# *Applied Electromagnetic Theory*

***(Part A)***

## *Classical Electromagnetism*

**ECET 3500 – *Survey of Electric Machines***

# Classical Electromagnetism

**Classical electromagnetic** (electrodynamic) **theory** describes the forces that exist between stationary electric charges and/or electric charges in motion (currents).

The foundation of this theory is defined by the **Lorentz Force Law** equation in conjunction with **Maxwell's Equations**.

# Lorentz Force Law

The **Lorentz Force Law** describes the force on a point-charge in the presence of electric and magnetic fields.

The **Lorentz Force Law** is used in many classical electro-magnetism textbooks to define both electric fields and magnetic fields as representations of the forces that exist on a point-charge that is either sitting stationary in a region or moving through a region.

For this reason, electric fields and magnetic fields are often referred to as "**force fields**".

# Lorentz Force Law

Given a **point charge** (***q***) existing at some location in space and traveling at a **velocity** (***v***), the charge will experience an (electromagnetic) **force** (***F***) which can be parameterized by two vectors, ***E*** and ***B***, in the form:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

where: ***E*** is the **electric field**, and
***B*** is the **magnetic field**

existing at that location in space.

# Lorentz Force Law

Based on the **1st term** in the equation:

$$\vec{F} = q\vec{E} + \cdots$$

- In the presence of an **electric field**, a **force** will be induced upon the point-charge, ***q***, independent of whether the charge is **stationary or moving**.
- The **vector-direction** of the **force** induced upon a positively-charged particle will be in the same direction as that of the electric field.

# Lorentz Force Law

Based on the **2nd term** in the equation:

$$\vec{F} = \cdots + q\vec{v} \times \vec{B}$$

- In the presence of a **magnetic field**, a **force** will be induced upon a point-charge, ***q***, only if the charge is **moving** in a direction that is **orthogonal** to the field.
- The **vector-direction** of the **force** induced upon the point-charge will be orthogonal to both the vector-direction of the magnetic field and the velocity-vector of the particle.

# Maxwell's Equations

**Maxwell's Equations** describe how **stationary electric charges** and **moving electric charges** (currents) act as the sources of electric fields and magnetic fields.

Furthermore, the **Maxwell's Equations** describe how:

- **time-varying electric fields induce magnetic fields**, and
- **time-varying magnetic fields induce electric fields**.

# Maxwell's Equations

**Maxwell's Equations** are a set of four equations that first appeared in a series of papers published by James Maxwell in the **1860**s.

Although they may be expressed in various forms, the four individual equations are known as:

- **Gauss's Law**
- **Gauss's Law for Magnetism**
- **Faraday's Law (of Induction)**
- **Ampere's Law with Maxwell's Correction**

# Maxwell’s Equations

<u>**Gauss’s Law**</u> describes the **relationship between electric fields and their sources** (electric charge), which provides that:

“**electric field lines** begin only at positive electric charge and end only at negative electric charge.”

✦

<u>**Gauss’s Law for Magnetism**</u> states that there are **no positive or negative “magnetic charges”**, thus requiring that:

“**magnetic field lines** only form along <u>closed-loop paths</u>.

***(I.e. – magnetic field lines have no beginning and no end)***

# Maxwell’s Equations

<u>**Faraday’s Law**</u> (of Induction) states that **a time-varying magnetic field will induce an time-varying electric field** such that:

“the **electro-motive force** (**emf**) induced around any closed-loop path is proportional to the <u>instantaneous rate of change of the magnetic field</u> passing through the surface bounded by that path.”

# Maxwell's Equations

**<u>Ampere's Law</u>** describes the **relationship between magnetic fields and their sources** (electric charge in motion or current), which provides that:

> "the **integral of the magnetic field** around a closed-loop path is equal to the net <u>current</u> passing through the surface bounded by that path."

✦

**<u>Maxwell's Correction</u>** to Ampere's Law provides that a **time-varying electric field will be induced by a time-varying magnetic field**.

# Energy Conversion Devices

**Energy conversion devices** are devices that <u>convert energy from one form to another</u>.

**Example:** An electric motor converts electrical energy into mechanical energy (motion).

The fundamental mechanisms that provide for the theoretical operation of these devices are based on the complex electro-magnetic interactions defined by the **Lorentz Force Equation** and **Maxwell's Equations**.

# Energy Conversion Devices

Despite the complexity of the Lorentz Force Equation and Maxwell's Equations, the basic operation of many **energy conversion devices**, such as:

- **Transformers**
- **Motors**
- **Generators**

can often be explained or predicted by reducing those equations down into a simpler set of discrete equations, each of which define or describe a component of the device's operation.

