ECET 3410

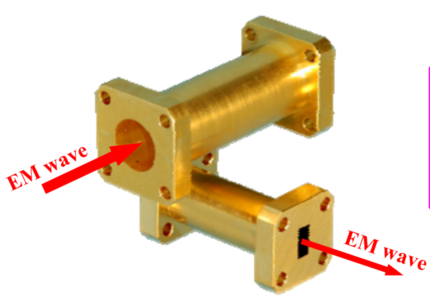
High Frequency Systems

Waveguides

Waveguides

A **waveguide** is a hollow physical structure that is used to convey high-frequency electromagnetic waves from one end to the other within the open space at the center of the waveguide.

Waveguides are typically constructed from a hollow section of copper, brass, or aluminum… similar to a metal pipe.

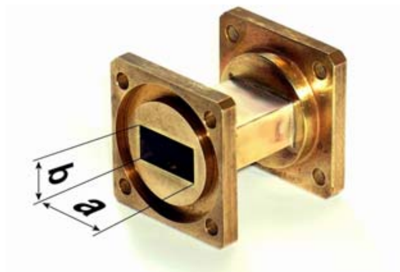


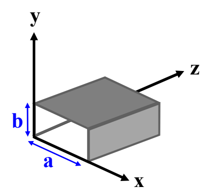
Waveguides are available in many different shapes and sizes and are selected based on the application and the required operational frequency.

Rectangular Waveguides

The most common type of guide is the **rectangular waveguide**, so named because of its shape.

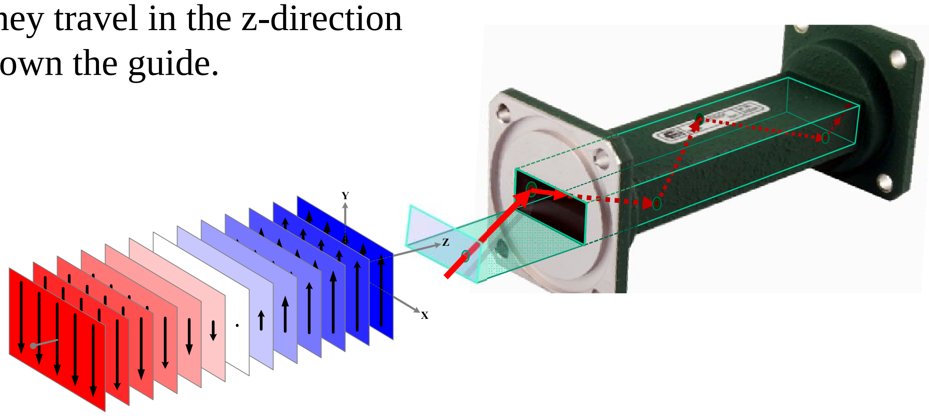
The **rectangular waveguide** is characterized by the size of the opening, such that "**a**" is the **length of the wider dimension** and "**b**" is the **length of the narrower dimension**.





Wave Propagation within a Waveguide

For the case of the rectangular waveguide, it is possible to base an analysis of the waveguide's fundamental operation on the concept of **plane waves** that reflect repeatedly between the side-walls as they travel in the z-direction down the guide.

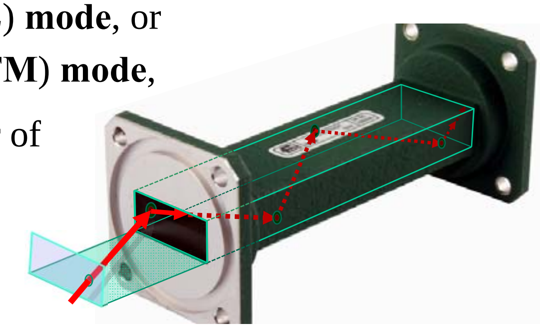


Modes of Propagation

But, unlike plane waves traveling through an open medium or waves traveling down a two-wire transmission line, both of which propagate in the TEM mode of (E⊥H⊥prop.direction), **waves within a waveguide** propagate in either:

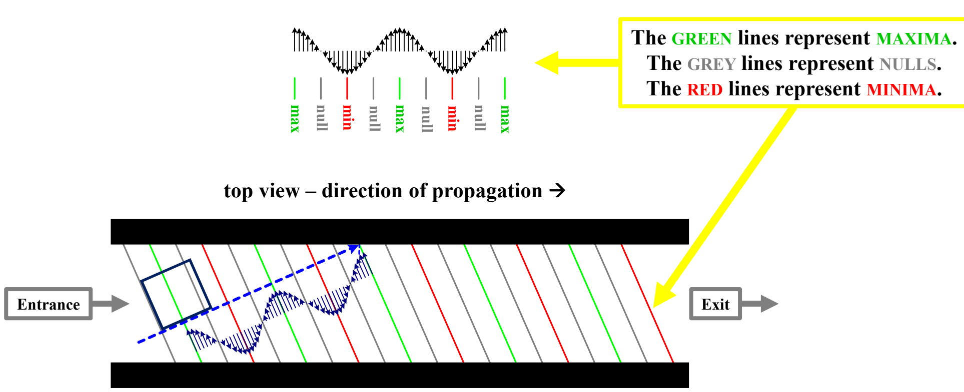
- the **Transverse Electric** (**TE**) **mode**, or
- the **Transverse Magnetic** (**TM**) **mode**,

as they travel through the center of the waveguide.



Wave Propagation within a Waveguide

If a "plane wave" is propagating (upward to the right) within the center region of a waveguide such that it is **incident** upon the side wall of the waveguide at some angle θ (from the ⊥)…

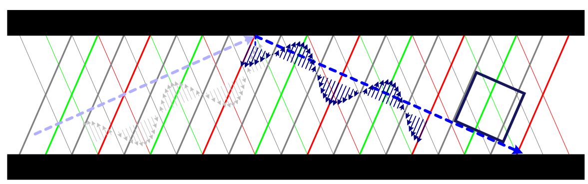


Wave Propagation within a Waveguide

If a "plane wave" is propagating (upward to the right) within the center region of a waveguide such that it is incident upon the side wall of the waveguide at some angle θ (from the ⊥)…

then the wave will **reflect** off the side wall at the same angle θ.

top view – direction of propagation →



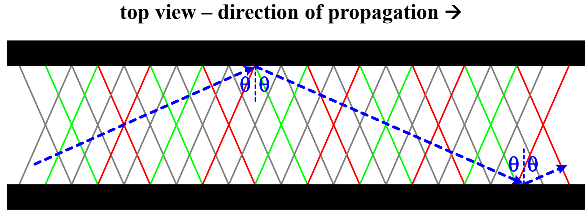
The angle of incidence will be equal to the angle of reflection.

Wave Propagation within a Waveguide

The **reflected** "plane wave" will continue propagating through the center region of a waveguide, but towards the opposite sidewall (downward to the right) at the same angle θ (from the ⊥).

Note that, when the wave hits the opposite sidewall, another reflection will occur such that the second reflection will be traveling in the same direction as the original incident wave.

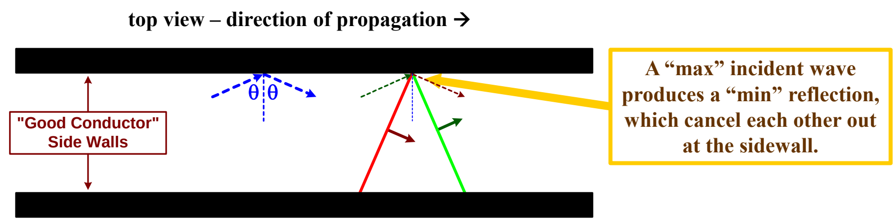
top view – direction of propagation →



Wave Propagation within a Waveguide

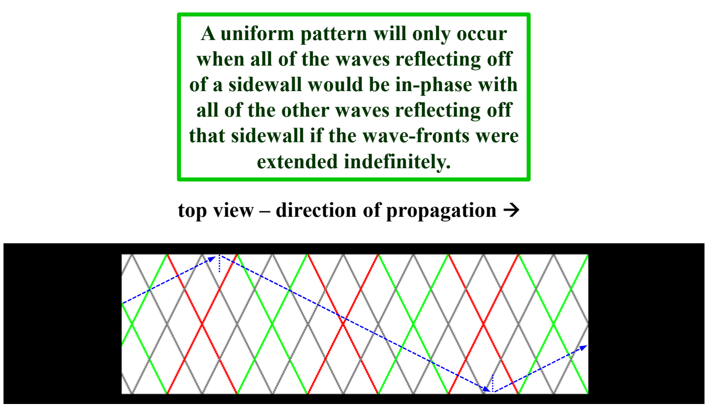
Assuming that the side walls of the waveguide are composed of an "good conductor", then the reflection coefficient for the sidewalls will be $\boldsymbol{\Gamma_R = -1}$.

