ECET 3410

High Frequency Systems

Transmission Line Analysis
using
Smith Charts

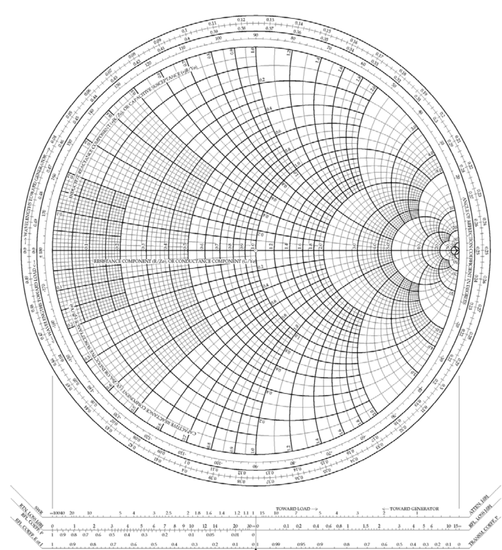
1

Transmission Line Analysis using Smith Charts

To find the **input impedance** of a lossless transmission-line using a Smith Chart:

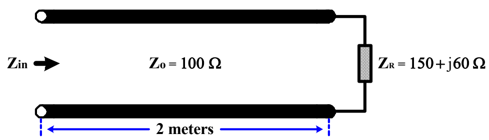
1) **Plot** the **normalized load** impedance.
2) **Draw** the **constant |Γ| circle**.
3) **Rotate** "Towards the Generator" (CW) the **length** of the line (**in wavelengths**) referenced to the ½−wavelength scale provided in the outer band.
4) **Read** the **normalized input** impedance.

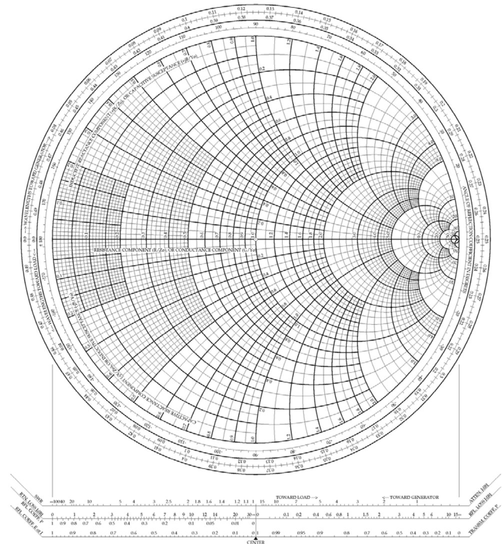
Note that the reverse procedure can be utilized to determine the load impedance from the input impedance.



2

Textbook Example 4-1

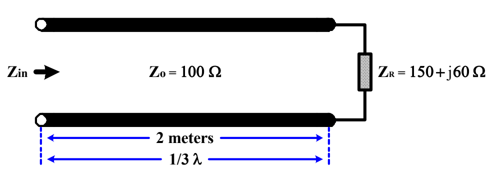


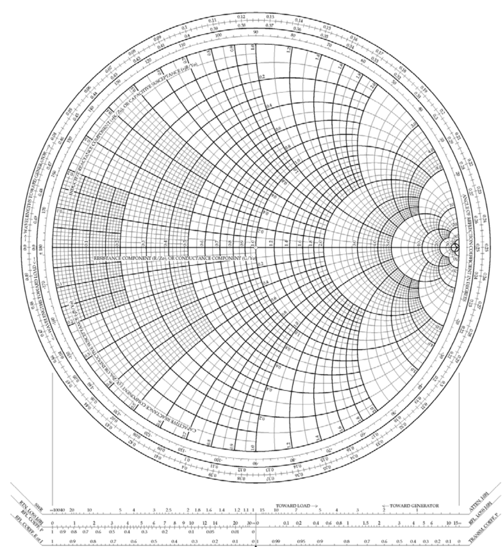


Determine the **input impedance** of a **2-meter** long, **air-filled**, **lossless**, **100Ω** transmission line that is terminated by a load $\mathbf{Z_R}$ **= (150 + j60)Ω** and supplied by a source that is operating as a frequency of **50MHz**.

3

Textbook Example 4-1





Step 0: If not expressed in wavelengths, convert the **line length** from physical units (meters) to electrical units (wavelengths).

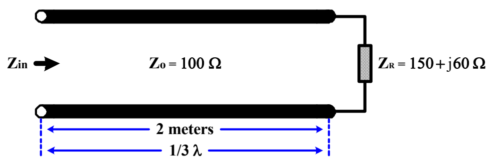
$$v_{air} = c = 3 \cdot 10^8 \ \tfrac{\text{meters}}{\text{sec}}$$

$$\lambda = \frac{v}{f} = \frac{3 \cdot 10^8}{50 \cdot 10^6} = 6 \text{ meters}$$

$$\text{L} = \frac{2 \text{ meters}}{6 \frac{\text{meters}}{\text{wavelength}}} = \frac{1}{3} \text{ wavelength}$$

4

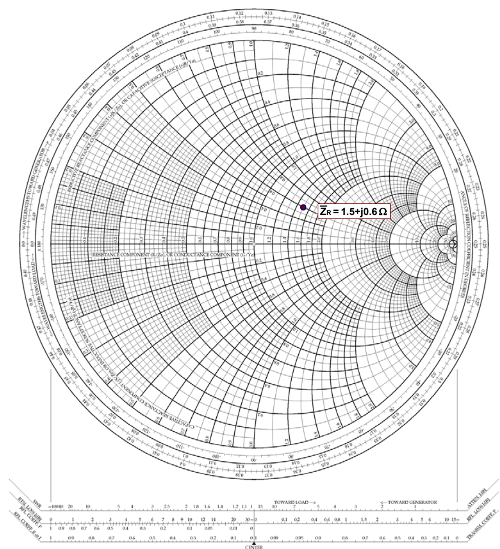
Textbook Example 4-1



Step 1: Normalize and **plot** the known (load) **impedance**.

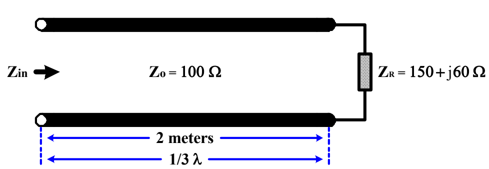
$$\overline{Z}_R = \frac{Z_R}{Z_o} = \frac{150 + j60}{100} = (1.5 + j0.6)\Omega$$

Since a Smith Chart with an impedance of 1 at its origin provides a mapping of normalized impedance to reflection coefficient, the load impedance must first be normalized and then plotted.



