ECET 3410

High Frequency Systems

Complex Number Calculations

on the TI-83/84/89 and the Casio fx-115/991

Equations Containing Complex Numbers

In **High Frequency Systems**, you will be expected to solve some difficult equations that contain **complex numbers**, such as:

$$\tilde{E}(x) = (E_0^+ \angle \theta_0^+) \cdot e^{-(\alpha+\beta i)\cdot x} + \left(\frac{(R_R + X_R i) - Z_0}{(R_R + X_R i) + Z_0} \right) \cdot (E_0^+ \angle \theta_0^+) \cdot e^{-2\cdot(\alpha+\beta i)\cdot L} \cdot e^{+(\alpha+\beta i)\cdot x}$$

Although these equations can be solved by means of a scientific calculator, this task can still be quite challenging.

This presentation will <u>not</u> cover the mathematical theory required to solve such equations. Instead, it will attempt to provide guidance regarding the use of scientific calculators in order to perform this function.

Calculators & Complex Numbers

Despite the fact that most scientific and/or graphing calculators can perform **complex number** calculations, getting the various calculators to correctly perform even "simple" calculations can be tricky.

Furthermore, many of the calculators have their own particular requirements or limitations for the manner in which complex number equations can be entered, making it hard to create a generic presentation on this topic.

Because of this, we will focus on the following platforms:

TI-83/84/89

Casio fx-115/991*

*** - Although not presented, the TI-36 will perform similar to the Casio calculators.**

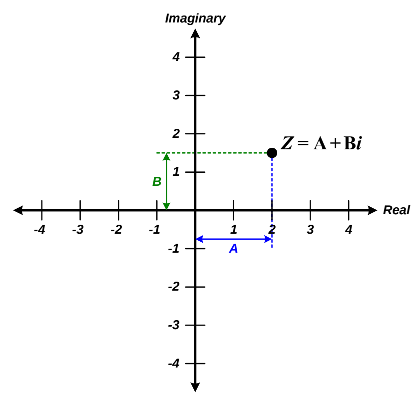
Quick Review of Complex Numbers

A **complex number**, expressed in **rectangular coordinates**:

$$Z = A + Bi$$

where: ***A*** is the "real" part of ***Z***, and ***B*** is the "imaginary" part of ***Z***,

can be used to represent the point on a rectangular coordinate plane when the **x-axis** defined as the **real** axis and the **y-axis** defined as the **imaginary** axis of the plane.



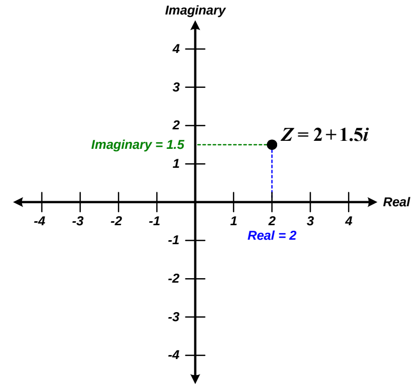
Quick Review of Complex Numbers

For example, the **complex number**

$$Z = 2 + 1.5i$$

is shown on the plot to the right, such that:

- the value of the point projected onto the **real** axis is **2**, and
- the value of the point projected onto the **imaginary** axis is **1.5**.



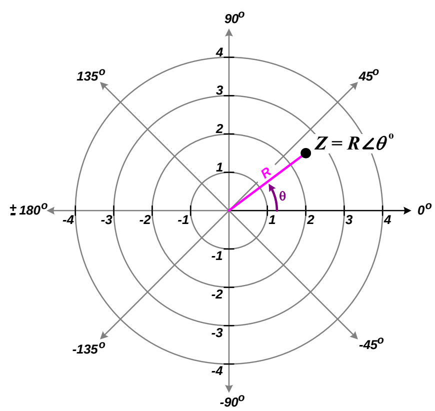
Quick Review of Complex Numbers

Similarly, a **complex number** that is expressed in **polar coordinates**:

$$Z = R\angle\theta°$$

where: ***R*** is the **distance** to the origin, and
θ is **angle** formed between the right half of the horizontal axis and a line passing through both the origin and point ***Z***,

can be used to represent a point on a polar coordinate plane.