Thus, when an incident wave hits a sidewall, a negative reflection will occur ($\mathbf{E^- = -E^+}$), resulting in a net field of zero at that location, since: $\mathbf{E_{sidewall} = E^+ + E^- = 0}$



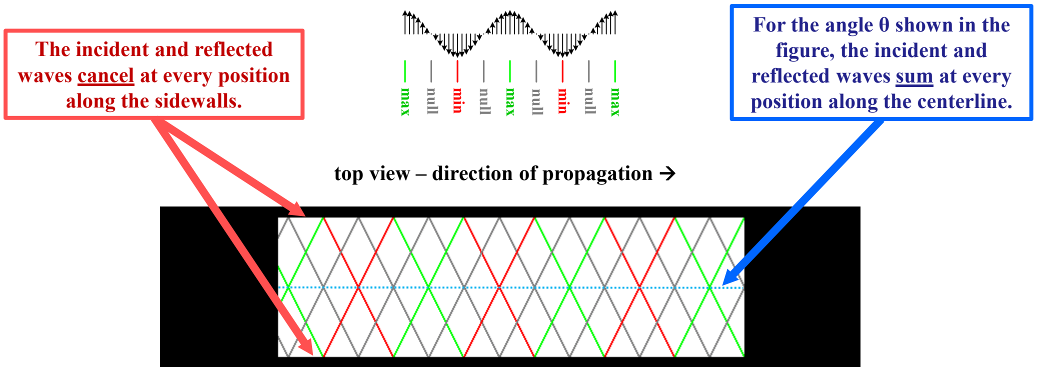
Wave Propagation within a Waveguide

At specific angles of θ, a **uniform field pattern** will occur within the waveguide as the plane waves continue to bounce back and forth between the sidewalls while traveling down the waveguide.



Wave Propagation within a Waveguide

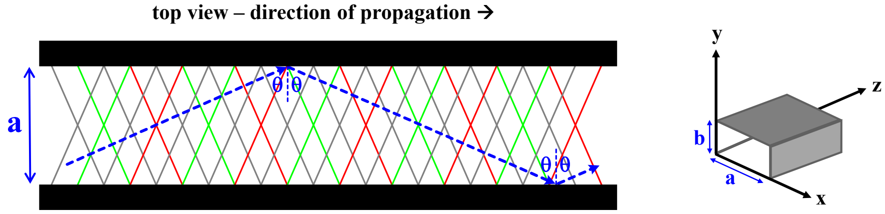
At specific angles of θ, a **uniform field pattern** will occur within the waveguide as the plane waves continue to bounce back and forth between the sidewalls while traveling down the waveguide.



Modes of Propagation

It turns out that energy will only propagate efficiently down the length of a guide while in a uniform field pattern, and that there are only a **finite number of θ angles** at which this will occur.

These angles relate to the various **modes of propagation** for which a wave is able to propagate within the waveguide, and they are directly related to the inner dimensions of the guide.



Modes of Propagation

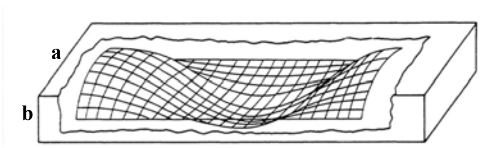
The different **modes** are characterized by two indexes, **m** and **n**, which are added to the type designation (**TE** or **TM**) to denote the specific mode that is being propagated:

$\mathbf{TE_{m,n}}$ and/or $\mathbf{TM_{m,n}}$

where: **m** denotes the number of **½-sine variations** of the transverse field in the "**a**" dimension, and

n denotes the number of **½-sine variations** of the transverse field in the "**b**" dimension.