# *Applied Magnetics*

# Magnetic Fields

**Magnetic Field** – a condition resulting from the **motion of electric charge** [*Ampere's Law*]

Note – although **Maxwell's Correction** to Ampere's Law provides that a time-varying magnetic field will be induced by a time-varying electric field, this presentation will focus on the magnetic fields that are derived from the motion of electric charge (current).

But, either way, just what is meant by:

"a **condition** resulting from the motion of electric charge?"

# Magnetic Fields

The concept of a **magnetic field** relates to the interaction that can occur between different charges in motion.

If a **charged particle** is **moving** through a region **in the presence of other moving charge**, an interaction between the charges may occur * that causes a force to be exerted upon the point charge in a direction that is orthogonal to its direction of motion.

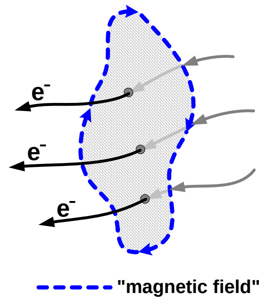
*** - depending on their respective directions of motion**

A **magnetic field** (***B***) may be defined as the vector-field necessary to make the Lorentz Force Law equation correctly describe the change in the motion of a charged particle in the presence of other moving charge.

# Magnetic Field Lines

**Magnetic fields** are often depicted by <u>magnetic field lines that form closed-loop paths</u>.

For a magnetic field line to exist, a **net amount of charge must pass through** the surface area whose boundary is defined by the closed-loop path of that field-line.




# Magnetic Field Lines

The <u>strength</u> of a **magnetic field** within a specific region is often indicated in a plot or drawing by:

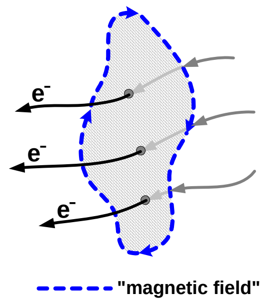
- the **thickness of the field-lines**,
- the **number of field-lines**, or
- the **distance between the field-lines**

that are shown within that specific region.

[For example – a **field strength** of **<u>1 Gauss</u>** is equivalent to **1** (magnetic field) **line per square centimeter**]

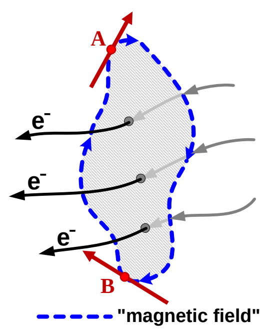
## Magnetic Field Lines

The following figure shows a **magnetic field** that is represented by a field-line drawn around a closed-loop path that defines the border of a surface-area through which electrons are passing.



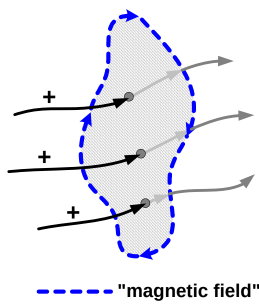

## Magnetic Field Lines

The value of the **magnetic field** shown at every point along the field-line (closed-loop) path can be represented by a **vector quantity** having both magnitude and direction.



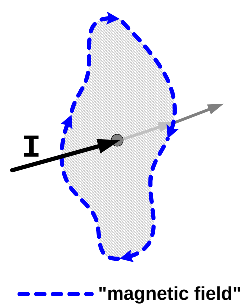
# Magnetic Field Lines

Note that, although it is typically electrons that are able to move from point to point in a system, classical electromagnetic theory is based on the concept of "**positive charge**" flow.




# Magnetic Field Lines

Since **current** is defined as the net amount of positive charge crossing a surface-area per second, the **magnetic field** can be characterized in terms of the current flowing through the center of the closed-loop field-line.



# Magnetic Sources

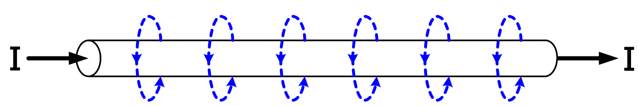
**Electro-Magnet** – a **magnetic source** whose field results from **current** flowing in a conductor.

**Permanent Magnet** – a **magnetic source** whose field results from a net uniformity of **electron orbits** around the atoms that form the physical material of the magnet.

Note that although permanent magnets are used in some electric machines, this presentation will focus on magnetic sources derived from the flow of current in conductors.

# Current-Sourced Magnetic Fields

Given the following section of a **linear conductor** through which a **current**, ***I***, is flowing:

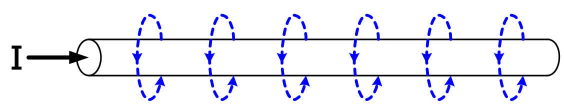


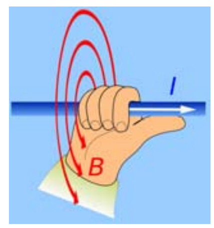

a **magnetic field** can be defined by field-lines in the region around the conductor, the vector-direction of which is based on the direction of current flow in the conductor.

