Applied Electromagnetic Theory

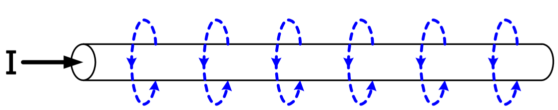
(Part B)

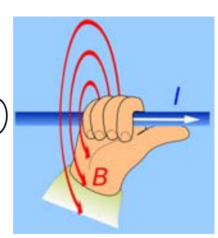
ECET 3500 – *Survey of Electric Machines*

Magnetic Fields

Around Linear Conductors

<u>Right-Hand-Rule</u> – Point the thumb of your right hand in the direction of current flow. The field lines form around the conductor in the direction that your fingers curl around the conductor if you grab it with your right hand.

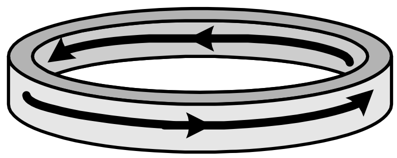


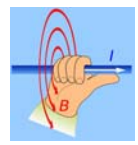


Original – http://en.wikipedia.org/wiki/File:Manoderecha.svg

Ring-shaped Conductors

Given a **ring-shaped conductor** around which a theoretical current is flowing, the right-hand-rule for linear conductors may still be applied around the ring incrementally in order to determine the pattern that the field lines form.

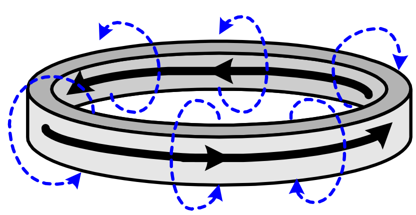


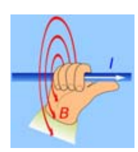


Ring-shaped Conductors

Applying the right-hand-rule at various locations around the ring provides the field pattern shown below.

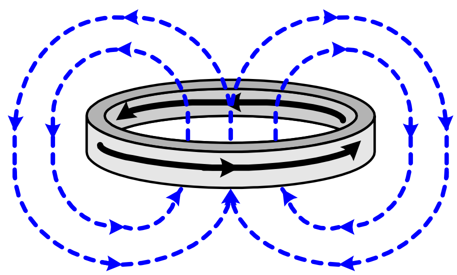
At every location around the ring, **all of the field lines point up through the center of the ring** and follow paths that continue over the top of the ring, down around the outside and back up through the center again to close the loops.





Ring-shaped Conductors

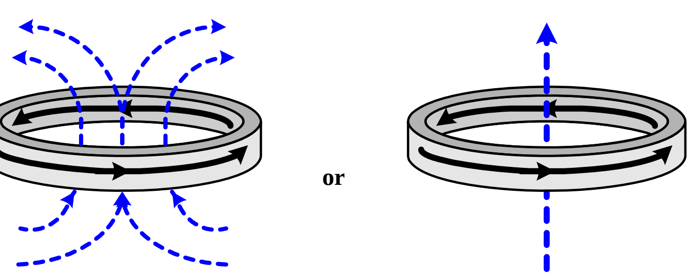
Since the closed-loop field lines all point up through the center of the ring, the overall field around the ring may be represented by the simplified two-dimensional field pattern shown below.



Ring-shaped Conductors

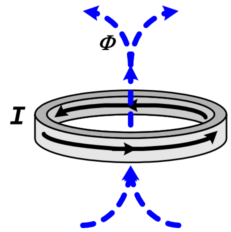
Furthermore, it is often the **direction of the field** as it passes <u>through the center of the ring</u> that is of primary concern.

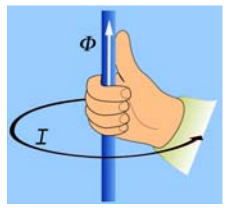
In this case, only the field lines in that area may shown, with the understanding that they must still form closed-loop paths.



Right-Hand-Rule
For Ring-Shaped Conductors

<u>Right-Hand-Rule</u> – Curl the fingers of your right hand in the direction of current flow around the ring. The field lines will point through the center of the ring in the direction that your thumb points.

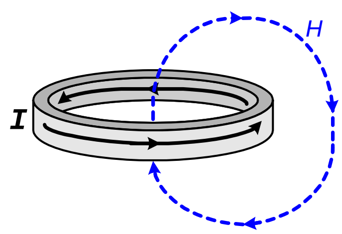




Original – http://en.wikipedia.org/wiki/File:Manoderecha.svg

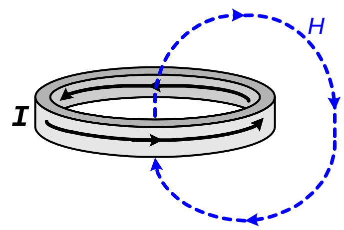
Magnetic Field Intensity
Around Ring-Shaped Conductors

The current-carrying, ring-shaped conductor shown below will produce a **magnetic field** whose <u>lines pass upward through the center of the ring</u> and then bend down around the outside of the ring, eventually curving back up towards the center to form closed-loops.