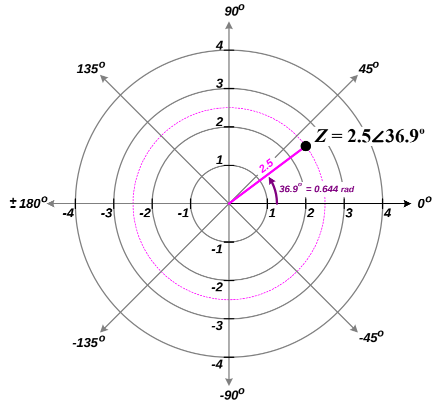
Quick Review of Complex Numbers

For example, the **complex number**

$$Z = 2.5\angle 36.9^\circ$$

is shown on the plot to the right, such that:

- the **distance** of point ***Z*** to the origin is **2.5**, and
- the **angle** formed between the right half of the horizontal axis and a line passing through both the origin and the point ***Z*** is **36.9°**.



Note that an angle of 36.9° is equivalent to an angle of 0.644radians since:

$$\text{radians} = \text{degrees} \cdot \frac{\pi}{180}$$

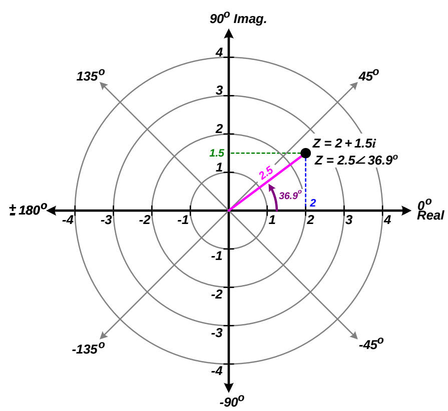
Rectangular Form ⇔ Polar Form

If the polar coordinate plot is overlaid on top of the rectangular coordinate plot, it can be seen that the values:

$$Z = 2 + 1.5i \qquad Z = 2.5\angle 36.9^\circ$$

relate to the same point.

Although there are a simple set of equations available to convert a complex number from one form into the other form, we will utilize a calculator to perform this task when necessary.



Polar Form vs. Exponential Form

It turns out that the expression of a **complex number** in **polar form**, such as:

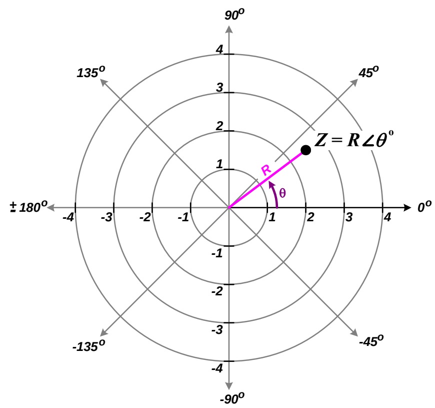
$$Z = R\angle\theta°$$

is just a shorthand notation for the **exponential form** of that number.

When expressed in **exponential form**, the complex number ***Z*** is written as:

$$Z = Re^{\theta i}$$

provided that the angle is expressed in **radians** instead of degrees.



Exponential form is technically the (mathematically) "proper" way to express a complex number in terms of a magnitude and an angle.

Complex #s in Electrical Engineering

Complex numbers are often utilized to represent quantities within electrical engineering, such as the **phasor** value of a sinusoidally-varying voltage:

$$v(t) = 100 \cdot \sin(377t + 30°) \iff \tilde{V} = \underbrace{70.7e^{j\frac{\pi}{6}}}_{\text{exponential form, angle in radians}} = \underbrace{70.7\angle 30°}_{\text{polar form, angle in degrees}} \text{ volts}$$

or the **impedance** of a 40Ω resistor connected in-series with a 79.6mH inductor when connected to a voltage source having the above defined voltage:

$$Z = R + j\omega L = 40 + j(377)(0.0796) = \underbrace{40 + j30}_{\text{rectangular form}}\ \Omega$$

