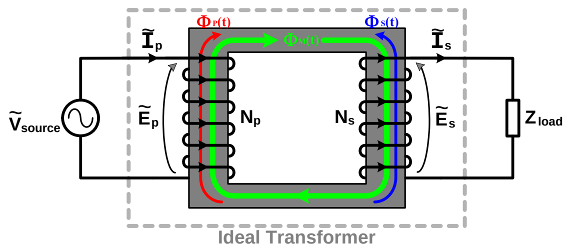
Based on the MMF relationship applied to both coils:

$$N \cdot i(t) = \Phi(t) \cdot \Re$$

the **ratio** of the **primary and secondary currents** must be:

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

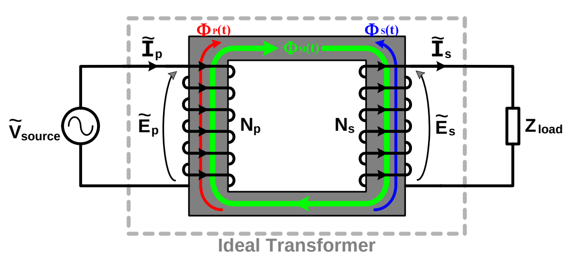
in order for their fluxes to cancel.



Overall Operation of Ideal Transformer

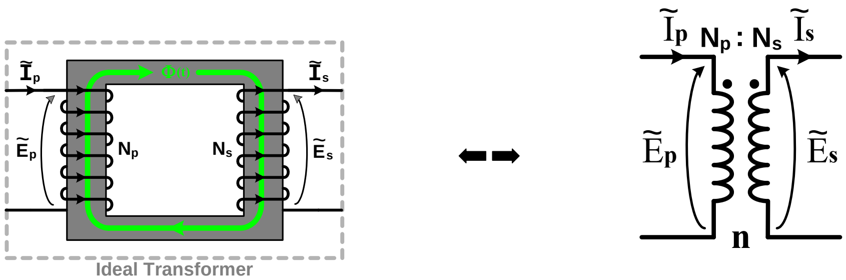
Thus, the overall operation of the **ideal transformer** that supplies a single load can be defined by the following set of equations:

turns ratio $a = \frac{N_p}{N_s}$ $\quad a = \frac{\tilde{E}_p}{\tilde{E}_s}$ $\quad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$ $\quad \tilde{I}_s = \frac{\tilde{E}_s}{Z_{load}}$



Ideal Transformer Equivalent Circuit

The following **equivalent circuit** will be used to represent an **ideal transformer**:



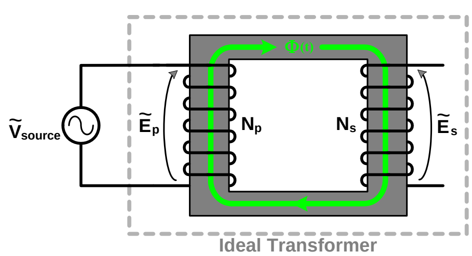
$$a = \frac{N_p}{N_s} \qquad \frac{\tilde{E}_p}{\tilde{E}_s} = a \qquad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a} \qquad a \equiv \textbf{turns ratio}$$

Ideal Transformer Definitions

Primary Winding ≡ the winding that creates the mutually-linked flux (I.e. – the sourced winding).

Secondary Winding ≡ the winding across which a voltage is induced (I.e. – the load winding).

Note – the primary & secondary winding designations can also be defined in terms of **power flow** direction (I.e. – **power into primary** and **power out of secondary**)

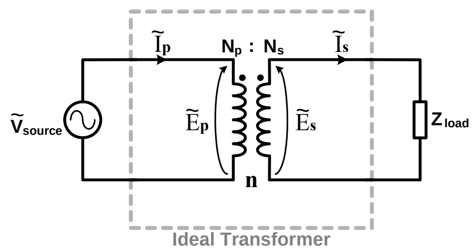


Ideal Transformer Definitions

High-Voltage Winding ≡ the winding with the larger voltage magnitude.
(I.e. – the coil with the larger number of turns)

Low-Voltage Winding ≡ the winding with the smaller voltage magnitude.
(I.e. – the coil with the smaller number of turns)

Note – the **high-voltage winding** will have the **larger number of turns** while the **low-voltage winding** will have the **smaller number of turns**.



