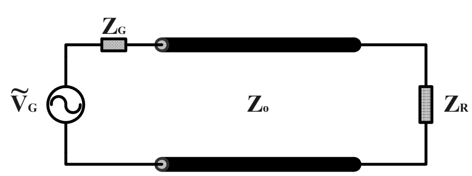
ECET 3410

High Frequency Systems

Impedance Matching

1

Maximum Power Transfer



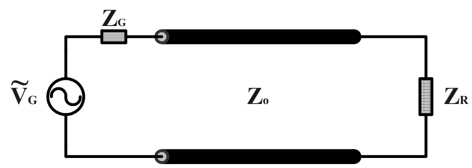
In order to deliver **maximum power** from the source to the load along a transmission line, both the source and load impedances must be **matched** to the characteristic impedance of the line (assuming all "real" values).

$$Z_G = Z_o = Z_R$$

This concept is equivalent to the Maximum Power Transfer Theorem that is typically presented during an introductory Circuits course.

2

Maximum Power Transfer



When a line is not matched to a source, less than maximum power will be transferred from the source onto the line.

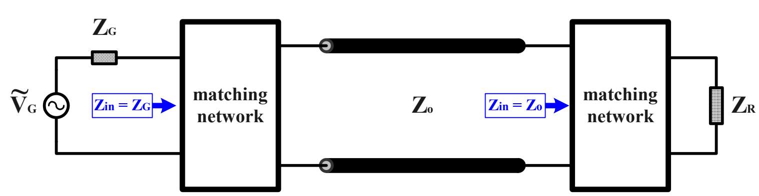
Commercial sources are typically well-matched to standard line impedances.

When a load is not matched to a line, some of the incident power will reflect back toward the source, resulting in less than maximum power being transferred to the load.

Devices connected to the end of a line are not always well-matched to the line.

3

Matching Networks



Whenever a transmission-line isn't fully matched, such that:

$$Z_G \neq Z_o \quad \text{and/or} \quad Z_R \neq Z_o$$

a **matching network** can be utilized at one or both ends of the line in order to transform the mismatched impedance(s) to the desired value(s), resulting in maximum power transfer from source → line → load.

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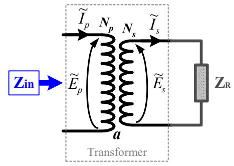
Impedance Matching Transformers

$Z_{in} = a^2 \cdot Z_R$



$a = \frac{N_p}{N_s} = \frac{\tilde{E}_p}{\tilde{E}_s} = \frac{\tilde{I}_s}{\tilde{I}_p}$

There are commercially available **impedance-matching transformers** that can be used to match "real" impedances at frequencies up to several hundred MHz.

Given an ideal transformer with turns-ratio "*a*" and load Z_R connected across its secondary terminals, the input impedance of the transformer is:

$$Z_{in} = a^2 \cdot Z_R$$

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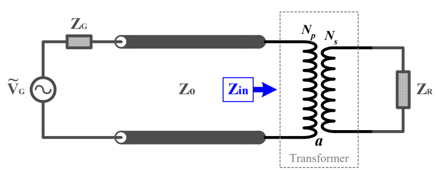
Impedance Matching Transformers

$Z_G = Z_o$



$Z_R \neq Z_o$

If a load $\boldsymbol{Z_R}$ is not matched to the line $\boldsymbol{Z_o}$, an impedance matching transformer with turns-ratio "***a***" can be placed between the load and the line, such that:

$$a = \sqrt{\frac{Z_o}{Z_R}}$$

Z_R and Z_o must both be real values.

resulting in:

$$Z_{in} = Z_o$$

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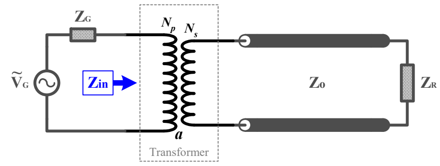
Impedance Matching Transformers



If a line Z_o is not matched to the source Z_G, an impedance matching transformer with turns-ratio "*a*" can be placed between the source and the line, such that:

$$a = \sqrt{\frac{Z_G}{Z_o}}$$

Z_G and Z_o must both be real values.