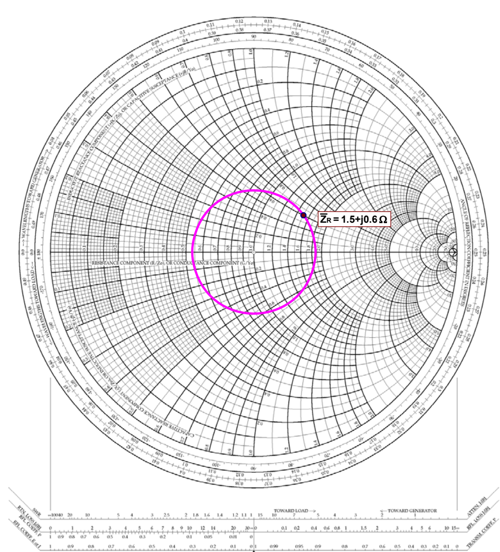
5

Textbook Example 4-1



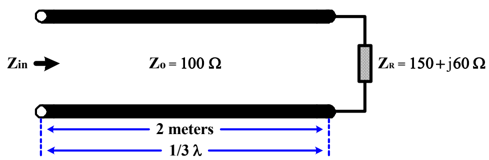
Step 2: Draw a **constant |Γ| circle** that is **centered at the origin** of the Smith Chart and **passes through** the plotted **impedance** $\overline{Z}_R$.

On a lossless line, $|\Gamma_{in}| = |\Gamma_R|$. Since the magnitude of the reflection coefficient is defined by the distance from the origin, the normalized load impedance and the normalized input impedance must both fall on a circle centered about the origin.



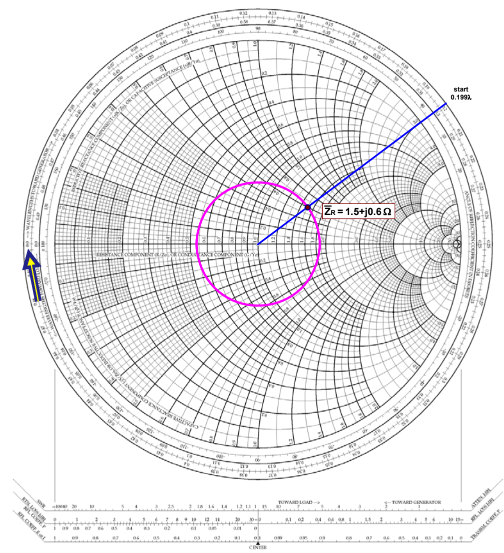
6

Textbook Example 4-1



Step 3a: Draw a **line** from the **origin**, **through** the known **impedance**, to the **1st band** and **determine** the **STARTING marker** position.

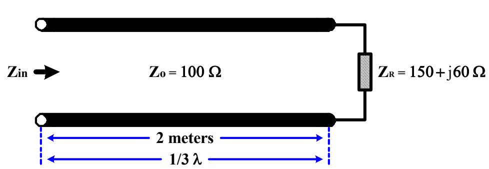
START 0.199



Since the input reflection coefficient, Γ_{in}, will rotate one complete revolution clockwise from Γ_R for each ½-wavelength of line-length, the 1st band (Wavelengths Toward Generator) can be utilized to determine the required rotation.

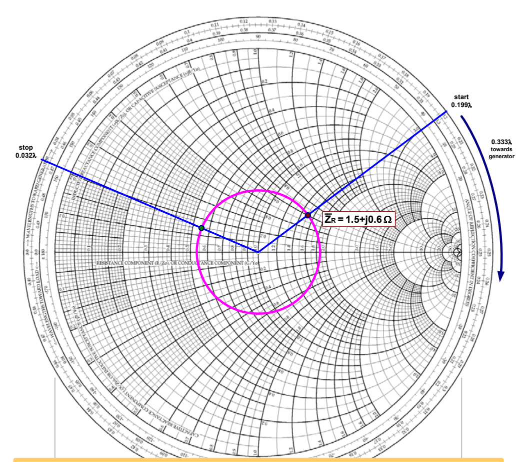
7

Textbook Example 4-1



Step 3b: Locate the **ENDING marker** position on the **1st band** and **draw** a **line** from the **origin** to the **ENDING marker**.

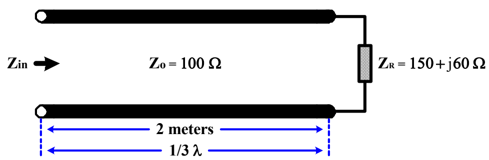
START		**0.199**
length	**+**	**0.333**
END		**0.532**
	-	**0.500**
Equivalent END		**0.032**



If the ending marker position is greater than 0.500, then subtract whole multiples of 0.5 to find an equivalent ending position since the scale on the 1st band resets back to 0.000 at the 0.500 marker.

8

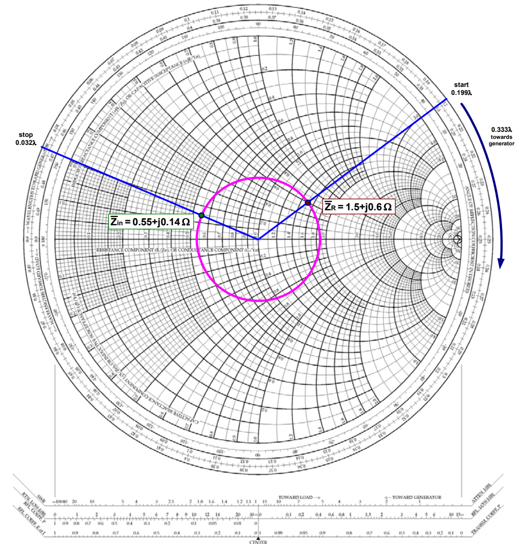
Textbook Example 4-1



Step 4: Read the **normalized** value of the unknown impedance ($\overline{Z}_{in}$) at the point where the **END line intersects** the **constant |Γ| circle**.

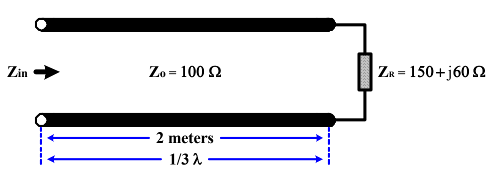
$$\overline{Z}_{in} = (0.55 + j0.14)\Omega$$

$$Z_{in} = \overline{Z}_{in} \cdot Z_o = (55 + j14)\Omega$$



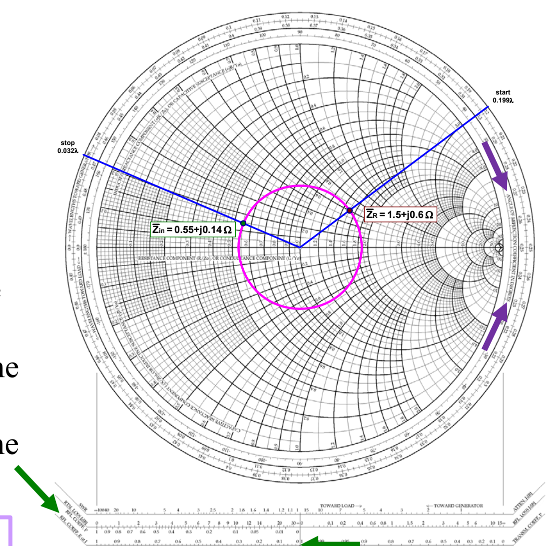
9

Textbook Example 4-1



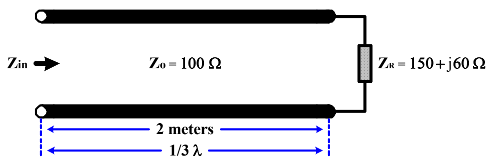
The **Reflection Coefficients** Γ_R and Γ_{in} on the line can be determined from the Smith Chart solution by using both the **3rd scale (RFL COEFF E or I)** provided at the **bottom left side** of the chart and the **3rd band (ANGLE OF RFL COEFF IN DEGS)** provided around the outer portion of the Smith Chart.