# Right-Hand-Rule (RHR)

## For Linear Conductors

<u>**Right-Hand-Rule**</u> – Point the **thumb** of your **right hand** in the **direction of current flow**. The field lines form around the conductor <u>in the direction that your fingers would curl around the conductor</u> if you grab it with your right hand.




Original – http://en.wikipedia.org/wiki/File:Manoderecha.svg

# Magnetic Field Analogies

Note – although **magnetic fields** have <u>no physical mass</u>, they are often referred to and described as if they are composed of particles "flowing" through a region or "filling" a region as water might fill the pores of a sponge.

*The usefulness of these analogies will be self-evident as we progress through this investigation of magnetic theory.*

# Defining Magnetic Fields

**<u>Magnetic Field</u>** – a condition resulting from the motion of electric charge

The term "**Magnetic Field**" is often used in an all-inclusive manner when discussing this "*condition resulting from the motion of electric charge*"… (i.e. – magnetism)

Although this is acceptable when casually discussing the topic of magnetism, the term "Magnetic Field" is inadequate when a more in-depth analysis of the topic is required.

# Defining Magnetic Fields

In order to proceed with this analysis, we need to introduce several other terms relating to magnetism, all of which are often referred to, in general, as "**Magnetic Fields**".

These include:

**Magnetic Flux**

**Magnetic Flux Density**

**Magnetic Field Intensity**

# Defining Magnetic Fields

**<u>Magnetic Flux</u>** (***$\Phi$***) – a measure of the net "magnetic field" developed by a magnetic source that passes through a specific (cross-sectional) surface area.

**<u>Magnetic Flux Density</u>** (***B***) – a measure of the magnetic flux passing through a unit sized cross-sectional surface area.

**<u>Magnetic Field Intensity</u>** (***H***) – a measure of the "force" developed along a closed-loop path by a "magnetic source" that tries to create a "magnetic field" along that path.

# Magnetic Flux

**<u>Magnetic Flux</u>** (***$\Phi$***) – a measure of the net "magnetic field" developed by a magnetic source that passes through a specific (cross-sectional) surface area.

The concept of Magnetic Flux basically provides a mechanism for quantizing the overall existence of a magnetic field.

The standard (SI) unit used to quantify magnetic flux is a **Weber** (**Wb**), which is equivalent to a *volt·second*.

(Other units include "Lines" and "Maxwells")

# Magnetic Flux Density

**Magnetic Flux Density** (***B***) – a measure of the magnetic flux passing through a unit sized cross-sectional surface area.

If the flux within a region is assumed to be evenly distributed across the region, then the flux density may be solved by:

$$B = \frac{\Phi}{A}$$

where: $A$ is the cross-sectional area of the region.

The standard (SI) unit used to quantify magnetic flux density is **Tesla** (**T**), which is equivalent to a *Weber/m*$^2$.

# Magnetic Field Intensity

**Magnetic Field Intensity** (***H***) – a measure of the "force" developed along a closed-loop path by a "magnetic source" that tries to create a "magnetic field" along that path.

The field intensity (***H***) is related to the total current ($I_T$), passing through the area bounded by that path, as follows:

$$I_T = \oint H \cdot dl$$

If the Magnetic Field Intensity along the path is constant, then:

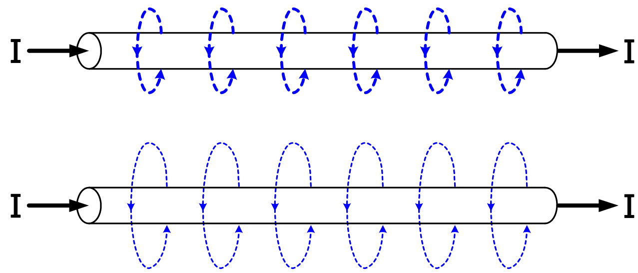
$$I_T = H \cdot L$$

where: $L$ is the length of the path.

# Magnetic Fields & Linear Conductors

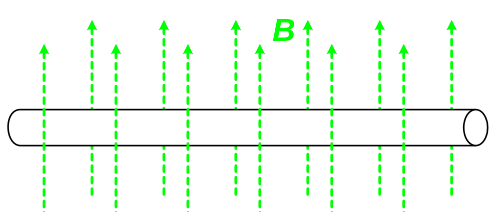
Thus, given a **linear conductor** carrying **current** (***$I_T$***), the magnitude of the **<u>magnetic field intensity</u>** (***H***) along a closed-loop path having **length** (***L***) will decrease as the **radial-distance** (***r***) from the path to the conductor increases, since:

$$H = \frac{I_T}{L} = \frac{I_T}{2 \cdot \pi \cdot r}$$




# Classical Electrodynamics
## Applied to Linear Conductors

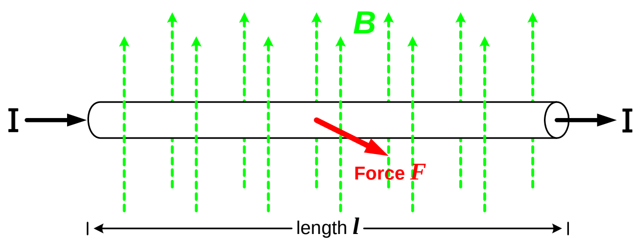
Although the topic of **motors and generators** will not be covered for several more weeks, the fundamental concepts relating to their operation can be seen by applying the classical theory to the case of a **linear conductor** that exists in a region that contains an <u>externally-sourced</u> **magnetic flux density** (***B***).



# The Lorentz Force Law
## Applied to Linear Conductors

Based on the **Lorentz Force Law**, if a current is flowing through the conductor, then a **force** will be induced upon the conductor, the magnitude of which is defined by:

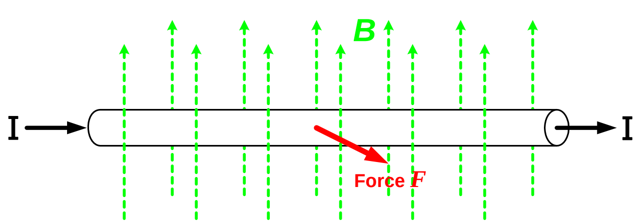
$$F = B \cdot l \cdot I$$




# The Lorentz Force Law
## Applied to Linear Conductors

Since the **force** is defined by a cross-product of the current and flux density vectors, the direction of the force must be orthogonal to both the current and the flux density vectors.

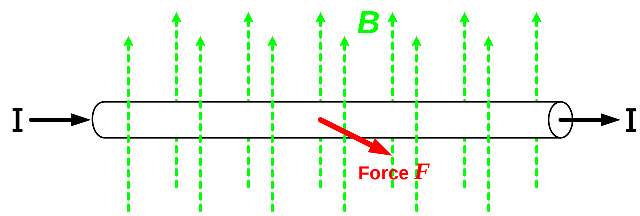
$$F = B \cdot l \cdot I$$



# The Lorentz Force Law
## Applied to Linear Conductors

The **force direction** can be determined in a visual manner by looking at the interaction between the externally-source flux and the flux created by the conductor current.

$$F = B \cdot l \cdot I$$




# Separately-Sourced Fields

If **two separately-sourced magnetic fields** exist in a region such that the directional-vectors of their field lines are parallel to each other, then a (mechanical) force will be induced upon the sources of those fields that will either:

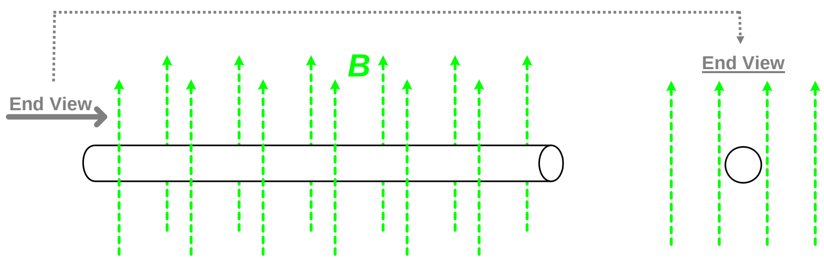
**1 –** **attract** the field sources towards that region if the **vectors** are pointing in **opposite directions** (canceling), or

**2 –** **repel** the field sources away from that region if the **vectors** are pointing in the **same direction** (adding).

# Field Interactions

## Separately-Sourced Fields

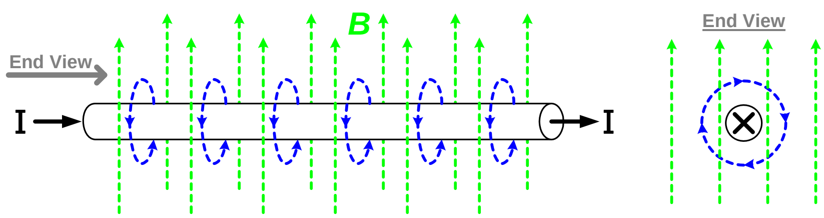
Shown below is the same **conductor** sitting within a region that contains an **externally-sourced magnetic field**. An end-view diagram of the conductor in the region (viewed from the left) has been added to the figure.

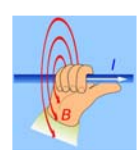



# Field Interactions

## Separately-Sourced Fields

If a **current** is flowing through the conductor then a second magnetic field will be created around the conductor, the direction of which can be determined using the right-hand-rule.