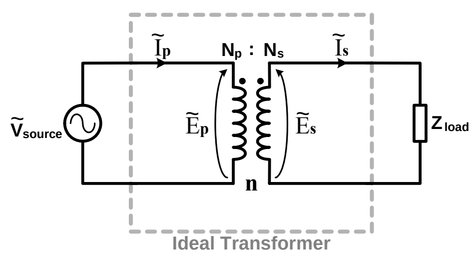
Ideal Transformer Definitions

Step-Up Transformer ≡ a transformer whose voltage increases from primary to secondary winding.

Step-Down Transformer ≡ a transformer whose voltage decreases from primary to secondary winding.

Notes: A **step-up** transformer's **turns ratio** will be **less than one** (***a*** **< 1**).

A **step-down** transformer's **turns ratio** will be **greater than one** (***a*** **> 1**).

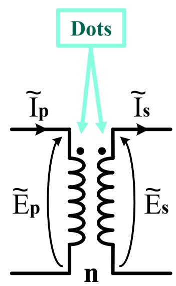


$$a = \frac{\tilde{E}_p}{\tilde{E}_s} = \frac{N_p}{N_s}$$

Polarity Relationship

The **polarity relationship** between the primary and secondary voltages depends on the direction that the coils are wrapped around the magnetic core.

The "**Dot Convention**" is often used to provide the polarity relationship for a specific transformer.

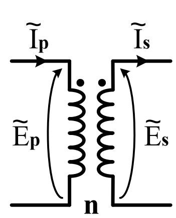


Equivalent Circuit "Dot Convention"

"**Dots**" are often included with the equivalent circuit to define the **polarity relationship** between the transformer windings.

1) An applied **primary voltage whose voltage-rise <u>points toward</u> the primary winding's dot** will induce a **secondary voltage whose voltage-rise <u>points toward</u> the secondary winding's dot**.

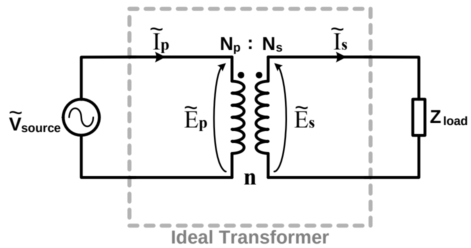
2) A **primary current will flow <u>into</u> the dot side of its winding** when a **secondary current flows <u>out of</u> the dot side of its winding**.



Turns-Ratio Consideration

It is important to note that the **turns ratio**, a, will change depending on which of the two windings are utilized as the primary winding.

$$a = \frac{N_p}{N_s} \qquad \frac{\tilde{E}_p}{\tilde{E}_s} = a \qquad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a} \qquad a = \textbf{turns ratio}$$



Either winding may be utilized as the primary winding. (I.e. – a transformer's operation does not have a required direction of power flow)

Turns-Ratio Consideration

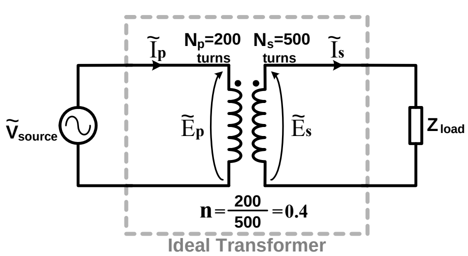
For example – Given a transformer with a 200-turn winding and a 500-turn winding:

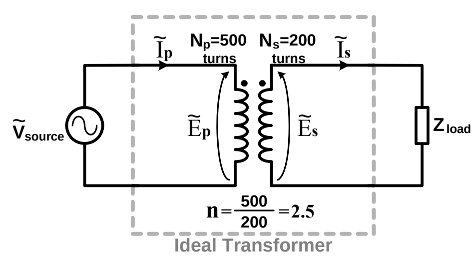
The transformer will have a **turns ratio $a = 0.4$** if the 200-turn winding is utilized as the primary, or

Step-Up Transformer

The transformer will have a **turns ratio $a = 2.5$** if the 500-turn winding is utilized as the primary.