Magnetic Field Intensity
Around Ring-Shaped Conductors

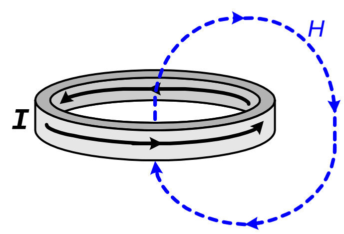
Since the ring passes through the center of the closed-loop path defined by the field-line, then the **magnitude of the current**, $\boldsymbol{I_T}$, that is flowing in the ring must equal the closed-loop integral of the **magnetic field intensity**, $\boldsymbol{H}$, along the close-loop field path.



$$I_T = \oint H \cdot dl$$

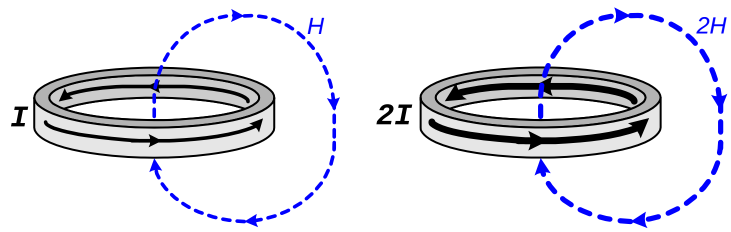
Magnetic Field Intensity
Around Ring-Shaped Conductors

Similarly, since the “source” of the magnetic field is the current in the ring, the magnitude of the **magnetic field intensity** at any point along a closed-loop path will be <u>proportional to the magnitude of the ring-current</u>.



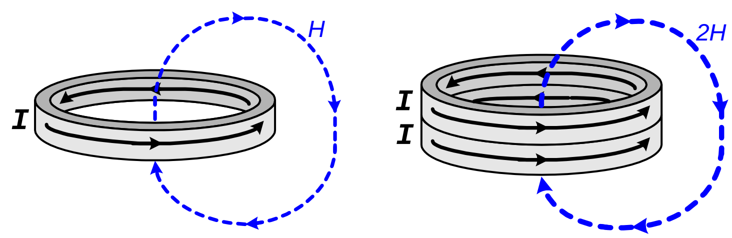
Magnetic Field Intensity
Around Ring-Shaped Conductors

Given this proportionality, if the magnitude of the **current** flowing in the ring **is doubled**, then the magnitude of the **magnetic field intensity** at each point along the closed-loop path **will also double**.



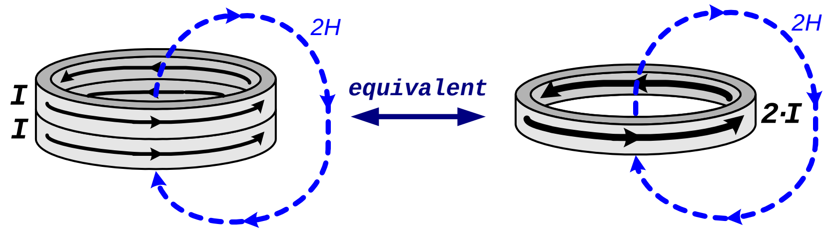
Magnetic Field Intensity
Around Ring-Shaped Conductors

Likewise, if a **second current-carrying ring** with current equal to the first is added such that <u>both rings pass through the center</u> of the closed-loop path, then the magnitude of the **magnetic field intensity** along that path **will again double**.



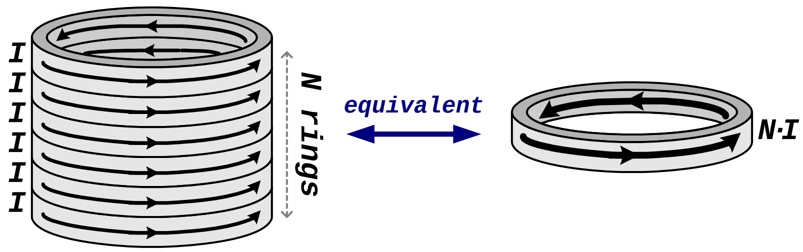
Magnetic Field Intensity
Around Ring-Shaped Conductors

Thus, in terms of their ability to create a magnetic field along a specific closed-loop path, **two parallel rings that both carry current *I* are equivalent to a single ring carrying current 2·*I*** provided that all of the rings pass through the center of the path.



Magnetic Field Intensity
Around Ring-Shaped Conductors

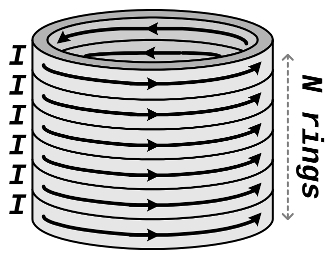
The same logic can be applied to a series of *N* **parallel rings** that **each carry current *I*** provided that all of the rings pass through the center of the path.



Ring-shaped Conductors

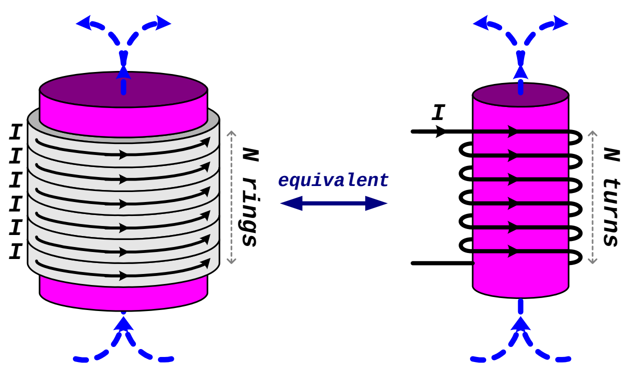
Although **ring-shaped conductors** provide a good <u>theoretical source</u> for magnetic fields, their practical use is limited because there is no mechanism provided to create the current-flow.

Instead, a slightly different structure will be utilized that offers the same characteristics as a set of current-carrying rings



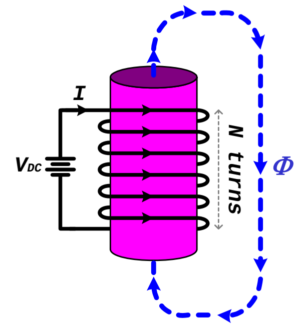
Conductive Coils

A set of ***N*** **parallel**, **current-carrying rings** is very similar in nature to a <u>current-carrying conductor</u> that is wrapped in the **shape of a coil having *N* "turns"** (loops) of conductor.