$TE_{1,0}$ mode ➔

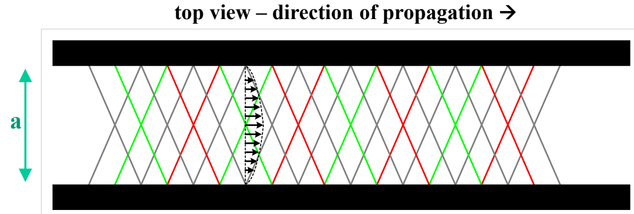


TE_{10} Mode of Propagation

The **lowest-frequency** (and most popular) **mode** is the **TE_{10} mode**, which is also called the **dominant mode**.

For the **TE_{10} mode**, there is **one (1) ½-sine variation** of the electric field in the "**a**" dimension and **zero (0) ½-sine variations** of the electric field in the "**b**" dimension.

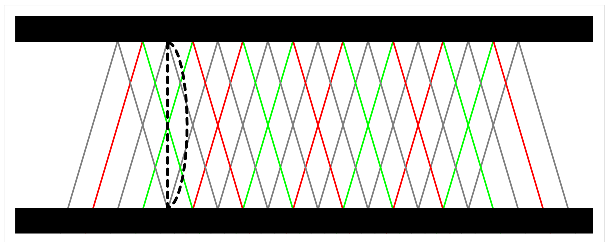
top view – direction of propagation →



TE_{10} Mode

Note that, depending of the wavelength of the plane waves and the width of the waveguide, other higher-order modes may possibly setup besides the **TE_{10} mode**;

top view – direction of propagation →

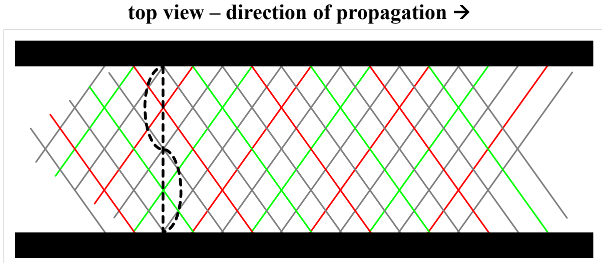


Higher-Order Modes

Note that, depending of the wavelength of the plane waves and the width of the waveguide, other **higher-order modes** may possibly setup besides the TE_{10} mode;

These modes include the **TE_{20} mode** of propagation…

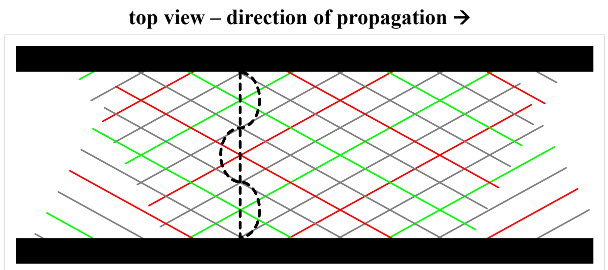
top view – direction of propagation →



Higher-Order Modes

Note that, depending of the wavelength of the plane waves and the width of the waveguide, other **higher-order modes** may possibly setup besides the TE_{10} mode;

These modes include the TE_{20} mode of propagation as well as the **TE_{30} mode** of propagation.



Higher-Order Modes

A waveguide can potentially support multiple propagation modes.

It is desired to only propagate **one mode** because it is difficult to couple energy into and out of the guide for different modes.

Usually, waveguides are operated only within the range of frequencies for which only the lowest-frequency, or **dominant, mode**, may propagate.

This is done by carefully choosing the dimensions of rectangular waveguides based on the desired frequency of operation.

Cutoff-Frequency

With respect to a mode of propagation, the **cutoff frequency** is the theoretical frequency, above which, a specific mode is able to propagate within a waveguide.

The cutoff frequency for a mode (having order **m,n**) is a function of both the order of the mode and the waveguide's dimensions.

Thus, given a waveguide having dimensions "**a**" and "**b**", the cutoff frequency for any specific mode may be determined.