Note that the lower-case "*j*" shown in the above examples is typically used in electrical engineering to represent the imaginary number "*i*", where:

$$j \equiv i = \sqrt{-1}$$

Complex #s in Electrical Engineering

But, looking back at the equation:

$$\tilde{E}(x) = (E_0^+ \angle \phi_0^+) \cdot e^{-(\alpha+\beta i)\cdot x} + \left(\frac{(R_R + X_R i) - Z_0}{(R_R + X_R i) + Z_0} \right) \cdot (E_0^+ \angle \phi_0^+) \cdot e^{-2\cdot(\alpha+\beta i)\cdot L} \cdot e^{+(\alpha+\beta i)\cdot x}$$

it can be seen that the equation contains complex numbers expressed in **rectangular**, **polar** and **exponential** forms.

This presents a problem because, while all of the calculators under consideration allow complex numbers to be expressed in rectangular form, most of the calculators only allow the numbers to be expressed in either polar or exponential form.

Thus, entering the equation in these calculators can be problematic.

Calculator Differences – Format

Given the following **complex number *Z*** (shown in various formats):

$$Z = \underset{\text{rectangular form}}{4+3i} = \underset{\text{polar form, angle in degrees}}{5\angle 36.9^\circ} = \underset{\text{polar form, angle in radians}}{5\angle 0.644\text{rad}} = \underset{\text{exponential form, angle in radians}}{5e^{i(0.644\text{rad})}}$$

these are all different versions of the same number

All of the **TI** and **Casio** platforms allow complex numbers to be entered in **rectangular form**.

The **TI-89*** and **Casio (115/991)** calculators allow complex numbers to be entered in **polar form** (**degrees** or **radians**).

* - TI-83/84s do not have the "∠" key on their keypads.

The **TI (83/84/89)** platforms allow the numbers to be entered in **exponential form**, but only with **angles** defined in **radians**.

Note – some **older TI-89s** (and the TI-85) even allow the number to be entered in **exponential form** with the **angle in degrees** instead of radians.

Complex Number Mode and Format

To perform complex number calculations on any of the calculators, they must first be set to "**complex number**" **mode**.

You must also choose the **format*** in which the calculator displays the complex number results, either:

* - the format determines how all final results will be displayed

a+**b*i*** (rectangular form) — useful when calculating complex impedances ($Z = R + jX$)

r∠**θ** (polar form)

re^θ*i* (exponential form)

useful when calculating phasor voltages and currents

both options may not be available

Additionally, all of the calculators allow the user to set the **units** by which angles are defined:

degrees, radians, or **gradians**. — not available on all calculators

(The **unit** setting can affect the calculator's ability to perform complex number calculations.)

Initial Settings on a TI Calculator

To set the **mode** of the calculator to **complex numbers**, with the results displayed in **rectangular form**, on a **TI-84***:

* - the TI-83/89s should configure similarly

- Press the **MODE** button. (A configuration menu will appear.)
- The following options will be available on one of the lines:

  the default setting is REAL — **REAL a**+**b*i*** **re^θ*i***

  Use the arrow keys to move down and highlight **a**+**b*i***, and then press the **ENTER** button. This simultaneously sets the calculator to ***complex number mode*** and ***rectangular format***.
- In the same menu, change the angle unit to **RADIAN** if degree is currently highlighted.
- Press **2ND** ⇒ **QUIT** to exit the configuration menu.

Initial Settings on a Casio Calculator

To set the **mode** of the calculator to **complex numbers**, with the results displayed in **rectangular form**, on a **<u>Casio 991</u>***:

* - the Casio 115 should configure similarly

- Press the **MENU** button. (A configuration menu will appear.)
- Press the **2** key [<u>or</u> use the arrow keys to highlight the second menu option i∠2 (2:Complex) and then press the = key]. This sets the calculator to ***complex number mode***.

  the default setting is option 1 (1:Calculate)

- Press **SHIFT** ⇒ **SETUP**. (A different configuration menu will appear.)
- Press the **down arrow** to scroll down to the 2nd page of options. Press the **2** key to select "**2:Complex**" and press the **1** key to select "**1:a+bi**". This sets the ***format*** to ***rectangular***.
- Similarly, press **SHIFT** ⇒ **SETUP**, press the **2** key to select "**2:Angle Unit**" and press the **2** key to select "**2:Radian**". This sets the angle unit.