Conductive Coils

Current-carrying coils are commonly used to create the magnetic field required for the operation of many different AC and DC devices such as transformers, relays, or motors.

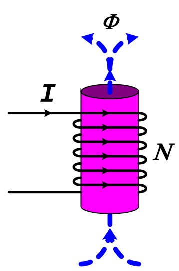


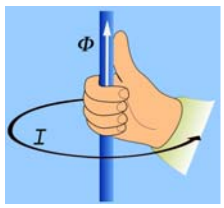
Coil-based magnetic sources are very customizable because the **strength of the field** that is created by the coil can be varied by adjusting either the **number of turns** of the coil and/or by adjusting the **magnitude of the current** flowing in the coil.

Right-Hand-Rule

For Current-Carrying Coils

<u>Right-Hand-Rule</u> – Curl the fingers of your right hand in the direction of current flow around the coil. The field lines will point through the center of the coil in the direction that your thumb points.

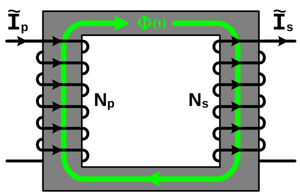


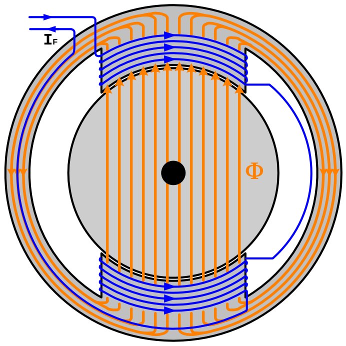


Original – http://en.wikipedia.org/wiki/File:Manoderecha.svg

Magnetic Circuits

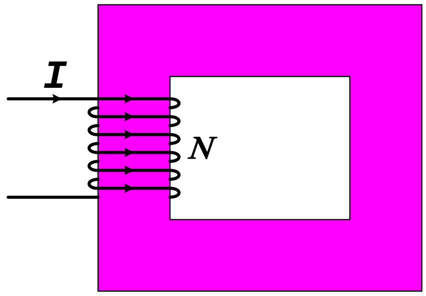
A **magnetic circuit** consists of one or more magnetic sources attached to a "magnetic core" structure that is used to guide the flux created by the magnetic source(s) along one or more closed-loop paths.





Simple Magnetic Circuit

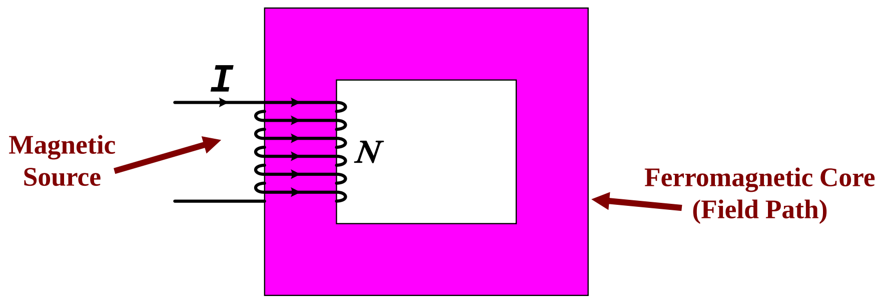
A **simple magnetic circuit** consisting of a single magnetic source and a uniform core that provides a single closed-loop field path is shown below:



Simple Magnetic Circuit

The **magnetic source** is an *N*-turn coil of wire through which a DC current ***I*** is flowing.

The **ferromagnetic core** provides a single closed-loop field path that passes through the center of the coil.



Ferromagnetic Core

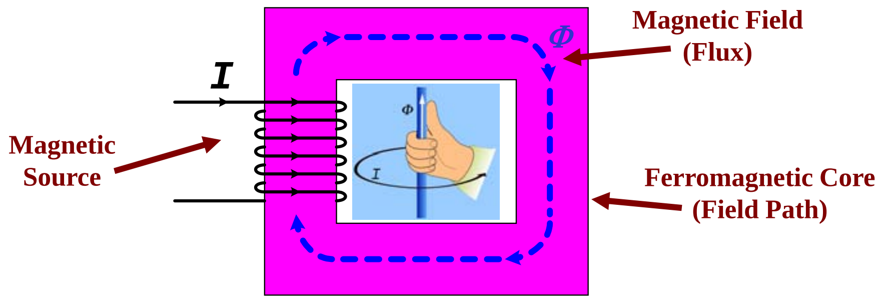
The **ferromagnetic core** is composed of high-permeability materials (typically iron-based) in which it is "easy" to form a strong magnetic field.

Compared to the flux created by a specific magnetic source within a region of air, the flux created by that same source within a good, closed-loop, ferromagnetic core can be 100s or 1000s of times larger in magnitude.

When a magnetic circuit contains a good, closed-loop, ferromagnetic core, it is often assumed that the flux created by the magnetic source(s) is constrained to the path(s) provided by the core.
(I.e. – the flux created in the surrounding air is negligible)

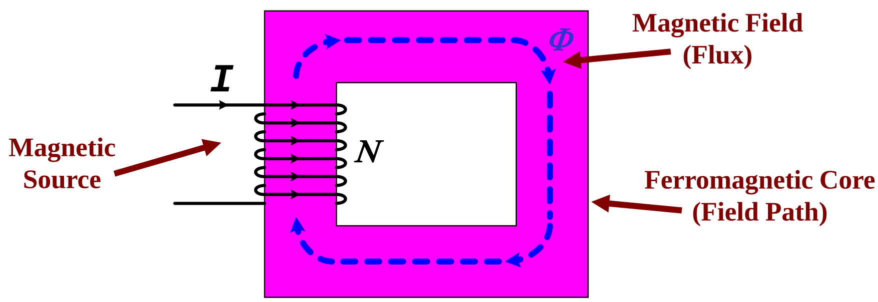
Simple Magnetic Circuit

The current flowing in the coil will create a **flux**, Φ, within the core that will, as defined by the right-hand-rule, point upwards through the center of the coil and thus in a clock-wise direction through (around) the magnetic core.