Note that the 4th band has been erased and a 4th scale has been removed from the Smith Charts shown in this presentation.



10

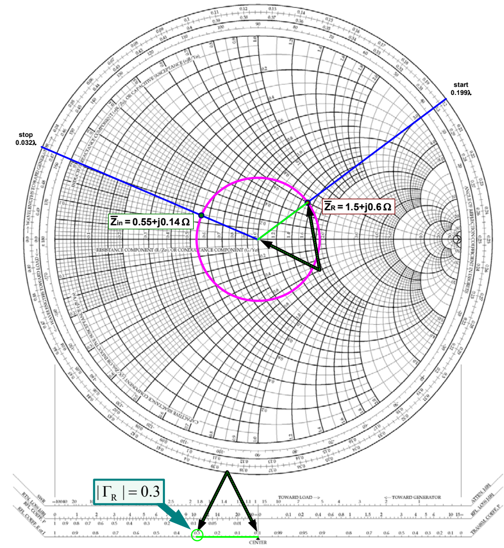
Textbook Example 4-1



The $|\Gamma_R|$ is found by **measuring** the **distance** from $\overline{Z}_R$ to the **origin** and then **reading** the **value** that corresponds to the same **distance** to the **left of center** on the **3rd scale (RFL COEFF E or I)** that is provided at the bottom left side of the Smith Chart.

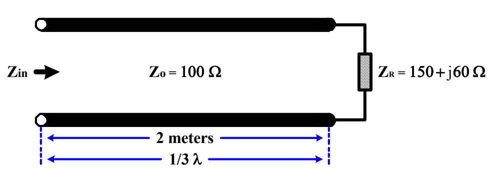
$$|\Gamma_R| = 0.3$$

$$\Theta_R = 36.8°$$



11

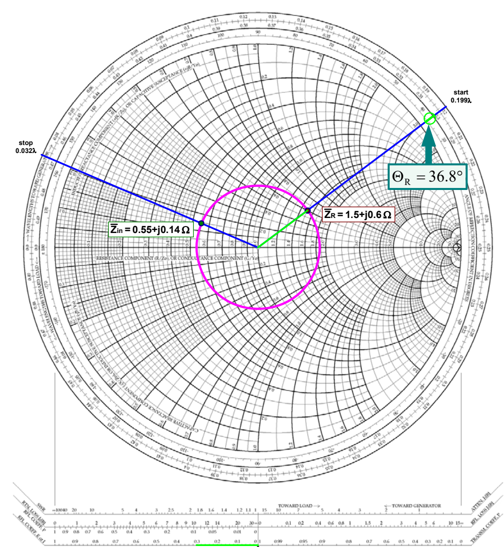
Textbook Example 4-1



The $\angle\Gamma_R$ is the **angle** that the **START line** (drawn from the **origin**, through $\overline{Z}_R$, to the outer bands) forms with the right-hand side of the horizontal axis.

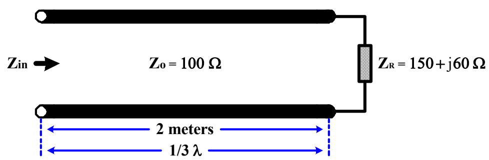
This **angle** can be found by **reading** the **value** corresponding to the point at which the **START line** crosses the 3rd band.

$$\Theta_R = 36.8°$$



12

Textbook Example 4-1

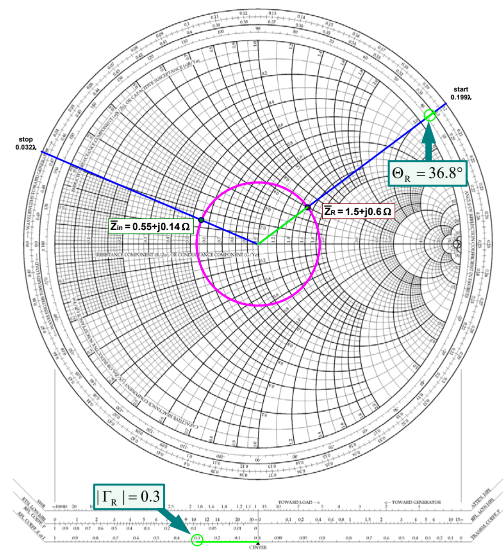


Thus, Γ_R, the **reflection coefficient** of the **load** is:

$$\Gamma_R = 0.3\angle 36.8°$$

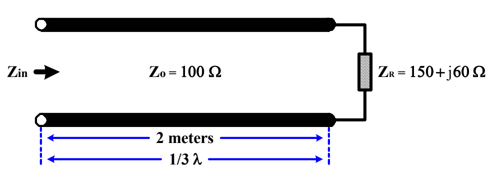
A **mathematical check** can be used to verify the accuracy of the result:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} = \frac{50 + j60}{250 + j60} = 0.304\angle 36.7°$$



13

Textbook Example 4-1

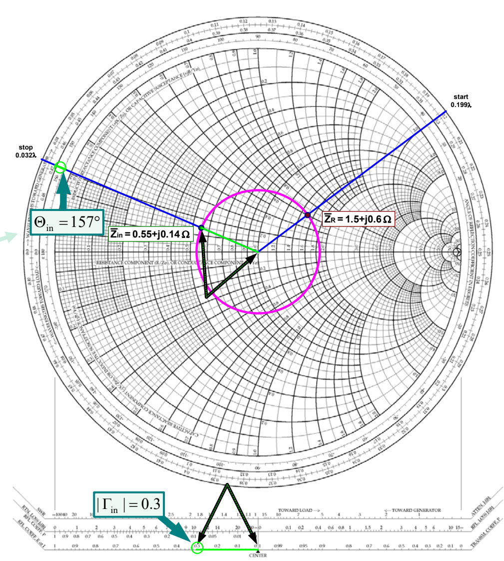


Similarly, Γ_{in}, the **reflection coefficient** at the **input** has the same magnitude as Γ_R but a different angle:

$$\Gamma_{in} = 0.3\angle 157°$$

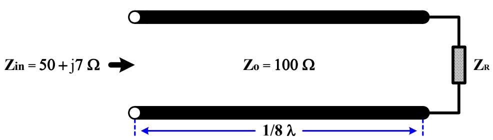
Checking the result:

