ECET 3410

High Frequency Systems

Introduction to
Smith Charts

1

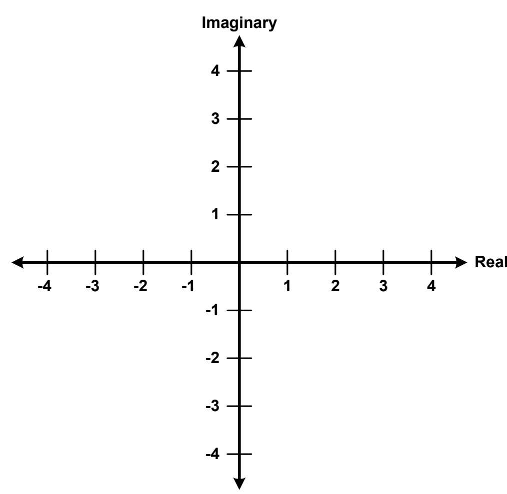
Rectangular Coordinates

A **rectangular coordinate plane** can be used to plot a complex number, **Z**, that is expressed in the form:

$$\mathbf{Z = a + jb}$$

where: **a** is the value of the **real** part of **Z**, and

b is the value of the **imaginary** part of **Z**.



2

Rectangular Coordinates

Note that, when in the form:

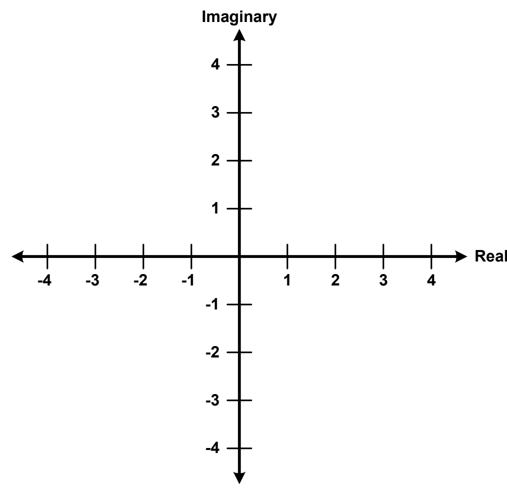
$$\mathbf{Z = a + jb}$$

the complex number **Z** is often referred to as being expressed in either:

rectangular coordinates

or

rectangular form.



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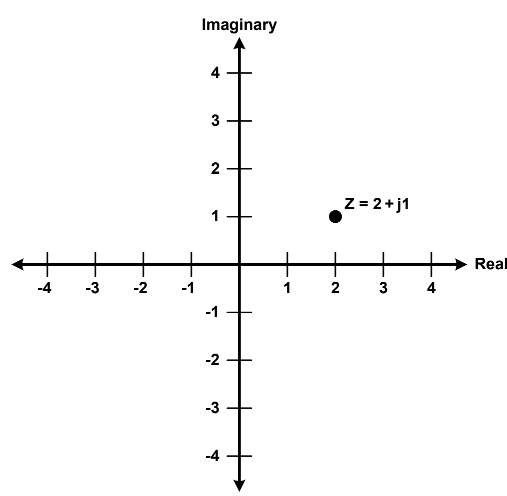
Rectangular Coordinates

When analyzing steady-state AC systems, **impedances** are often defined as complex numbers expressed in rectangular form.

Thus, given the impedance:

$$\mathbf{Z = 2 + j1}$$

the impedance may be plotted as a point on a rectangular coordinate plane as shown:



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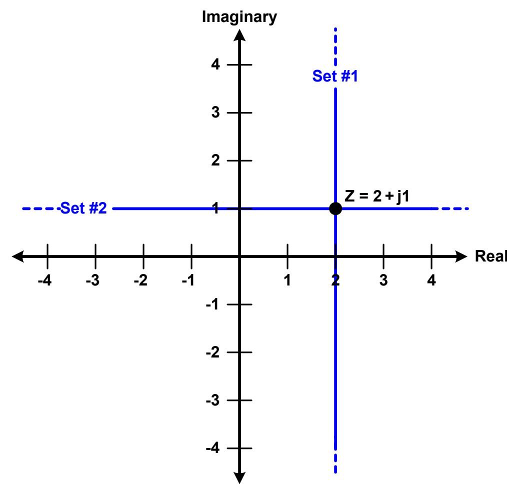
Rectangular Coordinates

The point **Z = 2 + j1** may be thought of as the intersection between two sets of points:

Set#1 – The set of all points having a real value of "2"

Set#2 – The set of all points having an imaginary value of "1"