resulting in:

$$Z_{in} = Z_G$$

provided that the load is matched to the line

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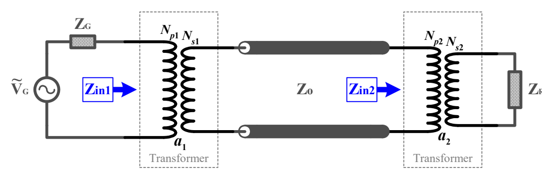
Impedance Matching Transformers



If the source Z_G **and** the load Z_R are not matched to the line Z_o, impedance matching transformers can be placed at ends of the line, such that:

$$a_1 = \sqrt{\frac{Z_G}{Z_o}} \qquad a_2 = \sqrt{\frac{Z_o}{Z_R}}$$

Z_R, Z_G and Z_o must all be real values.

resulting in:

$$Z_{in1} = Z_G \qquad Z_{in2} = Z_o$$

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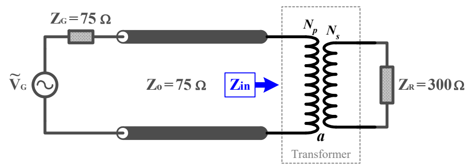
Impedance Matching Transformer Example



Determine the **turns-ratio "*a*"** for an impedance matching transformer that can be used to match a 300Ω load to a standard 75Ω line:

$$a = \sqrt{\frac{Z_o}{Z_R}} = \sqrt{\frac{75}{300}} = \frac{1}{2}$$

$$Z_{in} = a^2 \cdot Z_R = (\tfrac{1}{2})^2 \cdot 300 = 75\ \Omega$$

Since the primary terminals of the transformer are connected to the receiving-end of the line, a matched load of 75Ω appears to be terminating the line.

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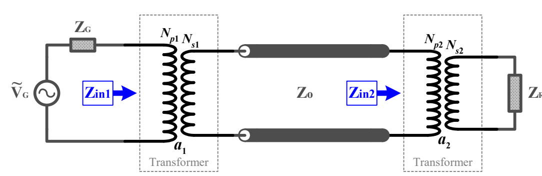
Impedance Matching Transformers



As previously noted, **impedance-matching transformers** that can be used to match "real" impedances at frequencies up to several hundred MHz are readily available.

But, above a few hundred MHz, the ferrite cores utilized in these transformers will often begin to saturate, causing the transformers to act in a non-linear manner, resulting in distortion of the applied signal or waveform.

The ferrite cores may also begin to saturate if the applied signal power becomes too large.

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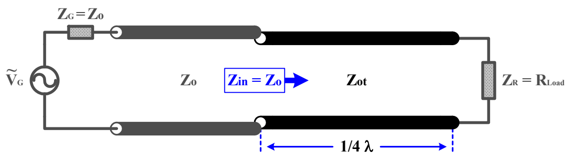
Quarter Wavelength Tuners



If Z_{ot} is not close to one of the standard characteristic impedances, it may be difficult to find a line section with the necessary impedance value.

A **Quarter Wavelength Tuner** can also be used to match a resistive load $\boldsymbol{Z_R}$ to any (purely real) line impedance value.

The tuner consists of a ¼–wavelength long section of line with characteristic impedance $\boldsymbol{Z_{ot}}$, such that:

Note that the ¼-λ Tuner is assumed to be lossless.

$$Z_{ot} = \sqrt{Z_{in} \cdot Z_R}$$

where: $\boldsymbol{Z_{in}}$ is the desired input impedance of the tuner. (i.e. $\boldsymbol{Z_{in}}=\boldsymbol{Z_o}$)

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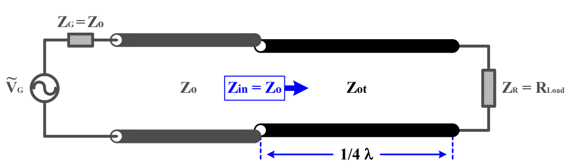
Quarter Wavelength Tuner Example