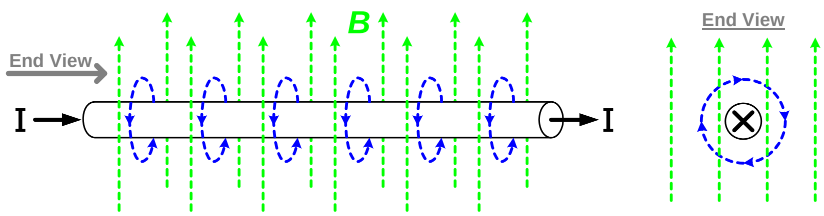
Right-Hand-Rule

# Field Interactions

## Separately-Sourced Fields

The current-carrying conductor is the source of the second field…

If the **vector-directions of the fields are parallel at any location** in the region, then the conductor (as the source of one field) will either be attracted-towards or repelled-from that location.

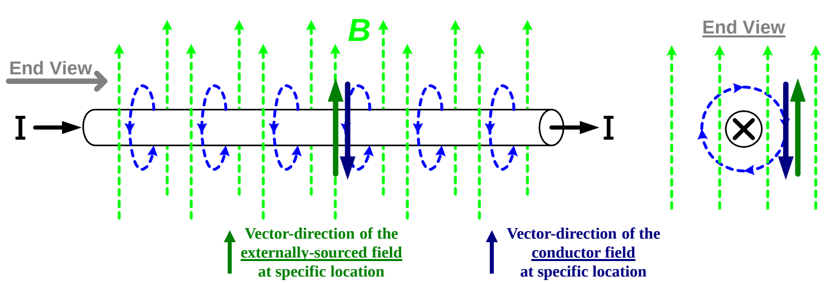



# Field Interactions

## Separately-Sourced Fields

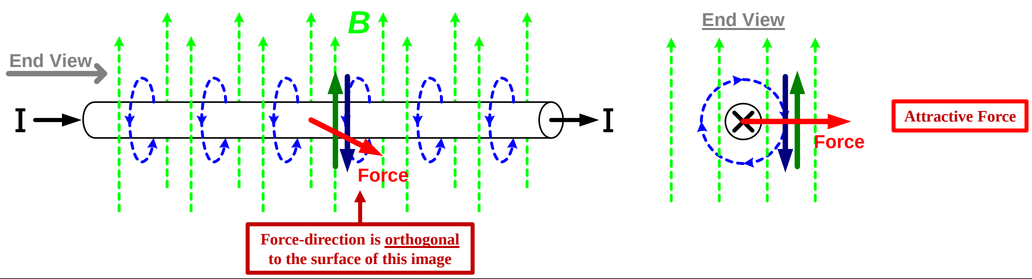
Examine the vector-directions of the two fields directly in "**front**" of the conductor (or to the "right-side" if shown in the End View)…

As can be seen, the two **fields** are **parallel** at this location and pointing in **opposite directions**.



# Field Interactions
## Separately-Sourced Fields

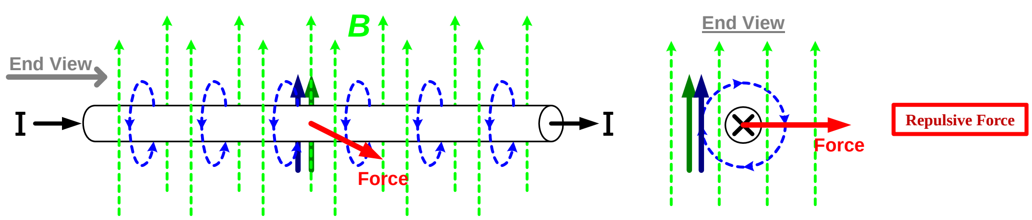
Since the vector-directions of the field-lines in front of the conductor are parallel but pointing in opposite directions, **the conductor will be attracted towards that location**.




# Field Interactions
## Separately-Sourced Fields

Similarly, the field lines in the area “**behind**” the conductor are **parallel** but pointing in the **same direction**.

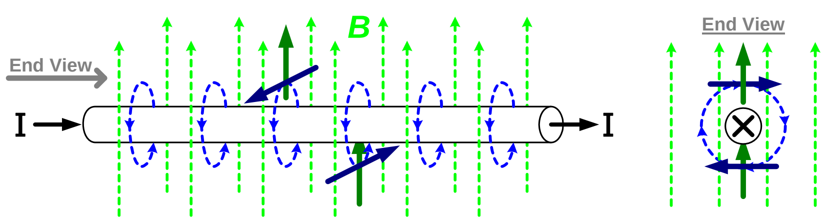
Thus, the **conductor will be repelled away from that location**.