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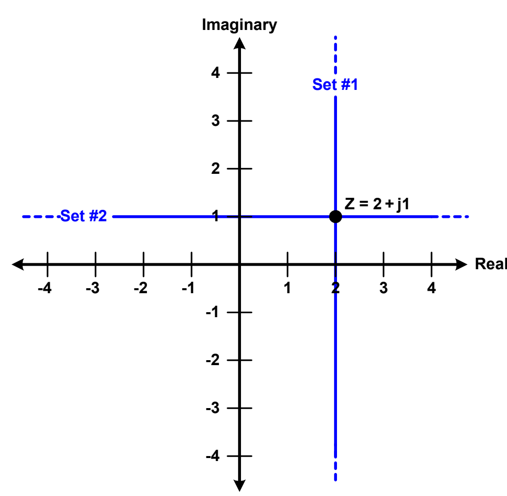
Rectangular Coordinates

Note that, when drawn on a rectangular coordinate plane:

- The set of points (Set#1) having a constant **real** value appears as a **vertical line**, and
- The set of points (Set#2) having a constant **imaginary** value appears as a **horizontal line**.



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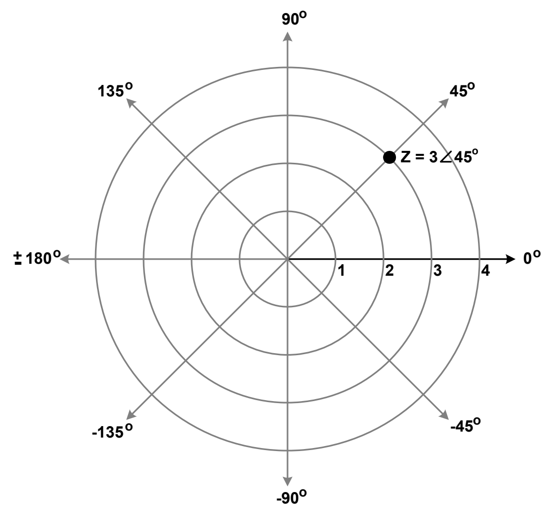
The Polar Coordinate System

The **polar coordinate system** provides another method for plotting complex numbers provided that they are expressed in "polar" form ($C\angle\theta$),

where: **C** is the distance of the point from the origin,

θ is the angle between the origin-to-point segment and the right-hand side of the horizontal axis.

For example: $\mathbf{Z = 3\angle 45°}$



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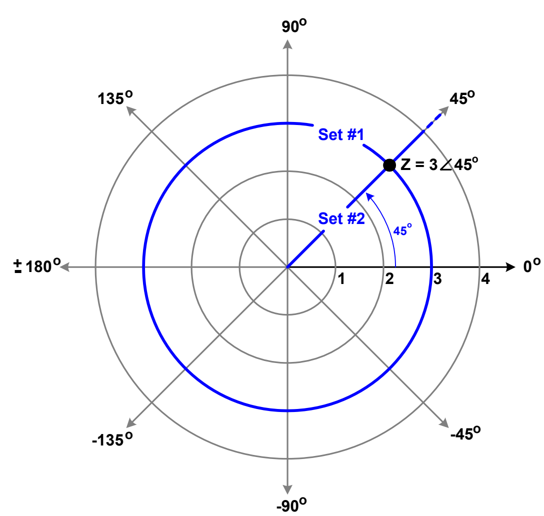
The Polar Coordinate System

The point $\mathbf{Z = 3\angle 45°}$ can be may be thought of as the intersection between two sets of points:

Set#1 – The set of all points having a distance of "**3**" from the origin

Set#2 – The set of all points that form an angle of "**45°**" with the right-hand side of the horizontal axis



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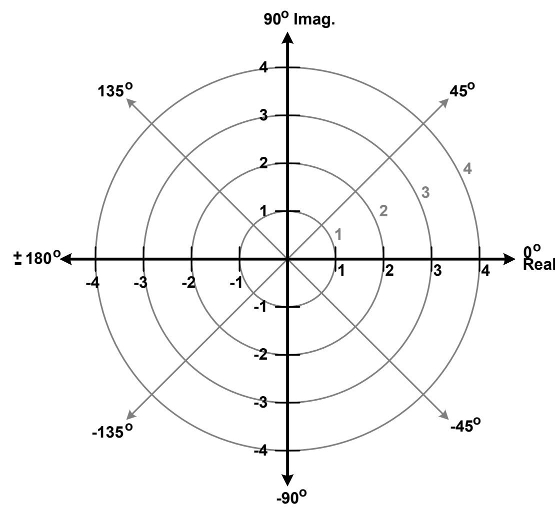
Rectangular ↔ Polar Coordinate Mapping

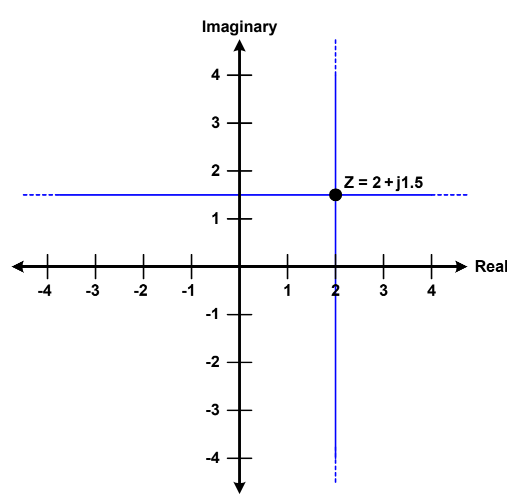
When overlaid such that they share a common origin and they are given the same scale, the rectangular and polar coordinate plots provide a **mapping** between complex numbers expressed in rectangular coordinates and complex numbers expressed in polar coordinates.