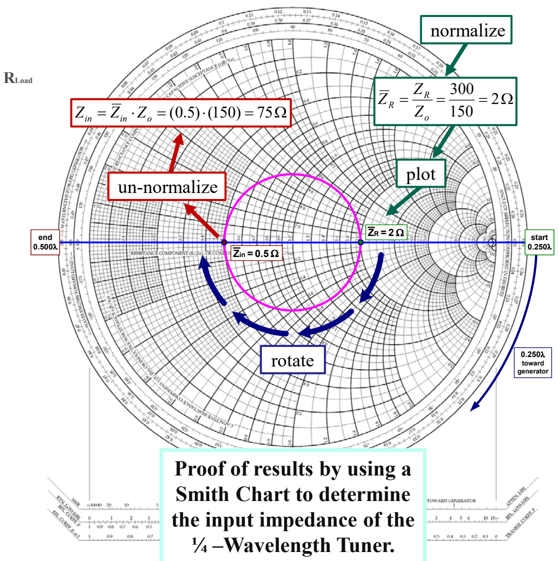


Design a **¼ Wavelength Tuner** to match a load of $\boldsymbol{Z_R}$ **= 300Ω** to a **75Ω system** using an **air-filled line** at **600MHz**.

$$Z_{ot} = \sqrt{Z_{in} \cdot Z_R} = \sqrt{75 \cdot 300} = \boxed{150\,\Omega}$$

$$\lambda = \frac{v}{f} = \frac{3 \cdot 10^8}{600 \cdot 10^6} = 0.5\,\text{m} = 50\,\text{cm}$$

$$L = 0.25\lambda = (0.25)\cdot(50) = \boxed{12.5\,\text{cm}}$$



Proof of results by using a Smith Chart to determine the input impedance of the ¼ –Wavelength Tuner.

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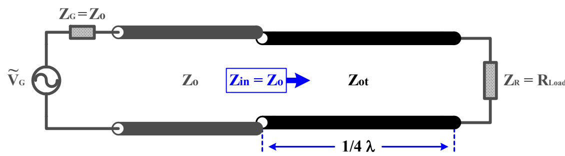
Quarter Wavelength Tuners



Since the actual (physical) length of a Quarter Wavelength Tuner is a function of the frequency of the applied waveform:

$$L = \tfrac{1}{4}\lambda = \tfrac{1}{4}\cdot\frac{v}{f}$$

each tuner is designed to operate at a specific frequency.

At both lower and higher frequencies, the length will no longer be ¼λ, resulting in a **limited operational bandwidth**.

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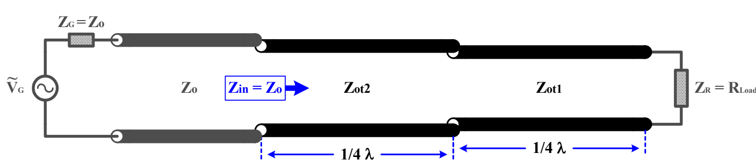
Multi-Stage ¼ Wavelength Tuners



Multi-stage ¼-λ Tuners will NOT appear on the course exams.

To reduce the narrow-band effect, multiple ¼–wavelength sections may be used, such as a 2-stage, ¼ Wavelength Tuner shown in the above figure.

Each stage requires a different characteristic impedance. One choice of impedances that yields good bandwidth results is:

$$\left(\frac{Z_o}{Z_{ot2}}\right)^2 = \frac{Z_{ot2}}{Z_{ot1}} = \left(\frac{Z_{ot1}}{Z_R}\right)^2$$

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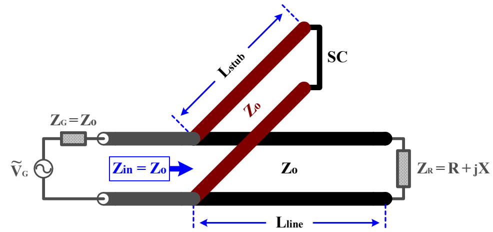
Single-Stub Tuners



A **Single-Stub Tuner** is used to match a complex load $\boldsymbol{Z_R = R + jX}$ to any (purely real) system impedance using line sections that have the same characteristic impedance $\boldsymbol{Z_o}$ as the system.

This tuner consists of a short-circuited "stub" in-parallel with the sending-end of a "line" that is terminated by the load, $\boldsymbol{Z_R}$.

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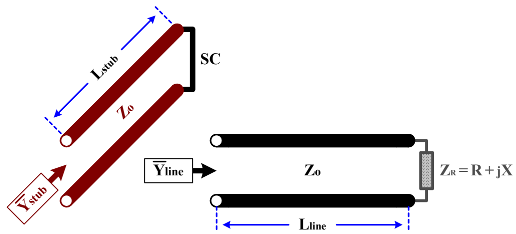
Single-Stub Tuners

Remember that admittances add for parallel-connected circuit elements.