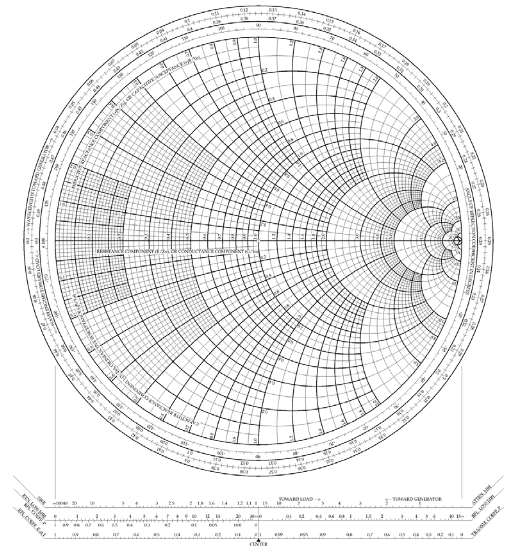
$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{-45 + j14}{155 + j14} = 0.303\angle 157.6°$$



14

Textbook Example 4-2



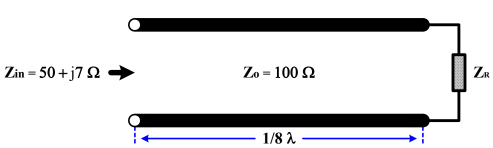


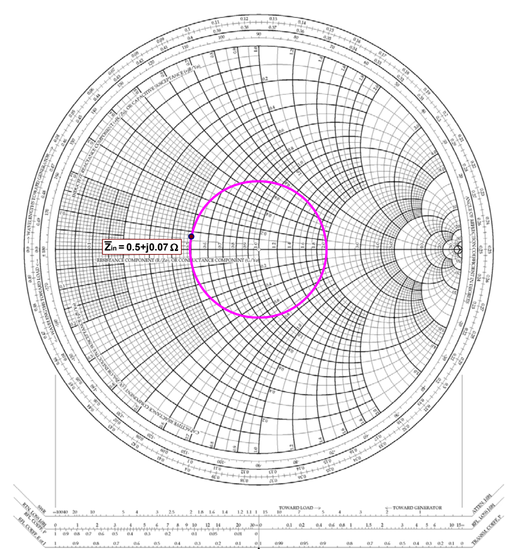
Determine the **load impedance** the terminates a 1/8-wavelength long, 100Ω, lossless line if the input impedance of the line is $Z_{in} = (50 + j7)\Omega$.

Since the length of the line is already expressed in wavelengths, neither frequency nor velocity is needed in order to solve this problem.

15

Textbook Example 4-2





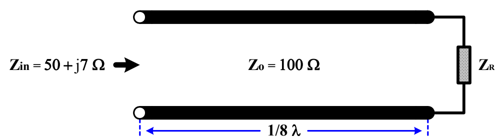
Step 1: Normalize and plot the known input impedance.

$$\overline{Z}_{in} = \frac{Z_{in}}{Z_o} = \frac{50 + j7}{100} = (0.50 + j0.07)\Omega$$

Step 2: Draw a constant |Γ| circle that passes through the normalized input impedance.

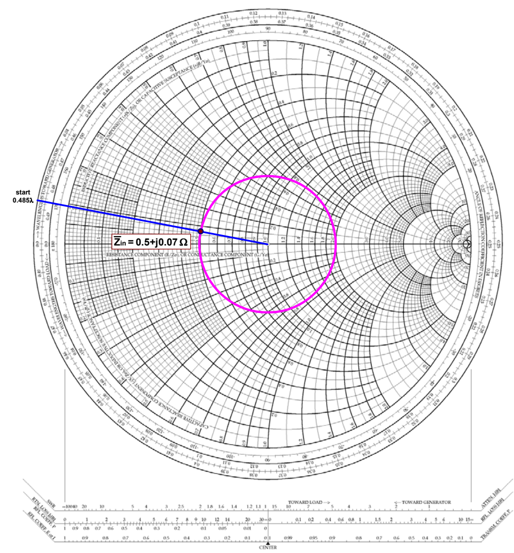
16

Textbook Example 4-2

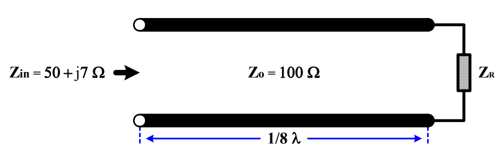


Step 3: Draw a line from the origin, through the known impedance, to the 2nd band and determine the starting marker position.

START 0.485

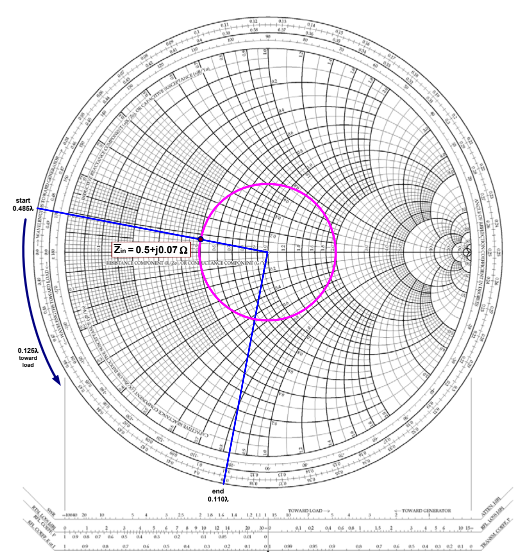


Textbook Example 4-2

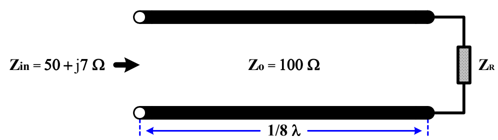


Step 4: Rotate the length of the line (toward generator) and draw a line from the origin to the ending marker.

START	**0.485**
length	**+ 0.125**
END	**0.610**
	- 0.500
Equivalent END	**0.110**



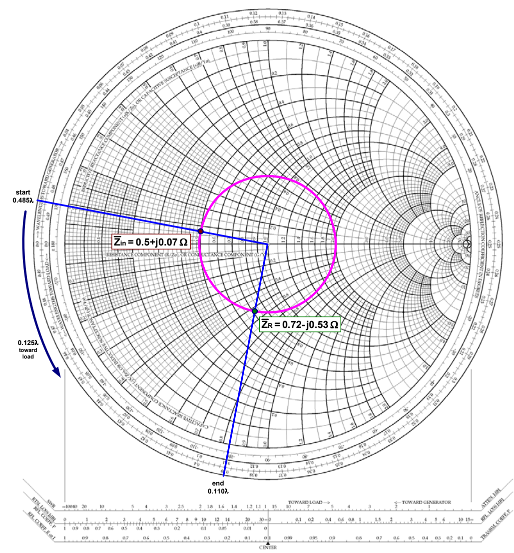
Textbook Example 4-2



Step 5: Read the value of the unknown impedance at the point where the ending line crosses the constant |Γ| circle.