Magnetic Circuit Analysis

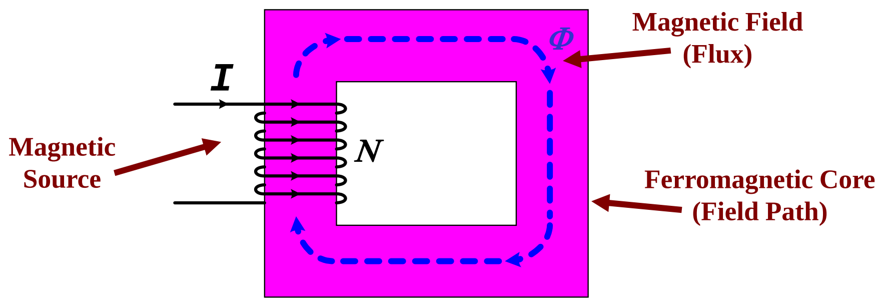
The number of turns, N, of the coil and the current magnitude, I, along with the size, shape, and composition of the core structure all affect the magnitude of the **flux** created by the coil.



Magnetic Circuit Analysis

In order to understand how all these different variables are related, we can apply the previously-introduced magnetic concepts to the analysis of the magnetic circuit.

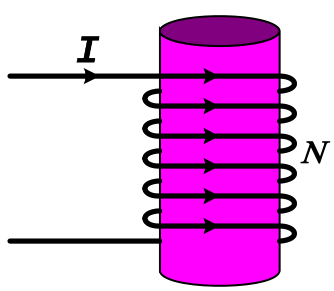
We will begin by characterizing our **magnetic source**…



Magneto-Motive Force (MMF)

Magneto-Motive Force ($\mathcal{F}$) – the overall force provided by a magnetic source that tries to create a magnetic flux.

The **MMF** created by a **coil** is proportional both to the number of turns of the coil and to the magnitude of the coil-current.



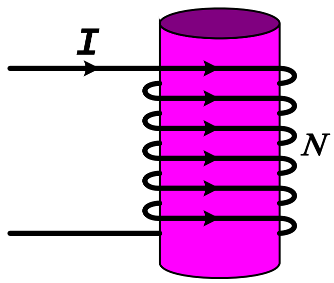
$$\mathcal{F} = N \cdot \boldsymbol{I}$$ (Amp·turns)

DC-Supplied Field Coils

Note that, when a coil is supplied by a **DC source**, the steady-state coil current is simply a function of the supply voltage and the coil resistance.

The relationship between the source voltage and the coil current is more complex for an <u>AC-supplied coil</u>. This will be discussed at the end of this presentation.

$$\mathcal{I}_{DC} = \frac{V_{DC}}{R_{coil}}$$

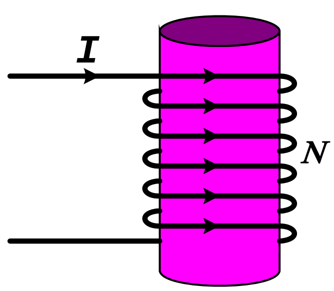


$$\mathcal{F} = N \cdot \mathcal{I}$$ (Amp·turns)

Magneto-Motive Force (MMF)

The **MMF** is basically a measure of the <u>total force</u> produced by a magnetic source that tries <u>to create a magnetic flux within a magnetic circuit</u>.

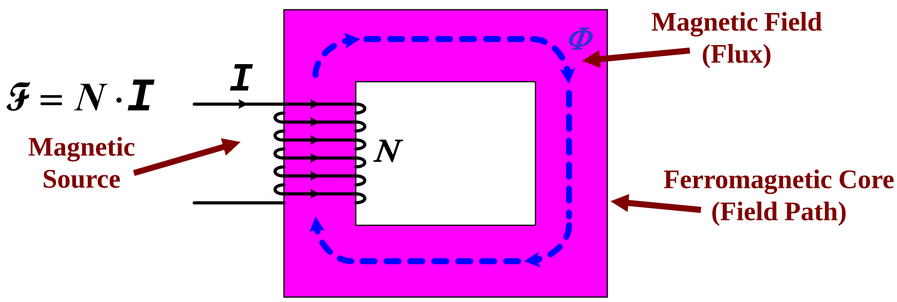
This is similar in concept to an <u>electro-motive force</u> (EMF), which provides a measure of the force that tries to push current through an electric circuit.



$$\mathcal{F} = N \cdot \mathcal{I}$$ (Amp·turns)

Magneto-Motive Force (MMF)

Yet, just as the magnitude of the current created by an EMF is affected by characteristics of the closed-loop current path, the **magnitude of the flux** created by an MMF is affected by the characteristics of the core structure that provides the closed-loop field path(s).

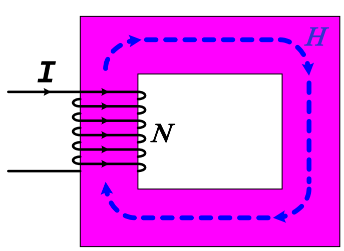


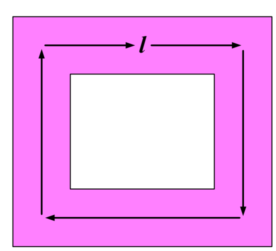
Magnetic Field Intensity

Magnetic Field Intensity (H) – the applied MMF (force) per unit length used to create a flux within the magnetic core.