Step-Down Transformer



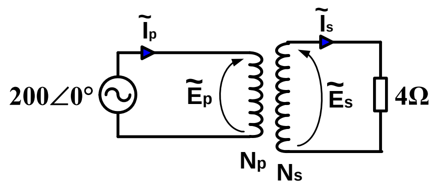


Ideal Transformer Example Problem

Given a transformer that contains windings having 50-turns and 500-turns:

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**,

Determine:
- The load voltage,
- The load current,
- The real power consumed by the load,
- The source current, and
- The real power produced by the source.

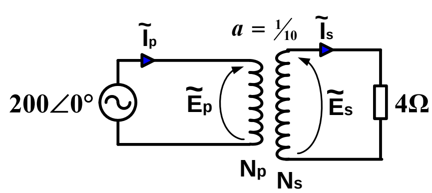


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **turns-ratio** for the transformer (as connected) is:

$$a = \frac{N_p}{N_s} = \frac{50}{500} = \frac{1}{10}$$



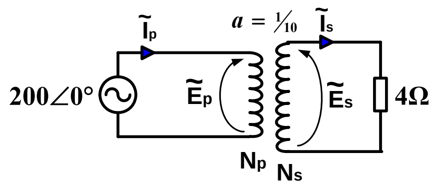
Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

Since the source is directly connected to the primary winding, the **primary voltage** $\tilde{E}_p$ is equal to the source voltage, thus:

$$\tilde{E}_p = 200\angle 0°$$

And, since the load is connected directly to the secondary winding, the load voltage and current are equal to $\tilde{E}_s$ and $\tilde{I}_s$ respectively.

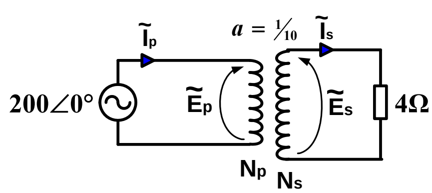


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **secondary** (load) **voltage** $\tilde{E}_s$ can be determined from the equation:

$$\tilde{V}_{load} = \tilde{E}_s = \frac{\tilde{E}_p}{a} = \frac{200\angle 0°}{\frac{1}{10}} = 2{,}000\angle 0° \text{ volts}$$

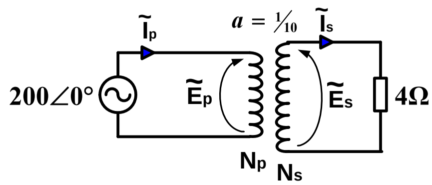


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The resultant **secondary** (load) **current** $\tilde{I}_S$ will be:

$$\tilde{I}_{load} = \tilde{I}_s = \frac{\tilde{V}_{load}}{Z_{load}} = \frac{2{,}000\angle 0°}{4} = 500\angle 0° \text{ amps}$$



Ideal Transformer Example Problem

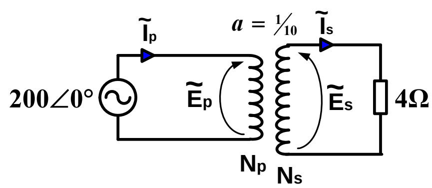
If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **complex power**, S_{load}, consumed by the load will be:

$$S_{load} = \tilde{V}_{load} \cdot \tilde{I}^*_{load} = (2{,}000\angle 0°)\cdot(500\angle 0°) = \boxed{1{,}000{,}000} + j0$$

from which the **load's real power** can be determined:

$$P_{load} = 1{,}000{,}000 \text{ watts}$$

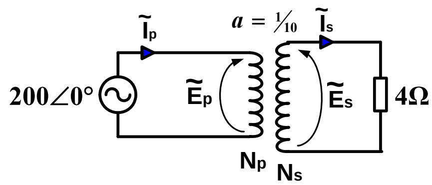


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **primary** (source) **current** $\tilde{I}_p$ can be determined from the equation:

$$\tilde{I}_{source} = \tilde{I}_p = \frac{\tilde{I}_s}{a} = \frac{500\angle 0^\circ}{\frac{1}{10}} = 5{,}000\angle 0^\circ \text{ amps}$$