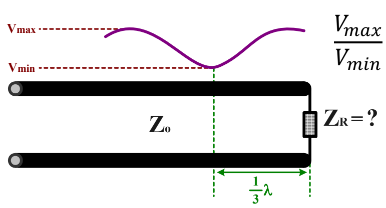
$$\overline{Z}_R = (0.72 - j0.53)\Omega$$

$$Z_R = \overline{Z}_R \cdot Z_o = (72 - j53)\Omega$$



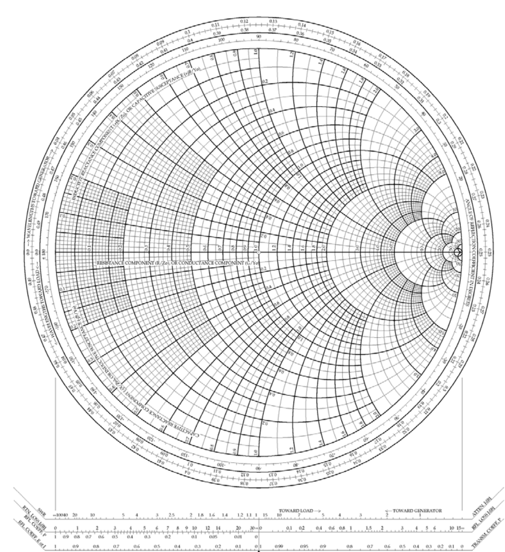
19

Textbook Example 4-3



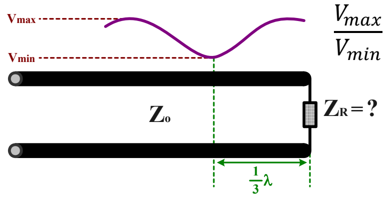
$$\frac{V_{max}}{V_{min}} = 5$$

If the **VSWR** is measured to be **5** on a **lossless, 50Ω line** and a voltage **minima** occurs **1/3 of a wavelength** from the load, determine the value of the load $\mathbf{Z_R}$.



20

Textbook Example 4-3

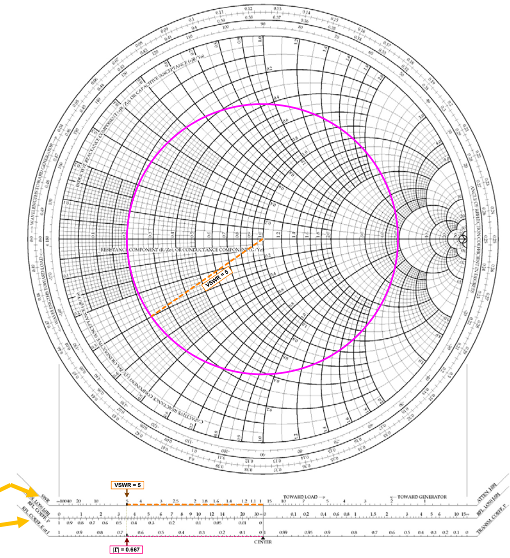


$$\frac{V_{max}}{V_{min}} = 5$$

If the **VSWR** is measured to be **5** on a **lossless**, **50Ω line** and a voltage **minima** occurs **1/3 of a wavelength** from the load, determine the value of the load $\mathbf{Z_R}$.

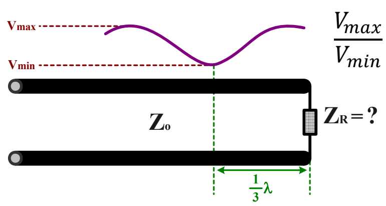
$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad |\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1} = \frac{5-1}{5+1} = \frac{4}{6} = 0.667$$



The 1st and 3rd bottom-left scales provide a mapping from |Γ| ↔ VSWR

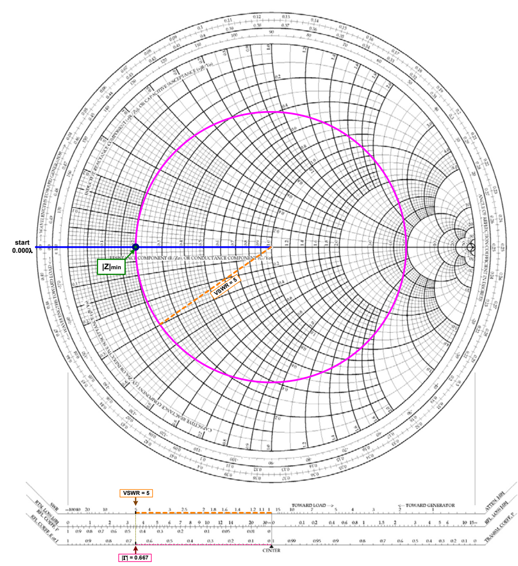
Textbook Example 4-3



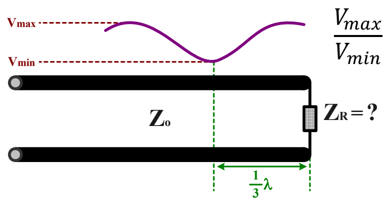
$$\frac{V_{max}}{V_{min}} = 5$$

If the **VSWR** is measured to be **5** on a **lossless**, **50Ω line** and a voltage **minima** occurs **1/3 of a wavelength** from the load, determine the value of the load $\mathbf{Z_R}$.

Note that a **voltage minimum** in the SWR pattern occurs wherever there is an **impedance minimum** on the line.



Textbook Example 4-3



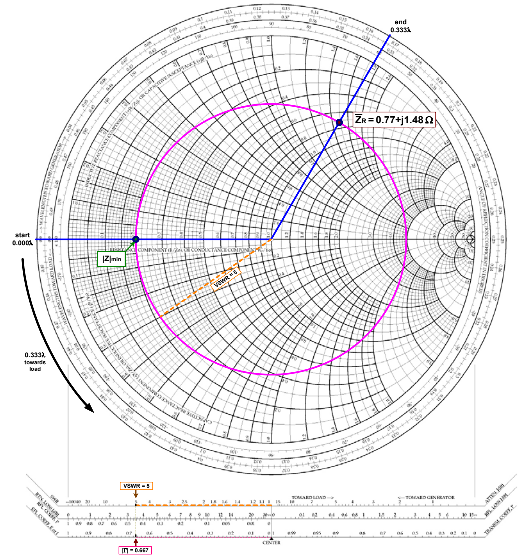
$$\frac{V_{max}}{V_{min}} = 5$$

If the **VSWR** is measured to be **5** on a **lossless**, **50Ω line** and a voltage **minima** occurs **1/3 of a wavelength** from the load, determine the value of the load $\mathbf{Z_R}$.