Complex Numbers in Rectangular Form

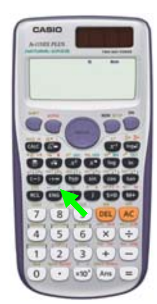
All of the calculators allow a **complex number** to be entered in rectangular form by using the ***i*** on their keypads:

$$4 + 3i$$

press 2ND ⇒ *i*
(2ND ⇒ . key)

press 2ND ⇒ *i*
(2ND ⇒ . key)

press 2ND ⇒ *i*
(2ND ⇒ CATALOG key)



the ENG key defaults to *i* when the mode is set to Complex

the ENG key defaults to *i* when the mode is set to Complex

Complex Numbers in Rectangular Form

All of the calculators allow a **complex number** to be entered in rectangular form by using the ***i*** on their keypads:

$$4+3i$$

Although not required on all of the calculator platforms, it is good practice to **enclose all complex numbers in parentheses** when entering the numbers into a calculator:

$$(4+3i)$$

Additionally, the TIs allow the ***i*** to be entered before or after the 3, while the Casio calculators <u>only</u> accept the ***i*** <u>after</u> the 3.

Note – if entered as (4 + ***i***3), the TIs will redisplay the number as (4 + 3***i***) while the Casios will display the message "Syntax ERROR".

Rectangular Form – Example #1

Entry the following into your calculator and press **ENTER** (TI) or = (Casio) to display the result:

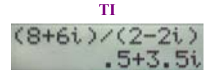
$$\frac{8+6i}{2-2i}$$

The **correct result** is:

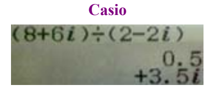
$$0.5+3.5i$$

If the Casio displays the result as: $\frac{1}{2}+\frac{7}{2}i$ press the S⇔D key to display decimals.

TI



Casio



which is achieved by entering $(8+6i)\div(2-2i)$.

But, if you entered the expression as $8+6i\div2-2i$ <u>without parentheses</u> around the complex numbers, your calculator will display the <u>incorrect result</u>:

$$8+i$$

The calculator evaluates this expression as: $8+\frac{6i}{2}-2i$

Rectangular Form – Example #2

Try to get the correct result for the following expression:

$$\frac{(3-4i)\cdot(1-i)}{(2+3i)-(3+2i)}$$

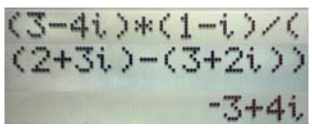
The **correct result** is:

$$-3+4i$$

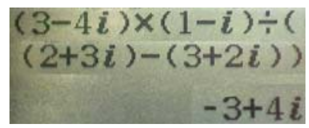
which is achieved by entering:

$$\big((3-4i)\cdot(1-i)\big)\div\big((2+3i)-(3+2i)\big)$$

TI



Casio



Polar Form and Exponential Form

There are vast differences in how the calculators handle complex numbers expressed in **polar form** and **exponential form**:

$$R\angle\theta \qquad Re^{\theta i}$$

The **TI-89** and **Casio 115/991** calculators allow complex #s to be entered in **polar form** with angles in either **degrees** or **radians**, but the **TI 83/84** do not allow numbers in polar form.

On the other hand, the **TI-83/84/89s** allow the numbers to be entered in **exponential form** (with the angle in **radians**), but the **Casio 115/991s** do not allow this form.

Thus, equations containing both forms may have to be rewritten to allow for entry in a calculator that only accepts one form.

Polar Form and Exponential Form

Note that the equation:

Incident Voltage on a transmission line as a function of position

$$\tilde{E}^+(x) = (E_0^+ \angle \phi_0^+) \cdot e^{-(\alpha+\beta i)\cdot x}$$

contains both a complex number in polar form and what appears to be a complex number expressed in exponential form.