Ideal Transformer Example Problem

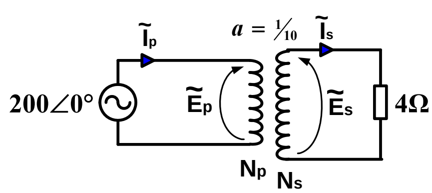
If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

Finally, the **complex power**, S_{source}, produced by the source will be:

$$S_{source} = \tilde{V}_{source} \cdot \tilde{I}^{*}_{source} = (200\angle 0^\circ)\cdot(5{,}000\angle 0^\circ) = \boxed{1{,}000{,}000} + j0$$

from which the **source's real power** can be determined:

$$P_{source} = 1{,}000{,}000 \text{ watts}$$

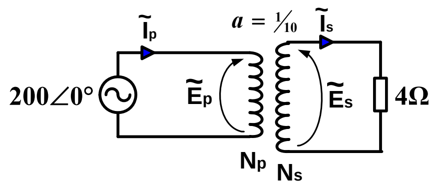


Ideal Transformer Example Problem

Given a transformer that contains windings having 50 and 500 turns:

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

- **Load Voltage**: $\tilde{V}_{load}$ **= 2,000∠0° volts**
- **Load Current**: $\tilde{I}_{load}$ **= 500∠0° amps**
- **Load Real Power**: P_{load} **= 1,000,000 watts**
- **Source Current**: $\tilde{I}_{source}$ **= 5,000∠0° amps**
- **Source Real Power**: P_{source} **= 1,000,000 watts**



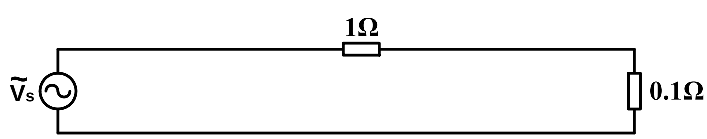
"Power System" Example Problem

A **load impedance Z_{load} = 0.1Ω** requires a **supply voltage of 100∠0° volts**.

Since the load is far from the actual voltage source, a long pair of wires are used to connect the load to the source.

If the wires have an overall resistance of $\boldsymbol{R_{wire}}$ **= 1Ω**,

Determine the required **source voltage** and the overall **system efficiency**.



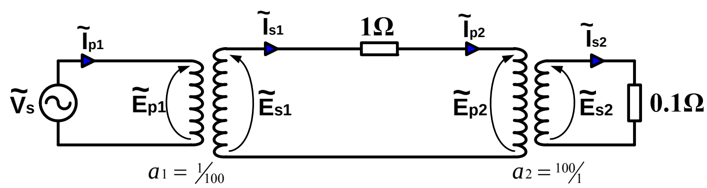
“Power System” Example Problem

A **load impedance Z_{load} = 0.1Ω** requires a **supply voltage of 100∠0° volts**.

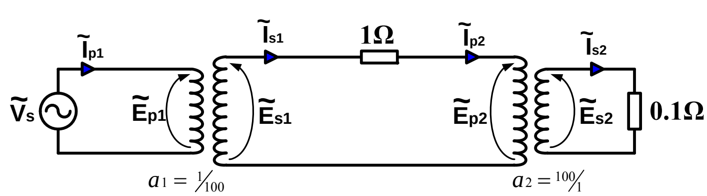
A long pair of wires are still used to connect the load to the source, but this time **ideal transformers** are placed at both the source-end and the load-end of the wires, the turns-ratios of which are $\frac{1}{100}$ and $\frac{100}{1}$ respectively.

If the wires have an overall resistance of R_{wire} **= 1Ω**,

Determine the required **source voltage** and the overall **system efficiency**.