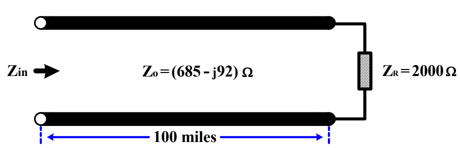
Therefore, rotate 1/3λ from Z_{min} towards the load to find Z_R.

$$\overline{Z}_R = (0.77 + j1.48)\Omega$$

$$Z_R = \overline{Z}_R \cdot Z_o = (38.5 + j74)\Omega$$



Textbook Example 4-5

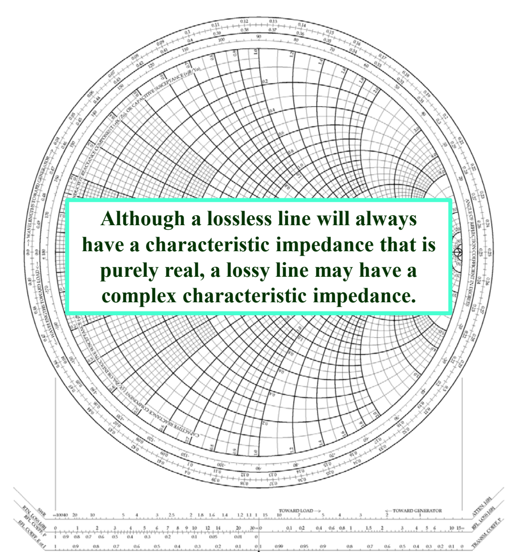


Determine the **input impedance** of a **100-mile** long, **lossy line** that is terminated with a **2000Ω load** if the line's **propagation constant** is:

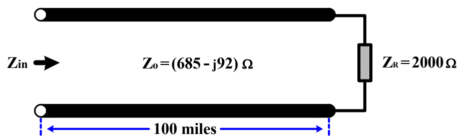
$$\gamma = \alpha + j\beta = 0.00497\ \tfrac{nepers}{mile} + j0.0352\ \tfrac{radians}{mile}$$

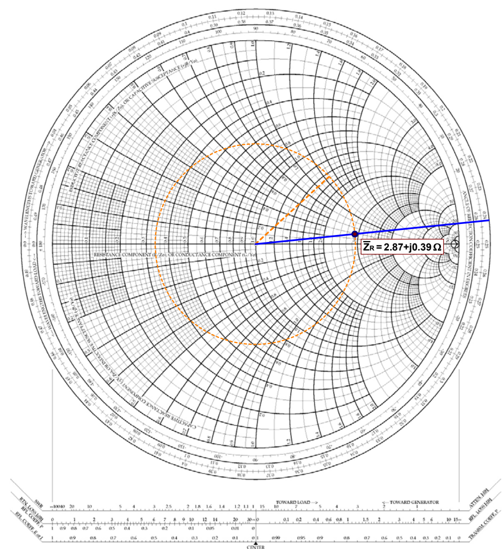
and its **characteristic impedance** is:

$$Z_o = (685 - j92)\Omega$$



Textbook Example 4-5





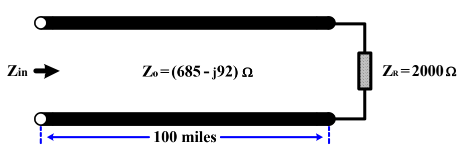
Step 2: Normalize and plot the known load impedance.

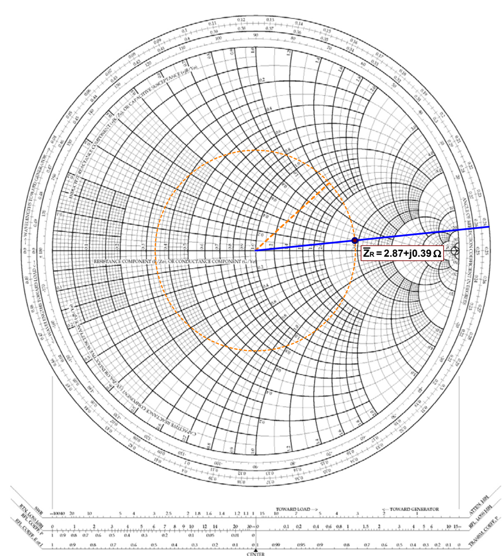
$$\overline{Z}_R = \frac{Z_R}{Z_o} = \frac{2000}{685 - j92} = (2.87 + j0.39)\Omega$$

Although it is not necessary to draw the constant |Γ| circle, it may help when performing the first part of the solution.

25

Textbook Example 4-5





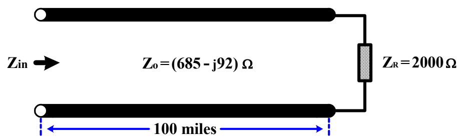
Step 3: Determine the length of the line in wavelengths:

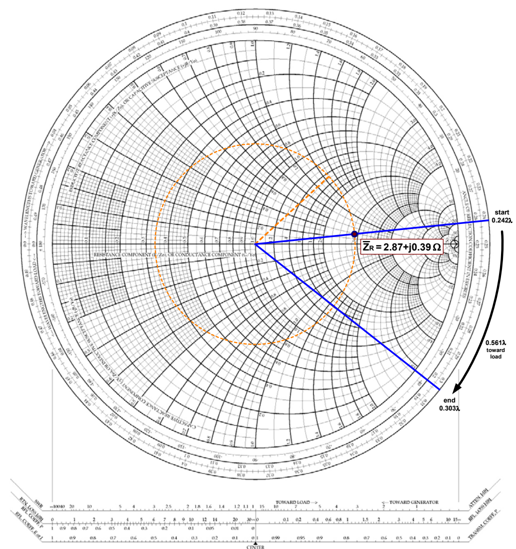
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0352} = 178.4 \text{ miles}$$

$$L = \frac{100 \text{ miles}}{178.4 \frac{\text{miles}}{\text{wavelength}}} = 0.561 \text{ wavelengths}$$

26

Textbook Example 4-5



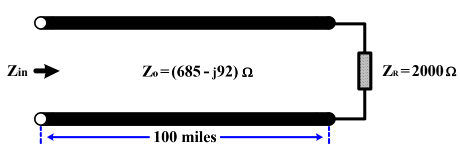


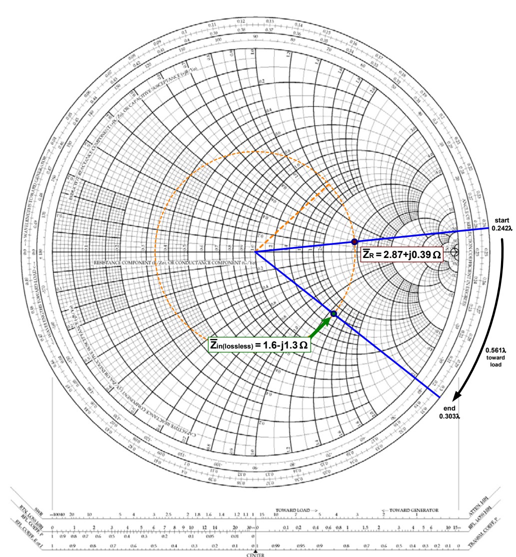
Step 4: Determine the starting and ending marker positions on the 1st band (Toward Generator).