The length, $\boldsymbol{L_{line}}$, is chosen such that the normalized input admittance of the "line" has a real value of one (**1**).

$$\overline{Y}_{line} = 1 + j\overline{B}_{line}$$

The length, $\boldsymbol{L_{stub}}$, is chosen such that the "stub's" normalized input admittance cancels the imaginary part of the "line's" input admittance:

$$\overline{Y}_{stub} = -j\overline{B}_{line}$$

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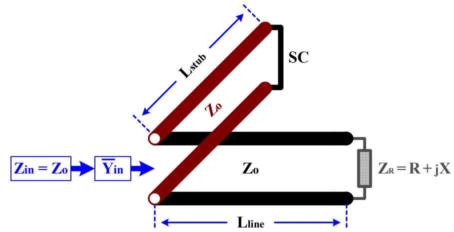
Single-Stub Tuners



Since **admittances** add in parallel, given:

$$\bar{Y}_{line} = 1 + j\bar{B}_{line} \qquad \bar{Y}_{stub} = -j\bar{B}_{line}$$

then:

$$\bar{Y}_{in} = \bar{Y}_{line} + \bar{Y}_{stub} = (1 + j\bar{B}_{line}) + (-j\bar{B}_{line}) = 1$$

$$\bar{Z}_{in} = \frac{1}{\bar{Y}_{in}} = 1 \quad \Rightarrow \quad Z_{in} = \bar{Z}_{in} \cdot Z_o = 1 \cdot Z_o = Z_o$$

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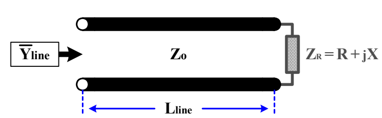
Single-Stub Tuners – "Line" Length

TO DETERMINE THE "LINE" LENGTH:

1) Plot the normalized load $\bar{Z}_R$ and reflect the impedance to the opposite side of a constant |Γ| circle to find the normalized load admittance $\bar{Y}_R$.



2) Rotate the normalized admittance around the constant |Γ| circle (towards generator) until it has a real value of <u>one</u>. This will be the normalized input admittance of the "line" section:

$$\bar{Y}_{line} = 1 + j\bar{B}_{line}$$

3) Set the length L_{line} equal to the distance that the "line" admittance was rotated around the constant |Γ| circle (in wavelengths).

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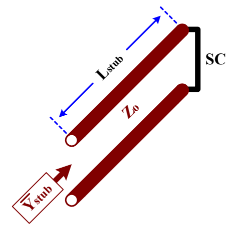
Single-Stub Tuners – "Stub" Length

TO DETERMINE THE "STUB" LENGTH:



1) **Determine the imaginary value of the normalized input admittance of the "line":**

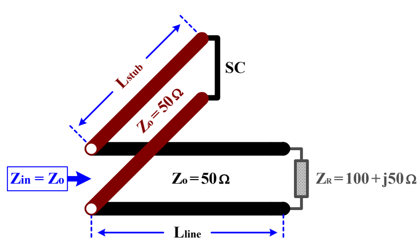
$$\overline{Y}_{line} = 1 + j\overline{B}_{line}$$

2) **Starting at the point $\overline{Y}_{SC} = \infty$, rotate around the outer circle of the Smith Chart until the normalized input admittance of the "stub" section is equal to:**

$$\overline{Y}_{stub} = -j\overline{B}_{line}$$

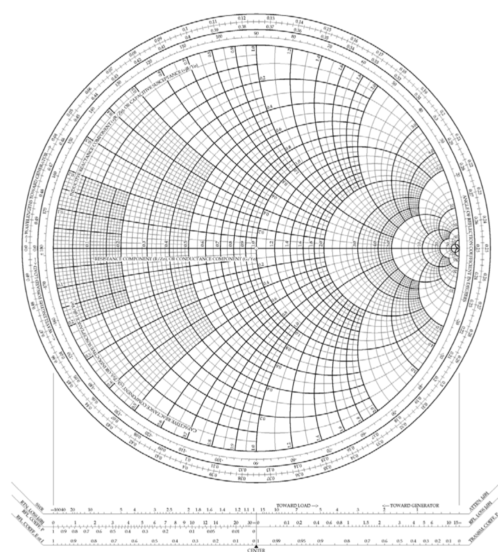
3) **Set the length L_{stub} equal to the distance that the "stub" admittance was rotated around the outer circle (in wavelengths).**

Textbook Example 6-1

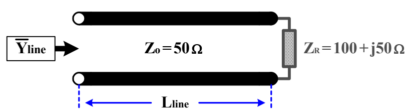


Design a **Single-Stub Tuner** to match a load of $\boldsymbol{Z_R}$ = (**100** + ***j*50**)Ω to a **50Ω source** using **50Ω transmission line**.