# Field Interactions
## Separately-Sourced Fields

Note that the field lines in the areas “**above**” and “**below**” the conductor are **orthogonal**…

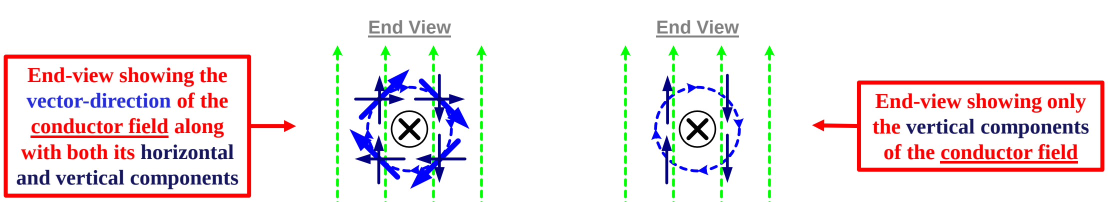
Thus, there is **no force** induced upon the conductor due to field interaction in these areas.




# Field Interactions
## Separately-Sourced Fields

At other locations around the conductor, the **vector-direction** of the **conductor-field has both horizontal and vertical components**.

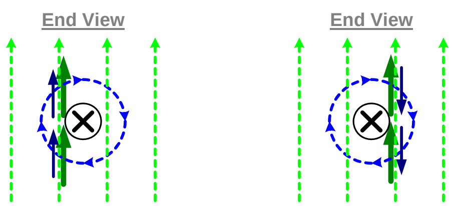
Only the vertical components will be considered since they are parallel to the external field.



# Field Interactions
## Separately-Sourced Fields

It is easiest to consider these locations in pairs…

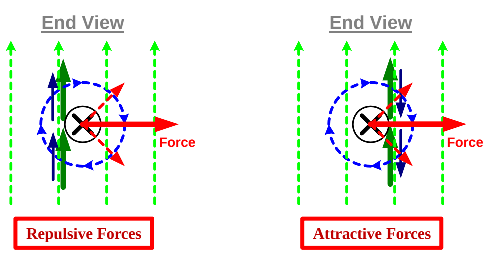
The **vertical components of the fields** are pointing in the same direction on one side of the conductor and in opposite directions on the other side of the conductor.




# Field Interactions
## Separately-Sourced Fields

**Forces** are induced upon the conductor due to the field interactions at each location…

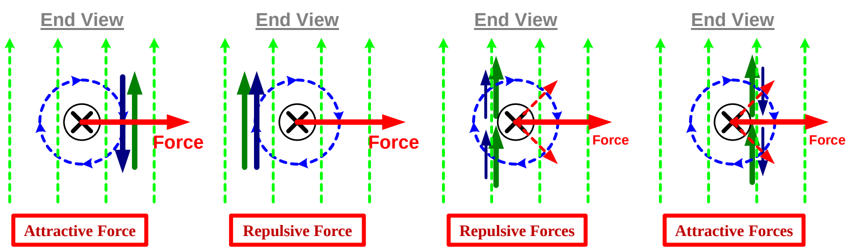
Since the forces are also vectors, each force-pair may be summed, with the following results:



# Field Interactions
## Separately-Sourced Fields

As can be seen, all of the field-interactions result in a **net force** being induced upon the conductor <u>in the same direction</u>.

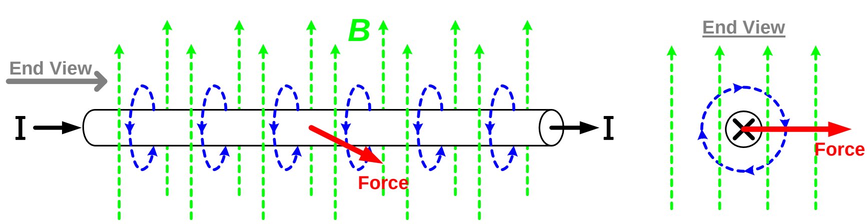
*(Note that the field-interactions "above" and "below" the conductor are not shown because the resulted in no additional force upon the conductor)*




# The Lorentz Force Law
## Applied to Linear Conductors

Thus, an **overall force** will be induced upon the conductor in a **direction** that is <u>orthogonal to both the external field and the direction of current-flow</u>, the **magnitude** of which is defined by:

$$F = B \cdot l \cdot I$$



# Rotating Electric Machines
## Motor Operation

A typical **rotating machine** contains a cylindrical rotor that is attached to a shaft, the ends of which are supported by bearings that allow the shaft to rotate.

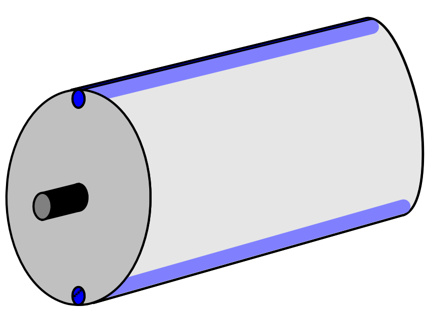
During **motor operation**, an electric source provides the current necessary to induce a **torque** (***rotational force***) upon the rotor that "tries" to accelerate (rotate) the rotor.