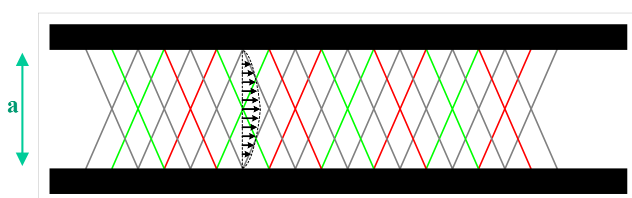
TE_{10} in Air-Filled Rectangular Waveguide

For the TE_{10} mode in a rectangular, air-filled waveguide, the **cutoff frequency** is:

$$f_{c10} = \frac{c}{\lambda_c} = \frac{c}{2a}$$

and the **cutoff wavelength** is:

$$\lambda_{c10} = \frac{c}{f_{c10}} = 2a$$



$TE_{m,n}$ Higher-Order Modes

The general expression for the **$TE_{m,n}$ cutoff frequency** in an air-filled rectangular waveguide is:

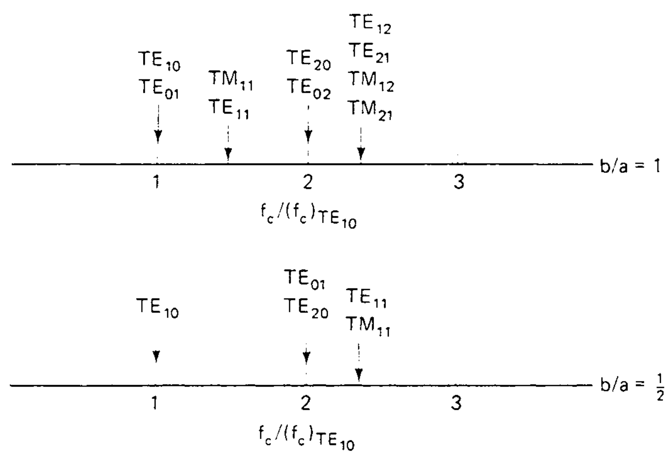
$$f_{cm,n} = \frac{1}{2\cdot\sqrt{\mu\cdot\varepsilon}}\cdot\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

Thus, the general expression for the **$TE_{m,n}$ wavelength** is:

$$\lambda_{c10} = \frac{c}{f_{cm,n}} = ???$$

$TE_{m,n}$ Higher-Order Modes

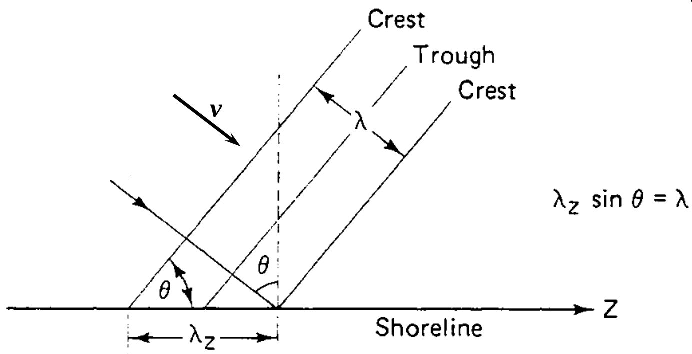
The following figure shows the **cutoff frequencies** for various modes in rectangular waveguides with two different ratios of b/a.



Phase and Group Velocity

Consider a water wave approaching a shoreline at an angle $\boldsymbol{\theta}$ as measured from the normal to the shoreline.

The approaching wave has **propagation velocity**: $v = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}} = \lambda \cdot f$



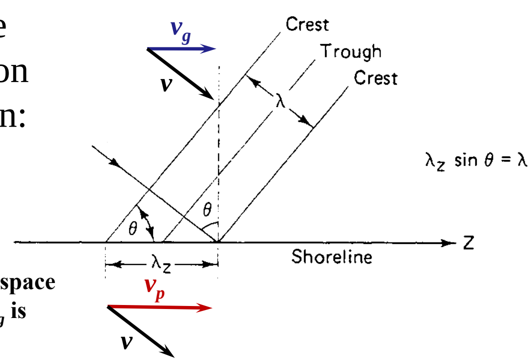
Phase and Group Velocity

Along the shoreline, the distance between the wave crests λ_z creates an apparent **phase velocity** $\boldsymbol{v_p}$:

$$v_p = \lambda_z \cdot f = \frac{\lambda}{\sin\theta} \cdot f = \frac{v}{\sin\theta}$$

whereas the **group velocity** $\boldsymbol{v_g}$ is the actual velocity of energy propagation along the shoreline in the **z** direction:

$$v_g = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \sin\theta$$



Note that the phase velocity v_p is greater than the free space propagation velocity v and that the group velocity v_g is always less than free space velocity v.