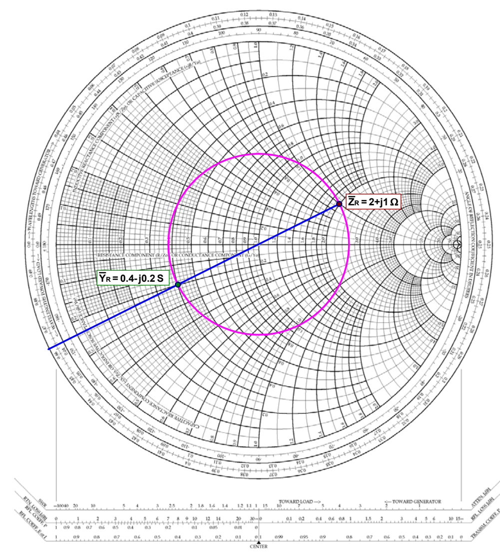
There are <u>two fundamental solutions</u> to this problem. One solution will be shown step-by-step in this presentation, followed by a summary of the results for the second solution.



Textbook Example 6-1



Step 1: Plot the normalized load $\overline{Z}_R$ and reflect the impedance to the opposite side of a constant |Γ| circle to find the normalized load admittance $\overline{Y}_R$.



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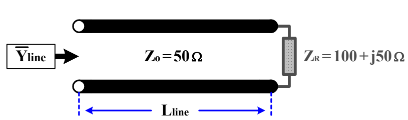
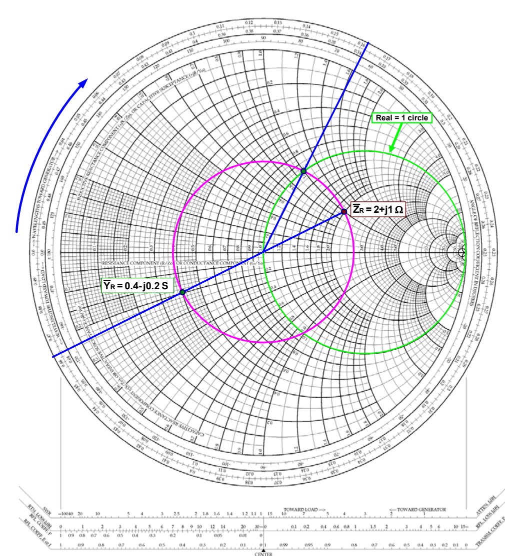
Textbook Example 6-1



Step 2: Rotate the normalized load admittance $\overline{Y}_R$ around the constant |Γ| circle it reaches the "Real = 1" circle such that:

$$\overline{Y}_{line} = 1 + j\overline{B}_{line}$$



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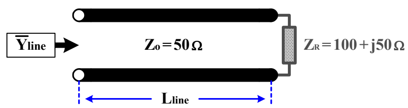
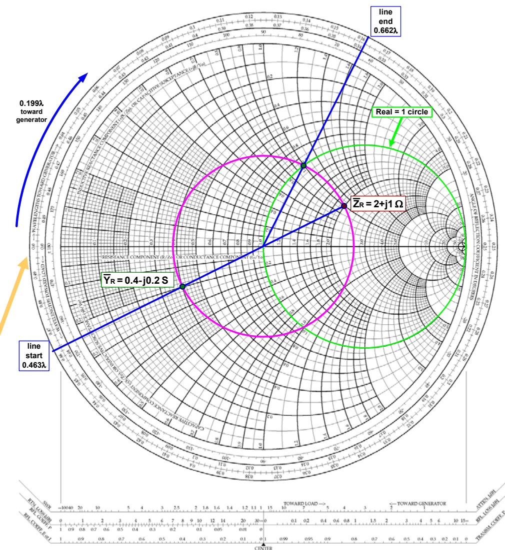
Textbook Example 6-1



Step 3: Determine the length of the "line" section L_{line}:

Line END		0.662
Line START	–	0.463
L_{line}	=	0.199 λ

Since the normalized load admittance crossed the 0.000/0.500 marker as it rotated to the "Real = 1" circle, 0.500 should be added to the ending marker value to determine an effective ending position.