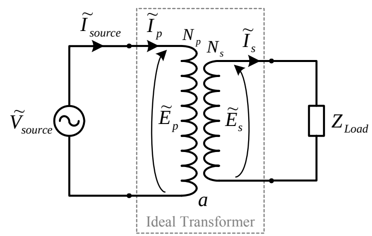
“Power System” Example Problem



Input Impedance

Given an ideal transformer with a source connected across the primary winding and a load connected across the secondary winding…

Determine the overall impedance "seen" by the source.
(I.e. – the **input impedance** of the ideal transformer)



$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

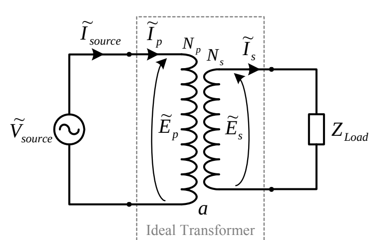
Input Impedance

The **input impedance** of an ideal transformer may be defined as:

$$\boldsymbol{Z_{in} = \frac{\tilde{E}_p}{\tilde{I}_p}}$$

If we substitute the following relations into the equation:

$$\boldsymbol{\tilde{E}_p = a \cdot \tilde{E}_s} \qquad \boldsymbol{\tilde{I}_p = \tfrac{1}{a} \cdot \tilde{I}_s}$$



$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

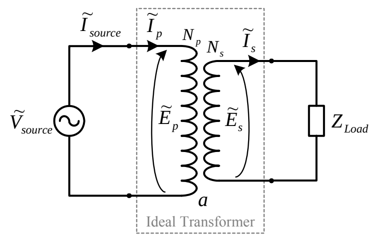
Input Impedance

Then the **input impedance** may be re-written as:

$$\boldsymbol{Z}_{in} = \frac{a \cdot \tilde{\boldsymbol{E}}_s}{\frac{1}{a} \cdot \tilde{\boldsymbol{I}}_s} = a^2 \cdot \frac{\tilde{\boldsymbol{E}}_s}{\tilde{\boldsymbol{I}}_s}$$

since $\tilde{E}_s$ and $\tilde{I}_s$ equal the load voltage and current respectively:

$$\frac{\tilde{\boldsymbol{E}}_s}{\tilde{\boldsymbol{I}}_s} = \frac{\tilde{\boldsymbol{V}}_{Load}}{\tilde{\boldsymbol{I}}_{Load}} = \boldsymbol{Z}_{Load}$$



$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

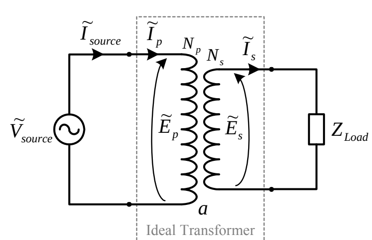
Input Impedance

If <u>expressed in terms of the load impedance</u>, the **input impedance** of the ideal transformer is:

$$\boldsymbol{Z}_{in} = a^2 \cdot \frac{\tilde{\boldsymbol{E}}_s}{\tilde{\boldsymbol{I}}_s} = a^2 \cdot \boldsymbol{Z}_{Load}$$

the turns-ratio squared times the connected load impedance:

$$\boldsymbol{Z}_{in} = a^2 \cdot \boldsymbol{Z}_{Load} = \boldsymbol{Z'}_{Load}$$



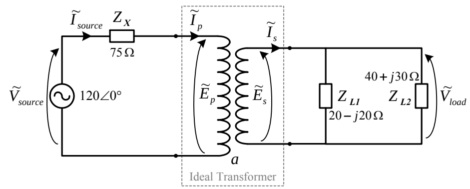
$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:

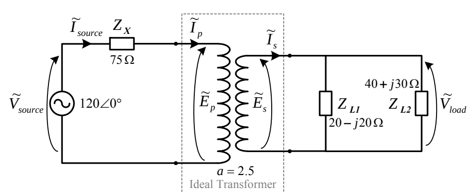


Assuming that the transformer is configured for **step-down** operation,

Determine:
- The source current,
- The load voltage,
- The complex power produced by the source, and
- The total complex power consumed by Z_{L1} and Z_{L2}.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