Phase and Group Velocity

Interestingly,

$$v_p \cdot v_g = \left(\frac{c}{\sqrt{\mu_r \varepsilon_r}} \right)^2$$

The TE_{10} waveguide **phase velocity v_p** as a function of guide **cutoff frequency f_c** is:

$$v_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \cdot \frac{1}{\sqrt{1-\left(f_c / f\right)^2}}$$

and the **guide wavelength λ_g** is:

$$\lambda_g = \frac{v_p}{f} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \cdot \frac{1}{\sqrt{f^2 - f_c^2}}$$

TE_{10} Mode of Propagation

From TE_{10} mode equations:

- No x and z components of E field
- y component of **E** field varies sinusoidally along x dimension
  - Value is 0 at waveguide boundaries x=0, x=a
- Traveling wave in z direction with phase constant β_g related to guide wavelength λ_g

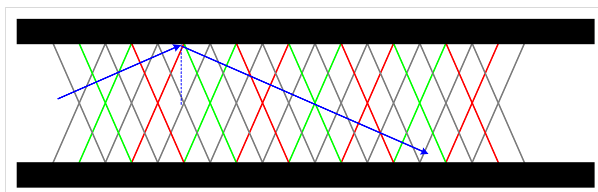
$$\beta_g = \frac{2\pi}{\lambda_g}$$

- Guide wavelength and free-space wavelength are <u>not</u> equal

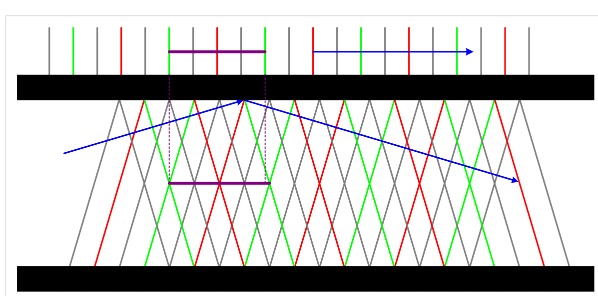
Phase and Group Velocity

Below the cutoff frequency, there is **no wave propagation** in the guide:

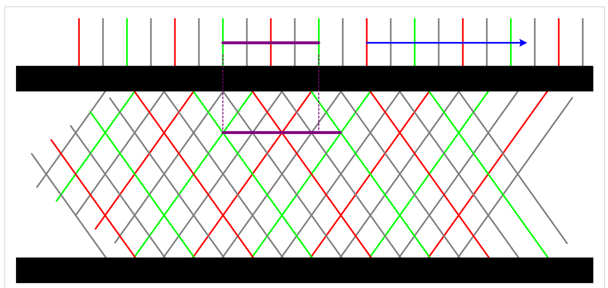
- As f decreases, the zig-zag angle θ decreases toward 0°
- Also λ_g approaches ∞ when f decreases toward f_c



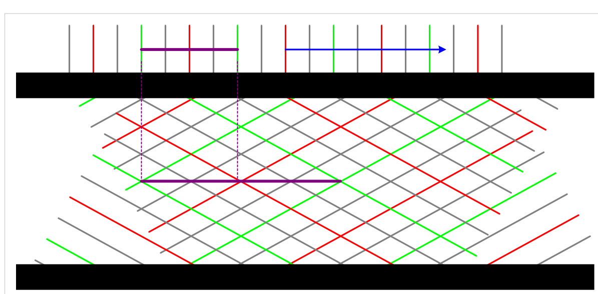
TE_{10} Mode of Propagation



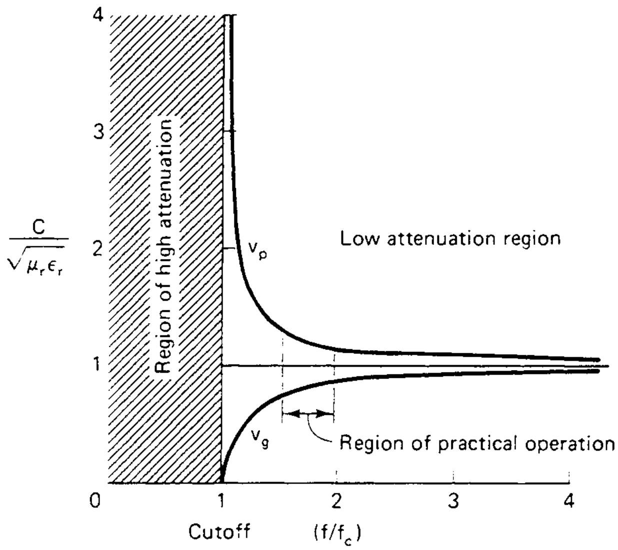
TE_{20} Mode of Propagation



TE_{30} Mode of Propagation



v_p and v_g as a Function of Frequency



TE_{10} in Air-Filled Rectangular Waveguide

The guide wavelength λ_g is:

$$\lambda_g = \frac{c}{\sqrt{f^2 - f_c^2}}$$

Relationship between wavelength in free space λ, wavelength in guide λ_g, and cutoff wavelength λ_c:

$$\lambda_g = \frac{\lambda \lambda_c}{\sqrt{\lambda_c^2 - \lambda^2}} \quad \text{and} \quad \lambda = \frac{\lambda_g \lambda_c}{\sqrt{\lambda_g^2 + \lambda_c^2}}$$

TE_{10} in Air-Filled Rectangular Waveguide

The last equation allows calculation of input signal wavelength (or frequency) if the guide wavelength is measured using a slotted line:

- Voltage minima in waveguide are separated by $\lambda_g/2$

The group velocity in the guide is:

$$v_g = \frac{\left(c/\sqrt{\mu_r \varepsilon_r}\right)^2}{v_p}$$

TE_{10} in Air-Filled Rectangular Waveguide

Example:

- Voltage minima in a WR-90 waveguide are separated by 2cm
- Determine λ_g, λ, and *f*

Solution:

- λ_g= 4cm
- From Appendix D, WR-90 waveguide has dimensions a=2.286cm and b=1.016cm, so
  - λ_c= 2a = 4.572cm

TE_{10} in Air-Filled Rectangular Waveguide

Example:

– Voltage minima in a WR-90 waveguide are separated by 2cm

– Determine λ_g, λ, and f

Solution:

– The free-space wavelength is

$$\lambda = \frac{\lambda_g \lambda_c}{\sqrt{\lambda_g^2 - \lambda_c^2}} = \frac{0.04(0.04572)}{\sqrt{0.04^2 - 0.04572^2}} = 0.03\ \text{m}$$

– The frequency is $f = \dfrac{3\times 10^8\ \text{m/s}}{0.03\ \text{m}} = 10\ \text{GHz}$

Practical Considerations For Waveguides

RF energy sent down a waveguide propagates at the **group velocity**, which is a function of frequency.

Thus, a pulse of RF energy (that contains energy at a variety of frequencies) experiences **dispersion**.

– The frequency components of the pulse travel at different group velocities

– The “Region of Practical Operation” shown in the previous figure represents a bandwidth that results in low dispersion

Practical Considerations For Waveguides

Note that the table of common rectangular waveguides in **Appendix D** has the ratio $a \cong 2b$

- This means that the frequency range in the left column will result in only TE_{10} mode propagation in the guide

Attenuation in Rectangular Waveguides

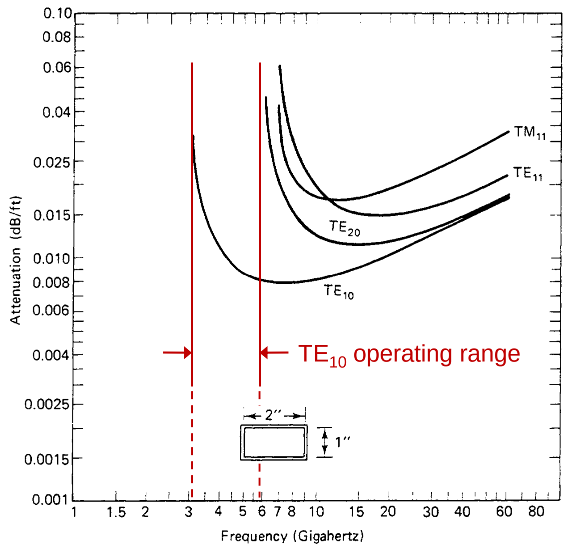
Waveguide walls cause some losses because the conductor has a small resistance

- It is not a perfect conductor

The figure on the next slide shows the losses for a 1” x 2” (2.54cm x 5.08cm) waveguide