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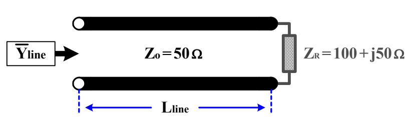
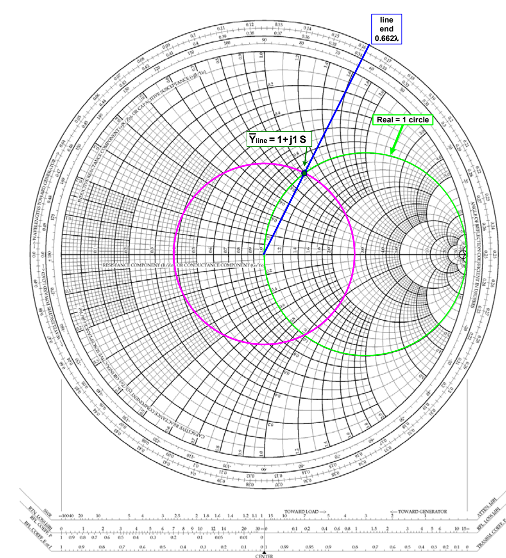
Textbook Example 6-1



Step 4: Determine the value of the normalized line admittance $\overline{Y}_{line}$.

$$\overline{Y}_{line} = 1 + j\overline{B}_{line}$$
$$= 1 + j1$$

Note that the constant |Γ| circle also crosses the "Real = 1" circle at effective ending position 0.838, resulting in a 2nd theoretical solution.



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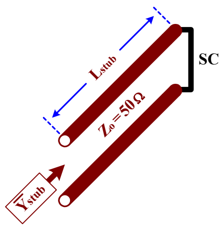
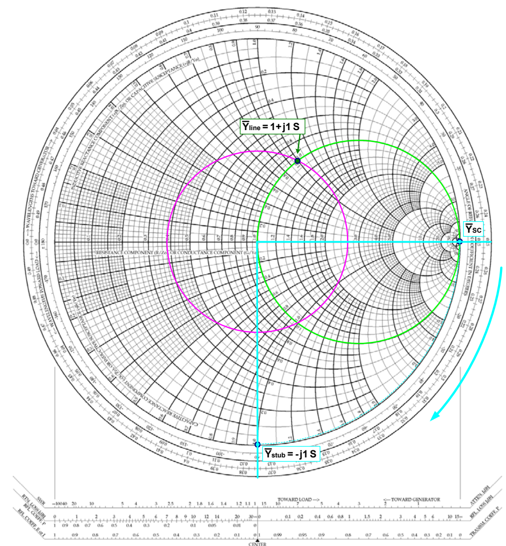
Textbook Example 6-1



Step 5: Starting at the point $\overline{Y}_{SC} = \infty$, rotate around the outer circle of the Smith Chart until the normalized "stub" admittance is equal to:

$$\overline{Y}_{stub} = -j\overline{B}_{line}$$
$$= -j1$$



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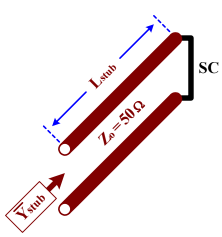
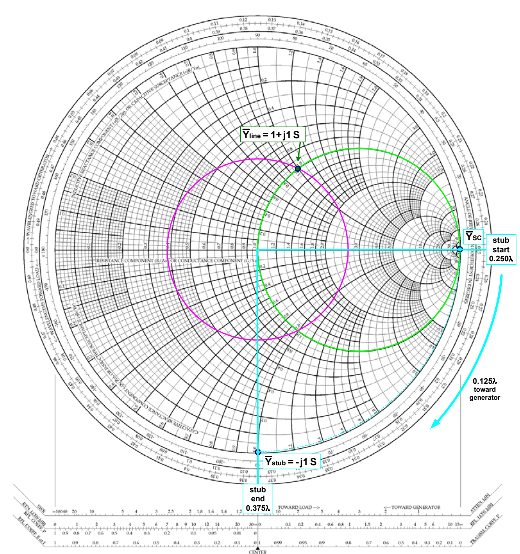
Textbook Example 6-1