To enter the equation into the **TI 83/84**, the polar must also be expressed in **exponential form**. And, if ϕ_0^+ is given in degrees, a unit <u>conversion</u> factor to **radians** must be included:

$$(E_0^+ \angle \phi_0^+) \quad \Rightarrow \quad \left(E_0^+ e^{\left(\phi_0^+ \cdot \frac{\pi}{180}\right)i}\right)$$

Thus:

$$\tilde{E}^+(x) = \left(E_0^+ e^{\left(\phi_0^+ \cdot \frac{\pi}{180}\right)i}\right) \cdot e^{-(\alpha+\beta i)\cdot x}$$

The TI-89 can utilize either version provided that the angle is in radians.

Polar Form and Exponential Form

But, the equation:

Incident Voltage on a transmission line as a function of position

$$\tilde{E}^+(x) = (E_0^+ \angle \phi_0^+) \cdot e^{-(\alpha+\beta i)\cdot x}$$

also contains a term that has a complex number in an exponent, which can be rewritten as follows:

$$e^{-(\alpha+\beta i)\cdot x} = e^{-\alpha x} \cdot e^{-\beta x i} = (e^{-\alpha x} \angle -\beta x)$$

such that the angle ***–βx*** is expressed in radians.

To enter the equation into the **Casio 115/991**, the angle must be converted to radians, thus:

$$\tilde{E}^+(x) = (E_0^+ \angle (\phi_0^+ \cdot \tfrac{\pi}{180})) \cdot (e^{(-\alpha \cdot x)} \angle (-\beta \cdot x))$$

The TI-89 can utilize either version provided that the angle is in radians.

Polar/Exponential Form – Example #1

Determine the phasor value of the **incident voltage** $\tilde{E}^{+}(5)$ that arrives at the receiving-end of a **5-meter** long transmission line if the initial incident voltage applied to the sending-end of the line is:

$$10\angle 45^{\circ} \text{ volts}$$

and the propagation constant for the transmission line is:

$$\gamma = \alpha + j\beta = 0.03 + j1.65$$

Be sure to change your calculator from rectangular form to either polar or exponential form: r∠θ or re^θi

Original Equation

$$\tilde{E}^{+}(x) = (E_0^{+}\angle\phi_0^{+})\cdot e^{-(\alpha+\beta i)\cdot x}$$

TI-83/84 version

$$\tilde{E}^{+}(x) = \left(E_0^{+} e^{\left(\phi_0^{+}\cdot\frac{\pi}{180}\right)i}\right)\cdot e^{-(\alpha+\beta i)\cdot x}$$

Casio 115/991 version

$$\tilde{E}^{+}(x) = (E_0^{+}\angle(\phi_0^{+}\cdot\tfrac{\pi}{180}))\cdot(e^{(-\alpha\cdot x)}\angle(-\beta\cdot x))$$

The **correct answer** is: $8.61\angle -1.18\text{rad} = 8.61e^{-1.18i}$ volts

Polar/Exponential Form – Example #1

Original Equation

$$\tilde{E}^{+}(x) = (E_0^{+}\angle\phi_0^{+})\cdot e^{-(\alpha+\beta i)\cdot x}$$

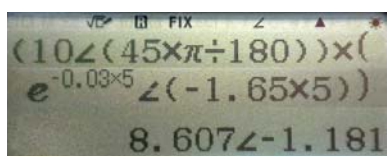
$x = 5$ meters $\quad \gamma = \alpha + j\beta = 0.03 + j1.65 \quad \tilde{E}_0^{+} = 10\angle 45^{\circ}$ volts

TI-83/84 version

$$\tilde{E}^{+}(x) = \left(E_0^{+} e^{\left(\phi_0^{+}\cdot\frac{\pi}{180}\right)i}\right)\cdot e^{-(\alpha+\beta i)\cdot x}$$

Casio 115/991 version

$$\tilde{E}^{+}(x) = (E_0^{+}\angle(\phi_0^{+}\cdot\tfrac{\pi}{180}))\cdot(e^{(-\alpha\cdot x)}\angle(-\beta\cdot x))$$



$$\tilde{E}^{+}(5) = 8.61\angle -1.18\text{rad} = 8.61e^{-1.18i} \text{ volts}$$