This **concept** is easily explained using two current-carrying conductors attached to a cylindrical rotor that is exposed to an externally-sourced magnetic field.

# Rotating Electric Machines
## Motor Operation

Given the following **cylindrical rotor** in which **two conductors** are embedded within the surface;

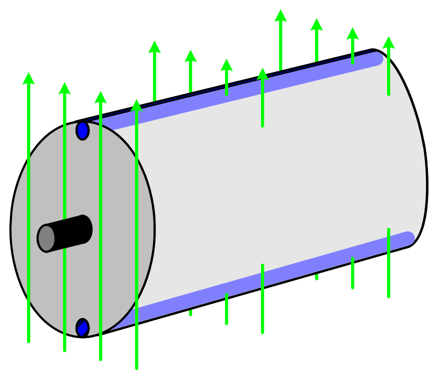
one lengthwise along the top and the other along the bottom…



# Rotating Electric Machines

## Motor Operation

Assume that the rotor is within a region that contains a uniform, **externally-sourced magnetic field**, the lines of which are all vertically oriented through the rotor.

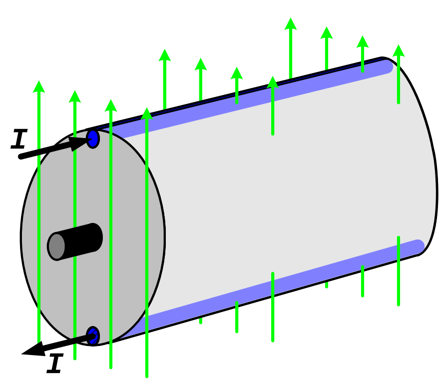


Note that the source of the external field is not shown in the figure

# Rotating Electric Machines

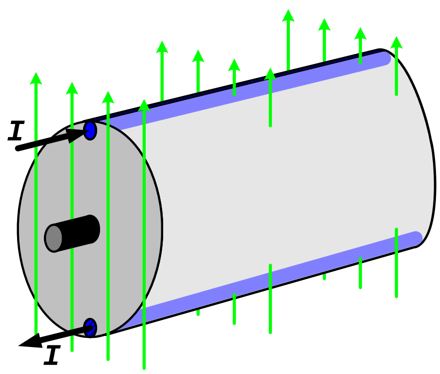
## Motor Operation

If currents flow in opposite directions through the conductors, then the **Lorentz Force Law** can be applied to determine the forces induced upon the conductors and, in-turn, the entire rotor.



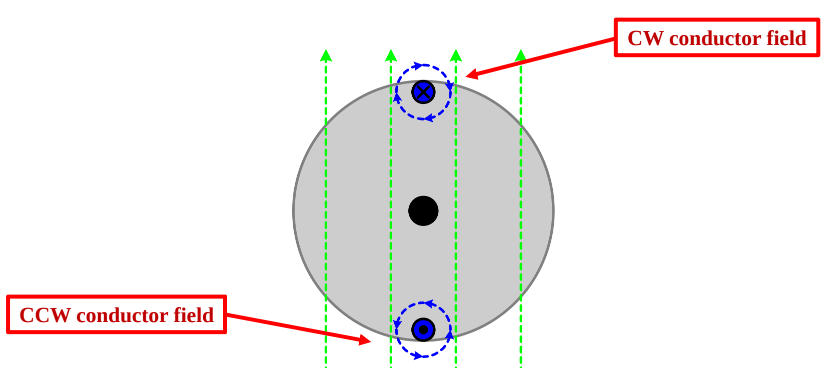
# Rotating Electric Machines
## Motor Operation

Of key importance is the **direction of the forces** induced upon the conductors. Thus, we will begin by examining the interactions between the conductor fields and the external field.



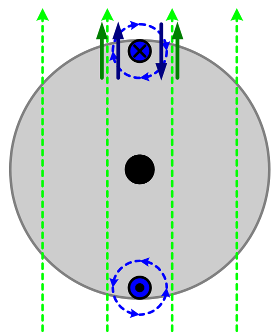

# Rotating Electric Machines
## Motor Operation

Since the conductor currents are flowing in opposite directions, the c**onductor-fields** that form in closed-loop paths around the conductors will point in opposite directions.



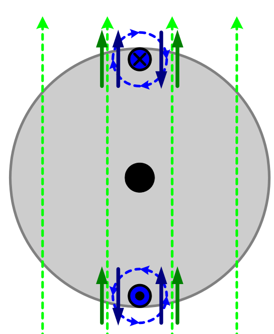
# Rotating Electric Machines
## Motor Operation

As shown below, the vector-directions of the **conductor field and the external field** are **parallel** in the same direction to the left of the upper conductor and in opposite directions to the right.