Step 6: Determine the length of the "stub" section L_{stub}:

Stub END		**0.374**
Stub START	**–**	**0.250**
L_{stub}	**=**	**0.125 λ**



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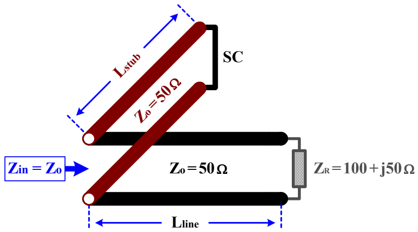
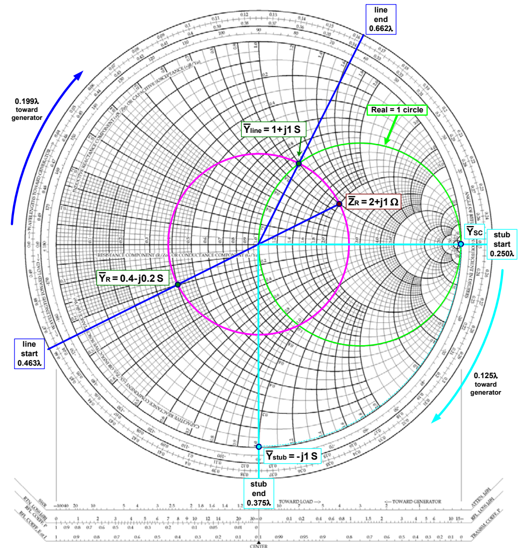
Textbook Example 6-1





Solution #1: $L_{line} = 0.199\,\lambda$

$L_{stub} = 0.125\,\lambda$

$$\overline{Y}_{line} = 1 + j1 \qquad \overline{Y}_{stub} = -j1$$

$$\overline{Y}_{in} = \overline{Y}_{line} + \overline{Y}_{stub} = (1 + j1) + (-j1) = 1$$

$$\overline{Z}_{in} = \frac{1}{\overline{Y}_{in}} = 1\,\Omega \qquad Z_{in} = \overline{Z}_{in} \cdot Z_o = 50\,\Omega$$

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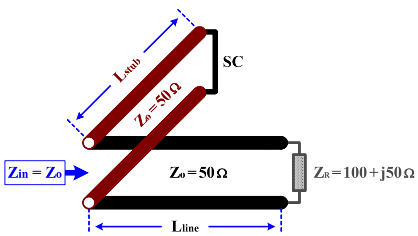
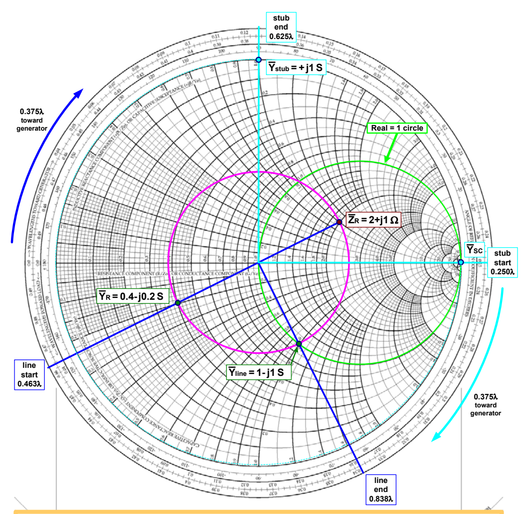
Textbook Example 6-1 Solution #2





The above Smith Chart shows the complete set of operations required to obtain Solution #2 for this problem.

Solution #2: $L_{line} = 0.375\,\lambda$

$L_{stub} = 0.375\,\lambda$

$$\overline{Y}_{line} = 1 - j1 \qquad \overline{Y}_{stub} = +j1$$

$$\overline{Y}_{in} = \overline{Y}_{line} + \overline{Y}_{stub} = (1 - j1) + (+j1) = 1$$

$$\overline{Z}_{in} = \frac{1}{\overline{Y}_{in}} = 1\,\Omega \qquad Z_{in} = \overline{Z}_{in} \cdot Z_o = 50\,\Omega$$

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