- Cutoff frequency is $f_c = c/2a = 2.95$ GHz

Attenuation in Rectangular Waveguides



Characteristic Impedance in Waveguides

The **characteristic wave impedance** is defined in terms of the components of the **E** and **H** fields transverse to the direction of propagation

$$Z_0 = \frac{E_{transverse}}{H_{transverse}} = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}}$$

Characteristic Impedance in Waveguides

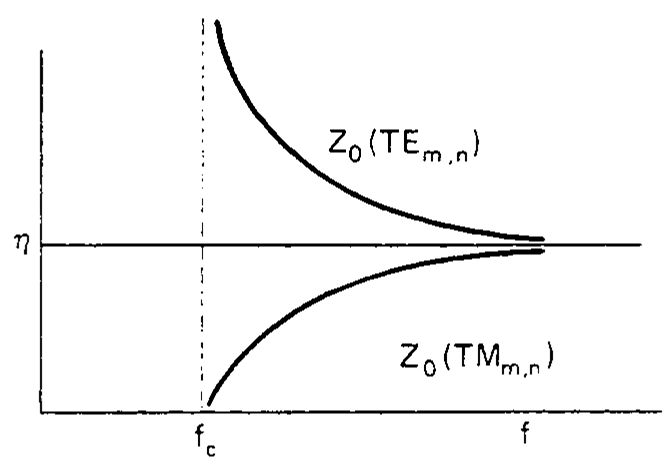
Based on the equations presented in **Appendix C** of the text, the **characteristic impedance** for the $TE_{m,n}$ modes simplifies to:

$$Z_{0(TE_{m,n})} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

For the $TM_{m,n}$ modes, the **characteristic impedance** is:

$$Z_{0(TM_{m,n})} = \eta\sqrt{1-(f_c/f)^2}$$

Characteristic Impedance in Waveguides



Characteristic Impedance in Waveguides

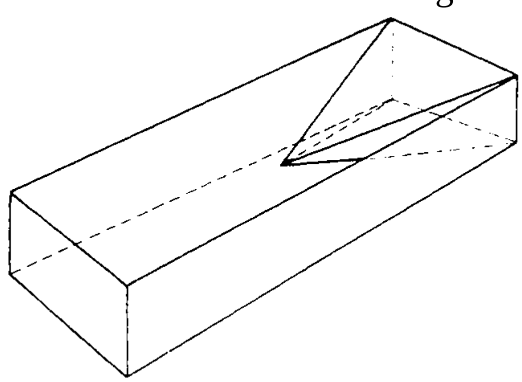
Impedance matching with waveguides must be done in some situations, but it is more complicated than with transmission lines.

- **Open circuit** waveguide stubs radiate and therefore <u>do not act</u> as "ideal" open circuits.
- **Short circuit** waveguide stubs can be used
- **Open circuits** can be simulated by attaching a $\lambda/4$ section to a short circuit stub.
- **Inductive or capacitive** waveguide irises can be used.

Characteristic Impedance in Waveguides

Impedance matching with waveguides must be done in some situations, but it is more complicated than with transmission lines.

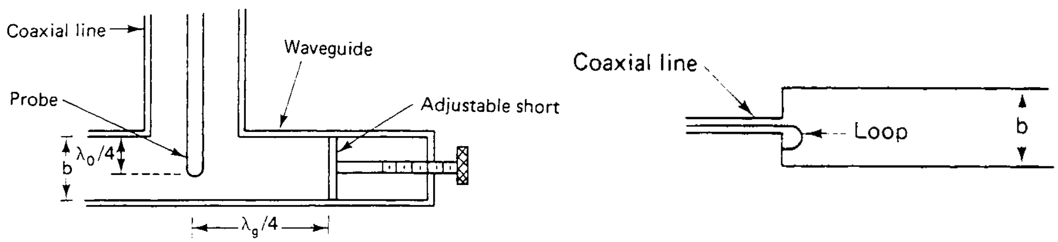
- **Matched loads** are created by inserting power absorbing material at the end of a section of waveguide.



Coupling to Waveguides

The two most common **coupling methods** are the **probe coupler** and the **loop coupler**, the shape of which depends on the modes to be excited.

TE_{10} mode probe and loop couplers:



Dual-Polarized Satellite Dish Antenna

Dual-Polarized Satellite Dish Antenna Feed