START	**0.242**
length	**+ 0.561**
END	**0.803**
	- 0.500
Equivalent END	**0.303**

27

Textbook Example 4-5



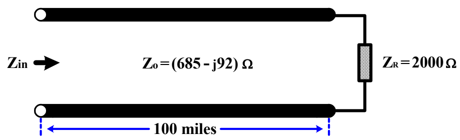


Note: If you determine the value of the impedance where the ending line crosses the constant |Γ| circle that passes through the normalized load, the result would be the normalized input impedance for a <u>lossless</u> line.

$$\overline{Z}_{in(lossless)} = (1.6 - j1.3)\Omega$$

28

Textbook Example 4-5



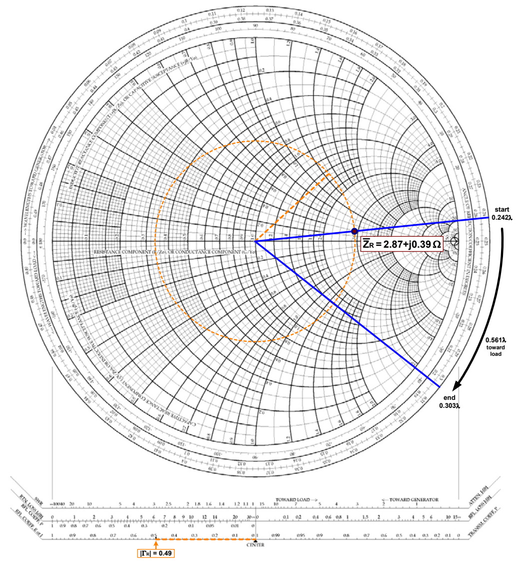
Step 5: Determine the $|\Gamma_R|$ using the 3rd scale and the calculate the $|\Gamma_{in}|$.

$$|\Gamma_R| = 0.49$$

$$|\Gamma_{in}| = |\Gamma_R| e^{-2\cdot\alpha\cdot L}$$

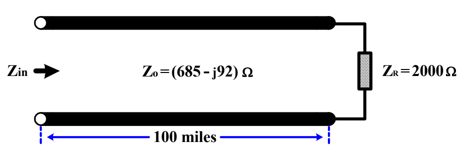
$$= (0.49)\cdot e^{-2\cdot(0.00497)\cdot(100)}$$

$$= (0.49)\cdot(0.37) = \boxed{0.18}$$



29

Textbook Example 4-5

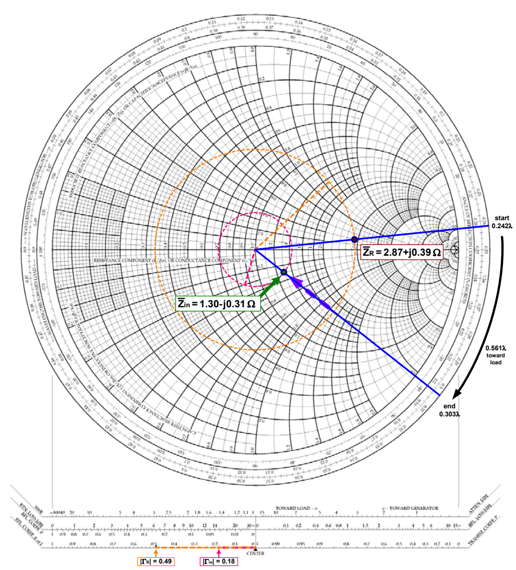


Step 6: Using the 3rd scale to determine the distance relating to $|\Gamma_{in}|$.

Z_{in} will be located this distance from the origin on the ending line.

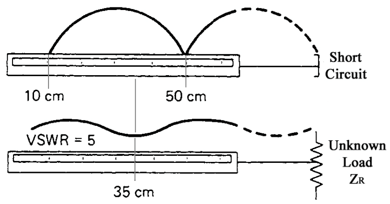
$$\overline{Z}_{in} = (1.3 - j0.31)\Omega$$

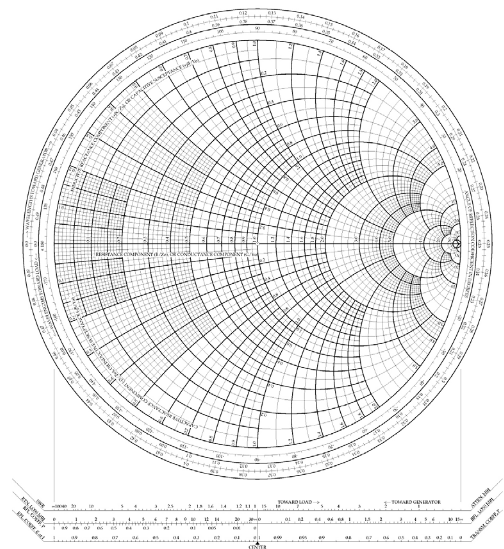
$$Z_{in} = \overline{Z}_{in}\cdot Z_o = (861 - j332)\Omega$$



30

Textbook Example 5-1





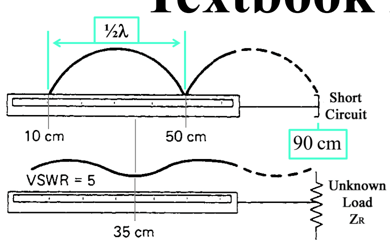
When a slotted-line is terminated with a short-circuit, voltage minima are detected at positions of 10cm and 50cm.

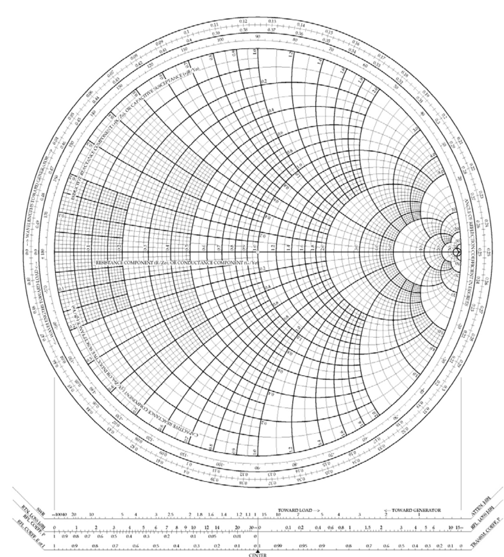
When the short-circuit is replaced by an unknown load, a voltage minimum is located at 35cm and the VSWR is 5.