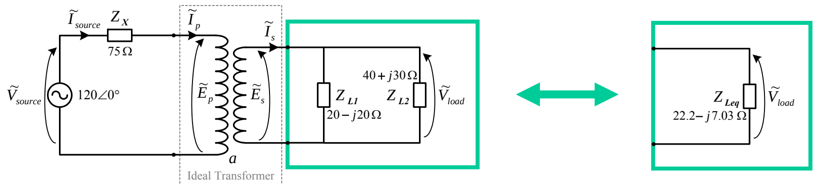
Since the transformer is configured for **step-down operation**, **the primary winding is the high-voltage winding** and **the secondary winding is the low-voltage winding**.

Thus, the operational **turns-ratio** of the transformer is:

$$a = \frac{V_{Rated(Pri)}}{V_{Rated(Sec)}} = \frac{\mathbf{120V}}{\mathbf{48V}} = \mathbf{2.5}$$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



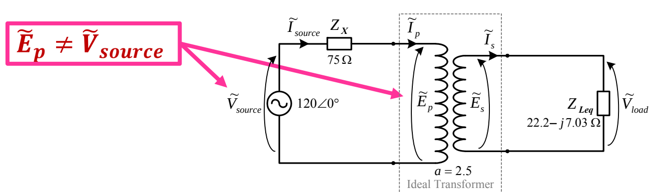
Since Z_{L1} is connected in parallel with Z_{L2}, the two impedances can be replaced by a single **equivalent impedance** Z_{Leq}:

$$Z_{Leq} = \left(\frac{1}{Z_{L1}} + \frac{1}{Z_{L2}}\right)^{-1} = \left(\frac{1}{20 - j20} + \frac{1}{40 + j30}\right)^{-1} = (22.2 - j7.03)\ \Omega$$

Note that the voltage across Z_{Leq} equals the original load voltage V_{load}.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



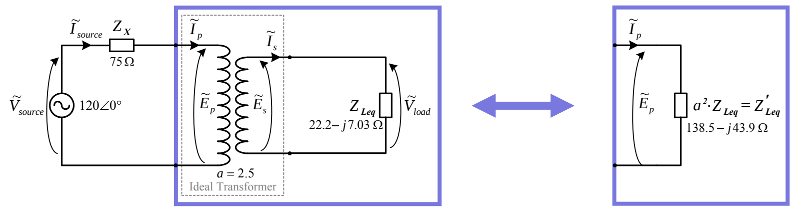
Since there is a 75Ω impedance connected in series between the source and the primary winding, the **primary voltage** E_p is a function of both the source voltage and the source current:

$$\tilde{E}_p = \tilde{V}_{source} - \tilde{I}_{source} \cdot Z_x$$

Because of this, the circuit cannot be easily analyzed in its current configuration.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:

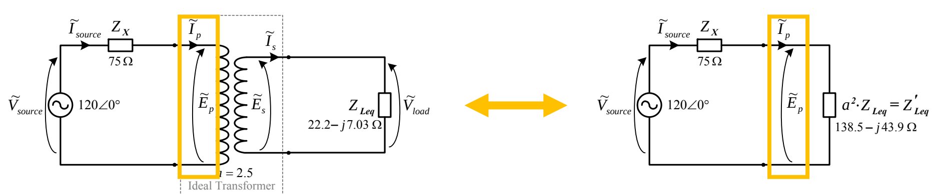


Thus, to facilitate the analysis of the circuit, the ideal transformer and load combination will initially be replaced by an overall equivalent impedance that equals the **input impedance** seen looking into the transformer's primary terminals:

$$\boldsymbol{Z_{in} = Z'_{Leq} = a^2 \cdot Z_{Leq} = 2.5^2 \cdot (22.2 - j7.03) = (138.5 - j43.9)\ \Omega}$$