For a simple uniform structure, the **magnetic field intensity** may be determined from:

$$H = \frac{\mathcal{F}}{l} = \frac{N \cdot I}{l} \quad \text{(A·t/m)}$$

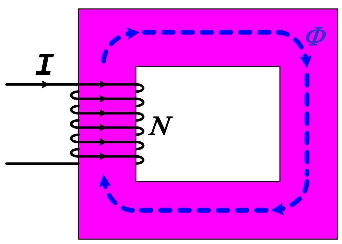




l is the average length of the field path.

Magnetic Flux

<u>Magnetic Flux</u> ($\boldsymbol{\Phi}$) – the total "magnetic field" created by the magnetic source that, under ideal conditions, will follow the closed-loop path provided by the magnetic core.

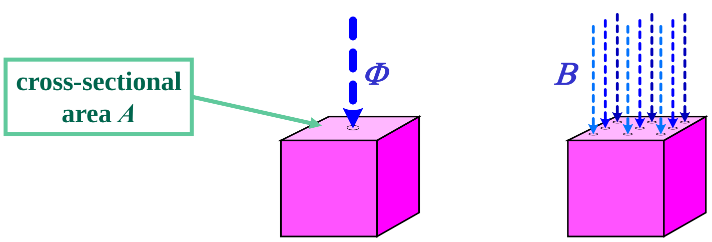


Magnetic Flux is given the units of Webers (Wb).

Magnetic Flux Density

<u>Magnetic Flux Density</u> ($\boldsymbol{B}$) – the magnetic flux per unit cross-sectional area created within a region by the magnetic field intensity applied across that region.

The **magnetic flux density** induced within a core is proportional to the <u>applied field intensity</u> and the <u>permeability of the core</u>.



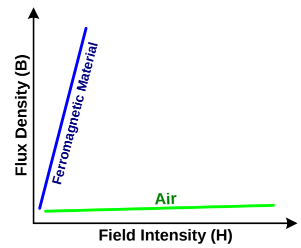
Magnetic Flux Density is given the units of Tesla (T).

Permeability

Permeability (μ) – the ratio of the flux density formed within a volume of the material compared to the magnetic field intensity applied across that volume of material.

$$\mu = \frac{B}{H}$$

Ferromagnetic materials have a **high permeability**, resulting in a relatively large increase in the flux density induced within the material due to an increase in the magnetic field intensity applied across the material.



Magnetic Saturation

Good ferromagnetic materials, such as iron and steel, facilitate the formation of a magnetic flux within those materials when they are exposed to an externally-sourced magnetic field intensity.

Yet, although they effectively "assist" in the formation of the flux, there is a practical limit to this ability for even the best materials.

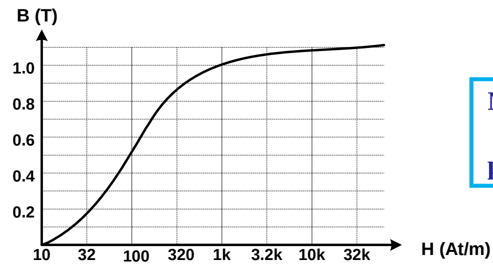
As the flux density present within a practical ferromagnetic material rises above a certain level, the material becomes **saturated**, resulting in a decrease in the material's ability to facilitate in the creation of any additional flux.

Magnetic Saturation

When the flux density within a ferromagnetic material rises to the point that the material becomes **saturated**, further increases in the externally-sourced magnetic field intensity (H) tend to produce relatively smaller increases in flux density (B).

This effect is represented in the following non-linear B-H curve:

$$\mu = \frac{B}{H}$$

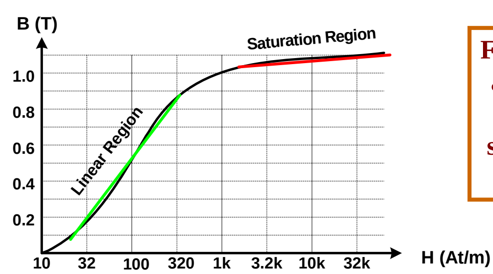


Note that, as a material becomes saturated, its permeability decreases.

Magnetic Saturation

Note that before saturation occurs, ferromagnetic materials often act in a relatively linear manner.

For this reason, many magnetic circuits are designed to operate such that their core material is kept within its **linear region**.



Ferromagnetic materials are often referred to as non-linear materials since their permeability is not a constant.

Relative Permeability

Relative Permeability (μ_r) – the ratio of the material's actual permeability compared to the permeability of "air" (free space):

$$\mu_r = \frac{\mu_{material}}{\mu_o}$$

where the permeability of air μ_o is:

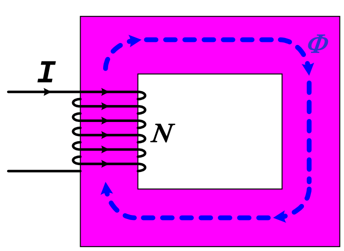
$$\mu_o = 4\pi \cdot 10^{-7}$$

Since "air" (free-space) has the smallest possible permeability, classifying materials by their relative permeability can provide useful insight into their quality as a potential ferromagnetic material.

Reluctance

Reluctance ($\mathscr{R}$) – reluctance is defined as the ratio of the applied MMF, $\mathscr{F}$, compared to the resultant flux, Φ.