Determine the value of the load $\mathbf{Z_R}$.

31

Textbook Example 5-1





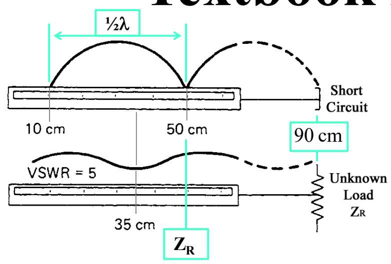
Since the standing-wave pattern repeats every ½ wavelength, then:

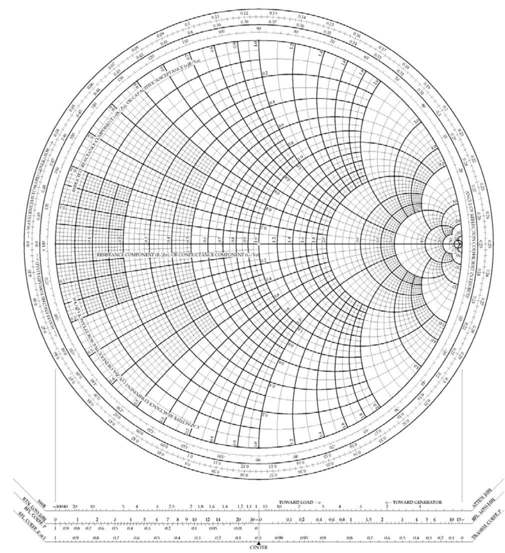
$$\lambda = 2 \cdot (50 - 10) = 80 \text{ cm}$$

Additionally, by extending the pattern in ½ λ increments, the location of the short-circuit is determined to be 90cm.

32

Textbook Example 5-1



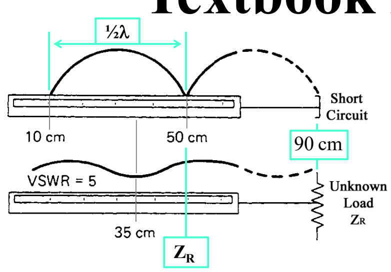


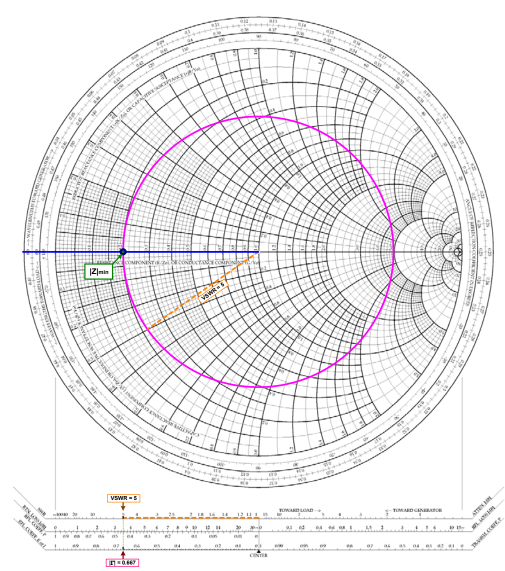
It is assumed that, when the short-circuit is replaced by the load Z_R, the load will also be located at the 90cm position.

Furthermore, based on transmission-line theory, the same impedance should also be seen every ½ wavelength down the line… (at 50cm and 10cm).

33

Textbook Example 5-1





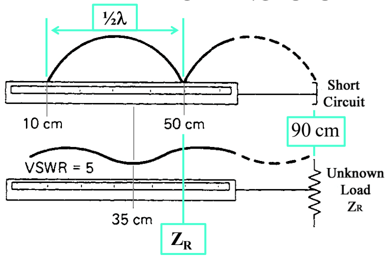
To determine the value of the load Z_R:

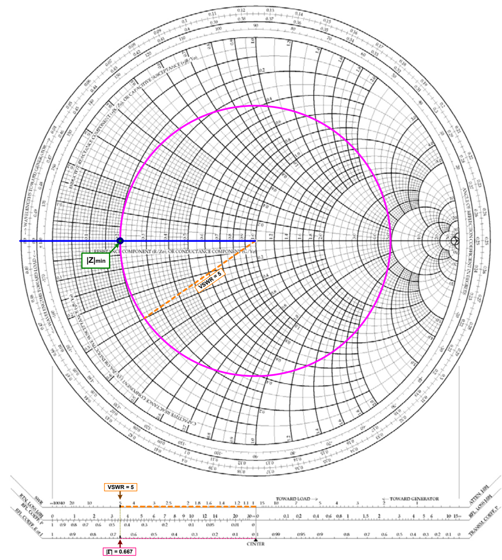
1) Draw a circle on the Smith Chart that relates to a VSWR of 5.

Note that a minimum in the SWR pattern occurs whenever the magnitude of the impedance on the line is at a minimum, which occurs at marker 0.000 on the SC.

34

Textbook Example 5-1





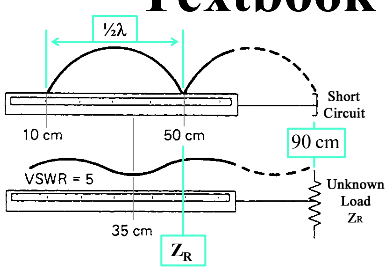
To determine the value of the load Z_R:

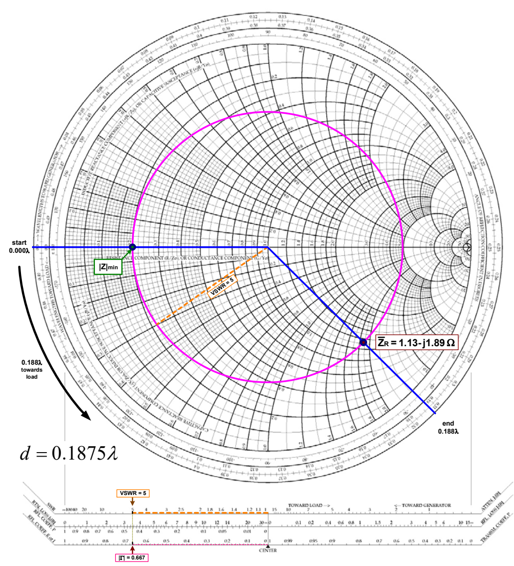
2) Determine the distance from the load minimum to the location of the load impedance in wavelengths:

$$d = (50 - 35) = 15\text{cm} = \frac{15\text{cm}}{80\,\frac{\text{cm}}{\lambda}} = 0.1875\lambda$$

35

Textbook Example 5-1





To determine the value of the load Z_R:

3) Beginning at the minimum, rotate that distance (Toward the Load) on the SC to determine the load value:

$$\overline{Z}_R = (1.13 - j1.89)\Omega$$

$$Z_R = \overline{Z}_R \cdot Z_o = (56.5 - j94.5)\Omega$$

36