Ideal Transformer Example Problem

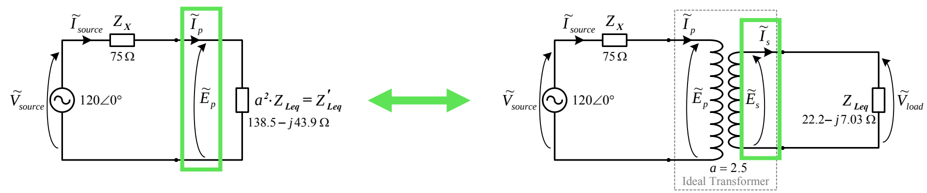
Given the following circuit that contains a **120V–48V** ideal transformer:



Note that, when the transformer–load combination is replaced by their equivalent impedance, the voltage across the impedance and the current flowing through the impedance are equal to the primary winding voltage and current respectively.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:

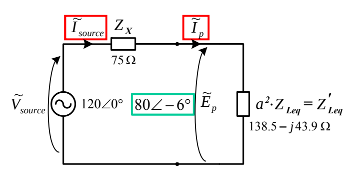


Now that the overall circuit has been simplified down to a relatively trivial circuit, the unknown voltages and currents remaining in the simplified circuit can be determined by applying basic circuit theory.

Additionally, once the remaining voltages and currents are determined, the basic turns-ratio equations can be utilized in order to relate the primary-side voltages and currents to the secondary-side quantities.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



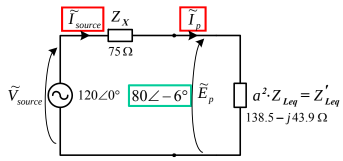
Since the circuit has been reduced-down to two series impedances, the remaining voltages and currents can be determined as follows:

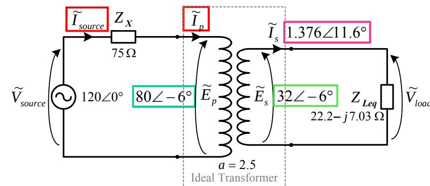
$$\tilde{I}_{source} = \frac{\tilde{V}_{source}}{(Z_x + Z'_{Leq})} = \frac{120\angle 0°}{75 + (138.5 - j43.9)} = 0.5505\angle 11.6°\ amps = \tilde{I}_p$$

$$\tilde{E}_p = \tilde{I}_p \cdot Z'_{Leq} = (0.5505\angle 11.6°) \cdot (138.5 - j43.9) = 80\angle -6°\ \text{volts}$$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:





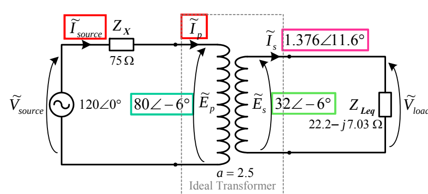
Since E_p and I_p in the reduced circuit equal E_p and I_p in the original circuit, the **secondary voltage** E_s and **secondary current** I_s can be determined:

$$\tilde{E}_s = \frac{\tilde{E}_p}{a} = \frac{80\angle{-6°}}{2.5} = \boxed{32\angle -6° \text{ volts}} \qquad \tilde{I}_s = a \cdot \tilde{I}_p = 2.5 \cdot 0.5505\angle 11.6° = \boxed{1.376\angle 11.6° \text{ amps}}$$

Note that the **load voltage** $\tilde{V}_{load} = \tilde{E}_s$ and the **total load current** $\tilde{I}_{load} = \tilde{I}_s$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



Now that the source and load voltages and currents are known, the **source and load complex powers** can be determined:

$$S_{source} = \tilde{V}_{source} \cdot \tilde{I}^*_{source} = (120\angle 0°) \cdot (0.5505\angle{-11.6°}) = 66.1\angle{-11.6°} = 64.7 - j13.3$$

$$S_{load} = \tilde{V}_{load} \cdot \tilde{I}^*_{load} = (32\angle{-6°}) \cdot (1.376\angle{-11.6°}) = 44.04\angle{-17.6°} = 42 - j13.3$$