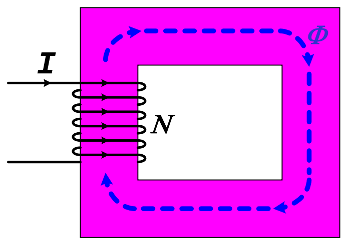
$$\mathscr{R} = \frac{MMF}{\Phi} = \frac{\mathscr{F}}{\Phi}$$



Reluctance

The **reluctance** of a field path is characterized by the amount of force ($\mathscr{F}$) required to create a specific flux (Φ) within the material region through which the path is defined.

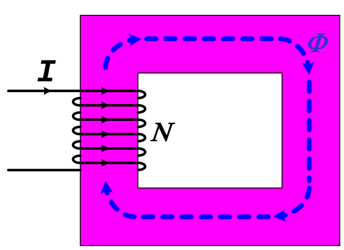
Thus, the reluctance of the core be viewed as a measure of the core's ability to oppose or "resist" the formation of that field.



Reluctance

Given the simple, single-path, uniform structure shown below, the **reluctance** ($\mathscr{R}$) of the overall field path (core) is:

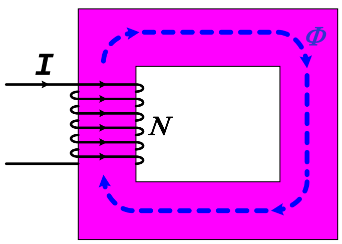
$$\mathscr{R} = \frac{\mathscr{F}}{\Phi} = \frac{N \cdot I}{\Phi} = \frac{H \cdot l}{B \cdot A} = \frac{l}{\mu \cdot A}$$

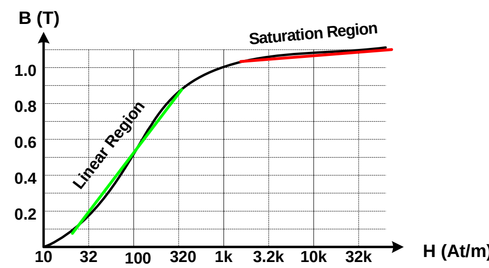


Saturation and Reluctance

Note that the **reluctance** of a field path <u>increases as the core material becomes saturated</u> since reluctance is inversely proportional to permeability.

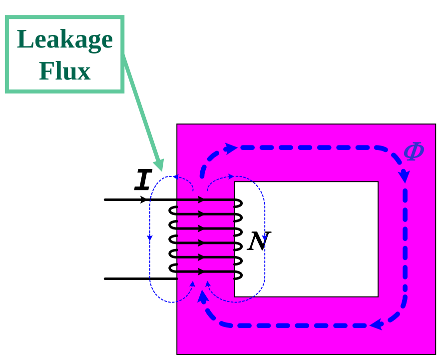
$$\mathcal{R} = \frac{l}{\mu \cdot A} \qquad \mu = \frac{B}{H}$$

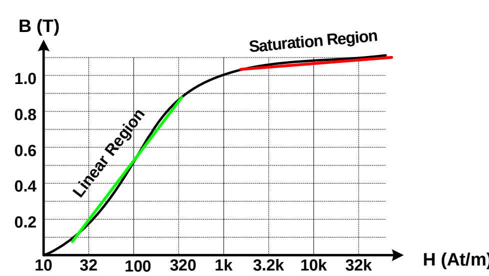




Saturation and Leakage Flux

Furthermore, as the core becomes **saturated**, the increased reluctance of the core will begin to push the flux out into the surrounding air, resulting in a <u>**leakage flux**</u>.

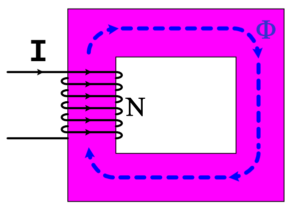


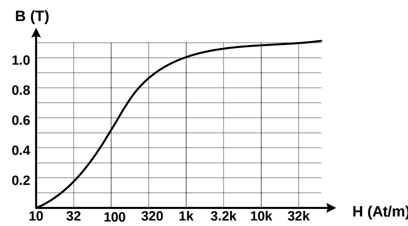


Magnetic Circuit Example

The simple, magnetic core shown below has a **13cm** average path length and a **3cm²** cross-sectional area. The B-H curve for the core material is also shown below.

If a **225-turn** coil is wound around the core and supplied with **400mA$_{(DC)}$**, determine the **magnitude of the flux** created within the core and the **relative permeability** of the core material.





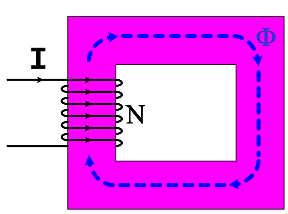
Magnetic Circuit Example

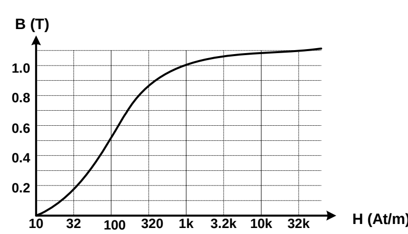
Given:

L = 13cm = 0.13m, A = 3cm² = 0.0003m², N = 225t, I = 400mA$_{DC}$ = 0.4A

Determine the **MMF** from the source coil:

$$\mathcal{F} = N \cdot I = (225\text{t}) \cdot (0.4\text{A}) = 90\text{A} \cdot \text{t}$$





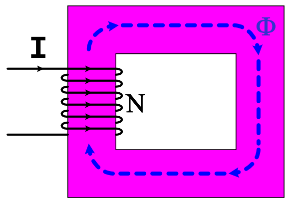
Magnetic Circuit Example

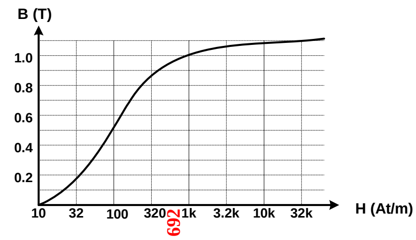
Given:

L = 13cm = 0.13m, A = 3cm² = 0.0003m², N = 225t, I = 400mA$_{DC}$ = 0.4A

Determine the **Magnetic Field Intensity** within the core:

$$H = \frac{\mathcal{F}}{L} = \frac{N \cdot I}{L} = \frac{90\text{A}\cdot\text{t}}{0.13\text{m}} = 692\,\frac{\text{A}\cdot\text{t}}{\text{m}}$$





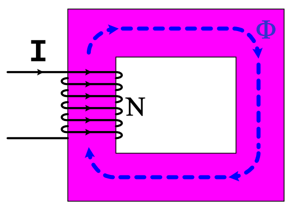
Magnetic Circuit Example

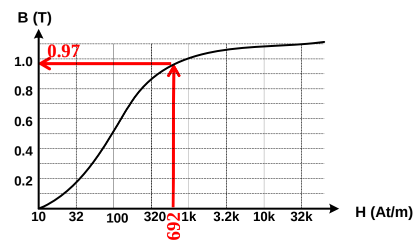
Given:

L = 13cm = 0.13m, A = 3cm² = 0.0003m², N = 225t, I = 400mA$_{DC}$ = 0.4A

Determine the **Magnetic Flux Density** from the B-H curve:

$$H = 692\,\frac{\text{A}\cdot\text{t}}{\text{m}} \quad \Rightarrow \quad B = 0.97\text{T}$$





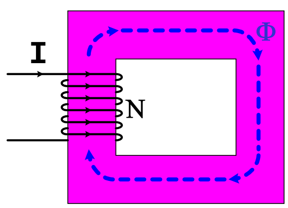
Magnetic Circuit Example

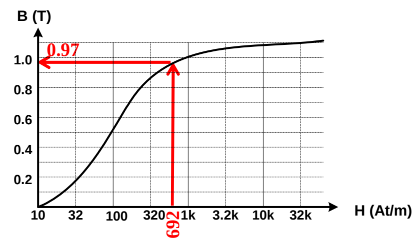
Given:

L = 13cm = 0.13m, A = 3cm² = 0.0003m², N = 225t, I = 400mA$_{DC}$ = 0.4A

Determine the **Permeability** of the core material:

$$\mu = \frac{B}{H} = \frac{0.97}{692} = 0.0014$$





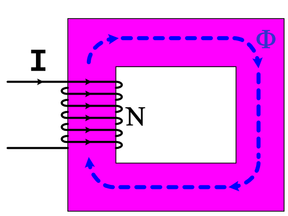
Magnetic Circuit Example

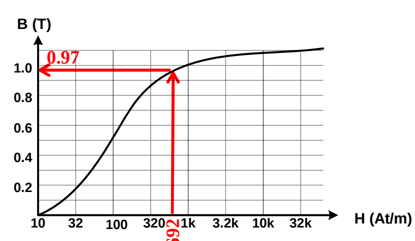
Given:

L = 13cm = 0.13m, A = 3cm² = 0.0003m², N = 225t, I = 400mA$_{DC}$ = 0.4A

Determine the **Relative Permeability** of the core material:

$$\mu_r = \frac{\mu_{material}}{\mu_o} = \frac{0.0014}{4\pi \cdot 10^{-7}} = 1114$$





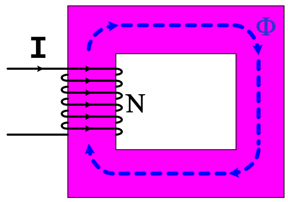
Magnetic Circuit Example

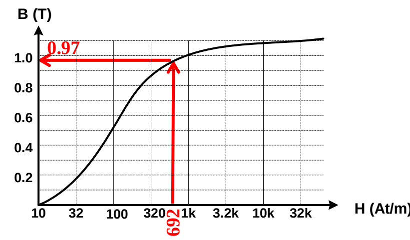
Given:

L = 13cm = 0.13m, A = 3cm² = 0.0003m², N = 225t, I = 400mA$_{DC}$ = 0.4A

Determine the total **Flux** created within the core material:

$$\Phi = B \cdot A = (0.97)\cdot(0.0003) = 0.000291\,\text{Wb} = 291\cdot 10^{-6}\,\text{Wb}$$



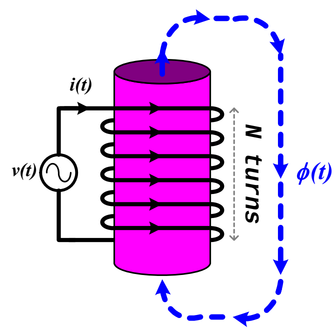


AC-Supplied Coils

When an **AC voltage source** is connected to the source coil of a magnetic circuit, an equal but opposite voltage must appear across the coil in order to satisfy Kirchhoff's Voltage Law.

$$v(t) = \sqrt{2}\cdot V \cdot \cos(\omega \cdot t)$$

Neglecting the coil's resistance, **Faraday's Law of Induction** provides the necessary mechanism for the creation of that counter-voltage.

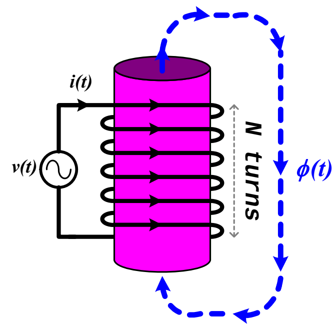


AC-Supplied Coils

Faraday's Law of Induction provides that a time-varying field passing through a coil will induce a voltage across the coil that is proportional to the rate of change of the field:

$$v(t) = N \cdot \frac{d\Phi(t)}{dt}$$

Thus, the time-varying flux required to induce the necessary counter-voltage can be determined by solving for **Φ(t)**.