# Rotating Electric Machines
## Motor Operation

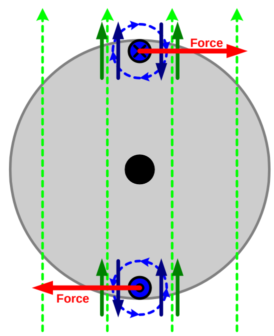
The conductor field of the lower conductor and the external field have the opposite directional relationships compared to those of the upper conductor.



# Rotating Electric Machines

## Motor Operation

Based upon the <u>field interactions</u>, a **force** will be induced on the <u>upper conductor pointing to the right</u>, while a **force** will be induced on the <u>lower conductor pointing to the left</u>.

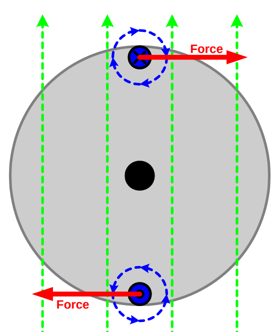



# Rotating Electric Machines

## Motor Operation

Although the forces point in opposite directions, if the same current flows in both conductors, then the <u>magnitude</u> of the **force** induced on each conductor is defined by:

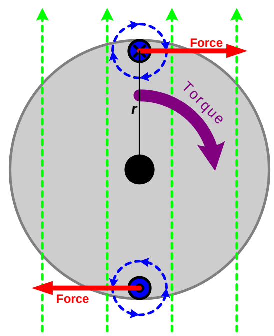
$$F = B \cdot l \cdot I$$



# Rotating Electric Machines
## Motor Operation

Furthermore, since they point in opposite directions with respect to rotation of the cylindrical rotor, both forces result in a net **clockwise torque** (rotational force) being developed on the rotor.

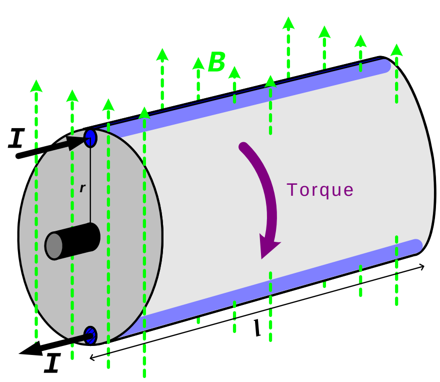
$$T = 2 \cdot F \cdot r$$




# Rotating Electric Machines
## Motor Operation

Thus, the currents flowing in the conductors result in a **torque** to be developed upon the rotor:

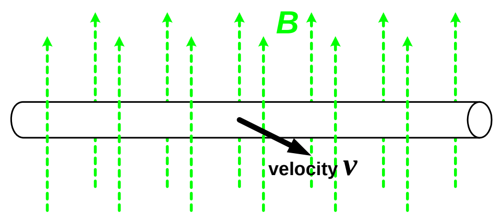
➡ **Motor Operation**




$$T = 2 \cdot B \cdot l \cdot I \cdot r$$

## Faraday’s Law of Induction
### Applied to Linear Conductors

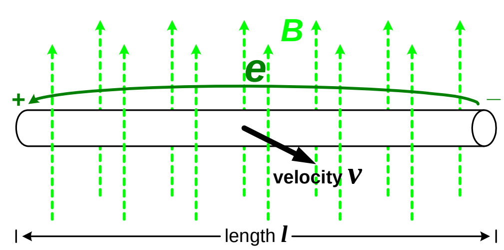
Once again, let’s consider a **linear conductor** in a region that contains an **externally-sourced magnetic flux**, but this time assume that the conductor is moving orthogonally through the field with velocity ***v***.




## Faraday’s Law of Induction
### Applied to Linear Conductors

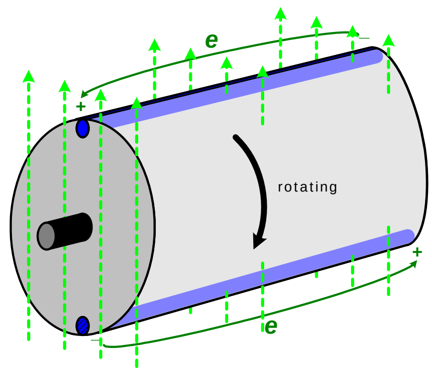
Based upon **Faraday’s Law of Induction**, if the conductor is moving orthogonally through the field, then a voltage will be induced across the conductor, the magnitude of which is defined by:

$$e = B \cdot l \cdot v$$



# Rotating Electric Machines
## Generator Operation

Similarly, if a pair of conductors are embedded within the surface of a rotor that is being rotated by some external means while exposed to a linear external magnetic field…




# Rotating Electric Machines
## Generator Operation

Then, a **voltage** will be induced across the conductors that is proportional to the rotational speed of the rotor.

➡ **Generator Operation**