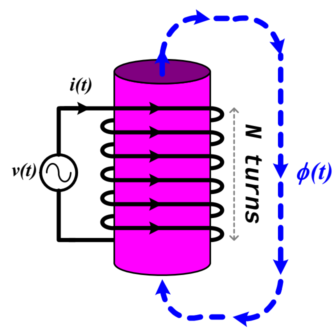


AC-Supplied Coils

Given the source voltage: $v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$

The required **flux**, **Φ(t)**, will be:

$$\begin{aligned}\Phi(t) &= \frac{1}{N} \cdot \int v(t)dt \\ &= \frac{1}{N} \cdot \int \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)dt \\ &= \sqrt{2} \cdot \frac{V}{\omega \cdot N} \cdot \sin(\omega \cdot t) \\ &= \sqrt{2} \cdot \frac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)\end{aligned}$$



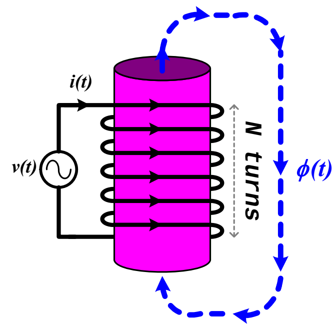
AC-Supplied Coils

Given the source voltage: $v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$

and the flux, **Φ(t)**: $\Phi(t) = \sqrt{2} \cdot \dfrac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)$

Assuming a simple magnetic circuit, the **coil-current** required to produce the flux may be determined from:

$$\mathscr{F} = N \cdot i(t) = \Phi(t) \cdot \mathscr{R}$$



AC-Supplied Coils

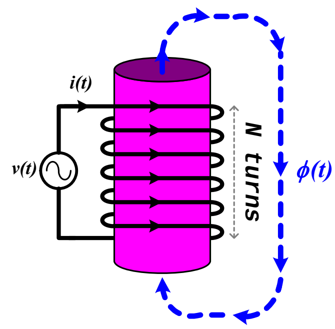
Given the source voltage: $v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$

and the flux, **Φ(t)**: $\Phi(t) = \sqrt{2} \cdot \dfrac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)$

The **coil-current** will be:

$$i(t) = \Phi(t) \cdot \frac{\mathscr{R}}{N}$$

$$= \sqrt{2} \cdot \frac{V \cdot \mathscr{R}}{\omega \cdot N^2} \cdot \cos(\omega \cdot t - 90°)$$



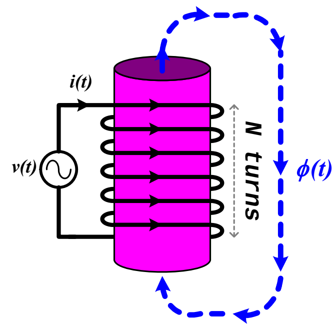
AC-Supplied Coils

Given the source voltage: $v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$

The resultant **current** may be rewritten as:

$$i(t) = \sqrt{2} \cdot \frac{V \cdot \mathcal{R}}{\omega \cdot N^2} \cdot \cos(\omega \cdot t - 90°)$$

$$= \sqrt{2} \cdot \frac{V}{\omega \cdot \frac{N^2}{\mathcal{R}}} \cdot \cos(\omega \cdot t - 90°)$$



AC-Supplied Coils

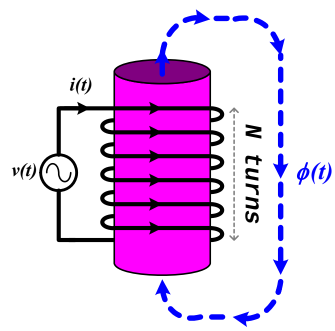
The **current** expression may be further simplified by defining a new variable ***L***:

$$L = \frac{N^2}{\mathcal{R}}$$

such that:

$$i(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot \frac{N^2}{\mathcal{R}}} \cdot \cos(\omega \cdot t - 90°)$$

$$= \sqrt{2} \cdot \frac{V}{\omega \cdot L} \cdot \cos(\omega \cdot t - 90°)$$



Self-Inductance of Coil

Given the source voltage: $v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$

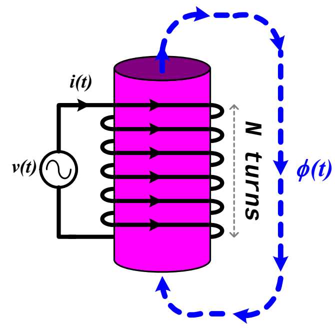
and the coil-current: $i(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot L} \cdot \cos(\omega \cdot t - 90°)$

The two waveforms adhere to the relationship:

$$v(t) = L \cdot \frac{di(t)}{dt}$$

where the coil **inductance**, L, equals:

$$L = \frac{N^2}{\mathcal{R}}$$

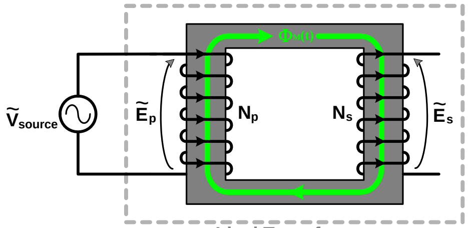


AC-Supplied Coils

Magnetic circuits containing AC-supplied source coils form the foundation for the next major lecture topic:

Chapter 2 – Transformers

which are covered in the next presentation.



Ideal Transformer