Rectangular ↔ Polar Coordinate Mapping

For example: **Z = 2 + j1.5**

The point **Z = 2 + j1.5** can be shown on a rectangular coordinate plot.



Rectangular ↔ Polar Coordinate Mapping

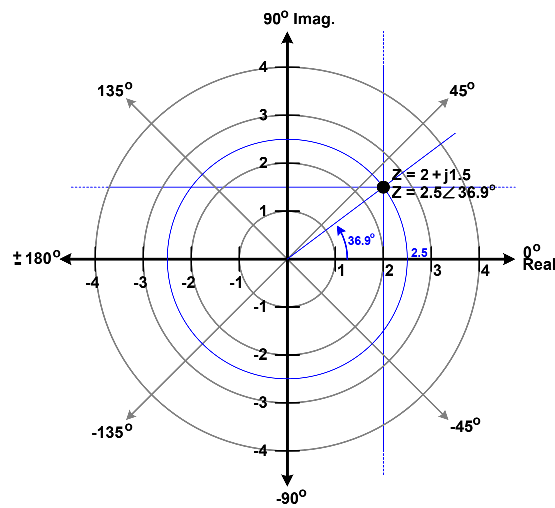
For example: **Z = 2 + j1.5**

The point **Z = 2 + j1.5** can be shown on a rectangular coordinate plot.

If a polar coordinate plot is overlaid on top of the rectangular plot and the point's value is read in polar coordinates, it value will be:

Z = 2.5∠36.9°

Thus: **Z = (2 + j1.5) = (2.5∠36.9°)**



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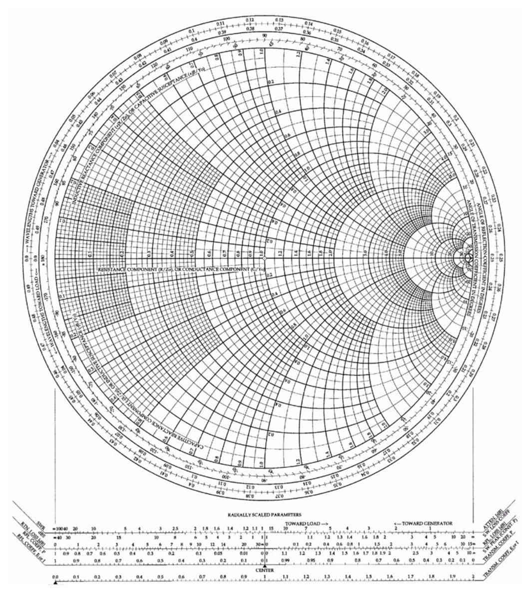
Introduction to Smith Charts

A ***Smith Chart*** may be thought of as a plot for impedances that are expressed as complex numbers in rectangular form, similar to a rectangular coordinate plane but with a completely different coordinate system.



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Introduction to Smith Charts

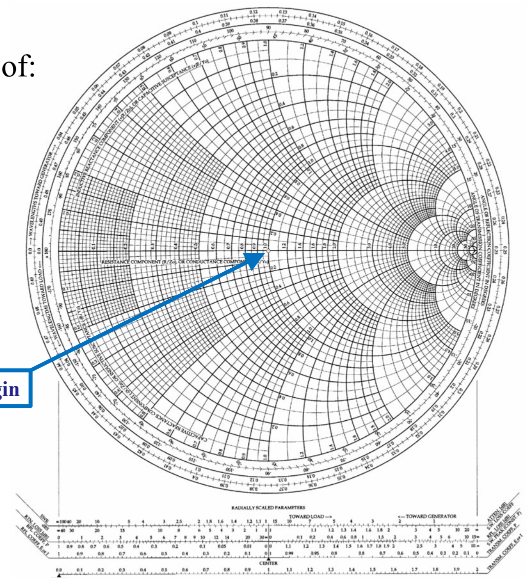
Smith Charts actually provide a **mapping** of:

Reflection Coefficient
(in Polar Form)
to
Load Impedance
(in Rectangular Form),

based on the equation:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

provided the impedance plane is scaled such that its <u>origin</u> is equal to Z_o, the characteristic impedance of the line.



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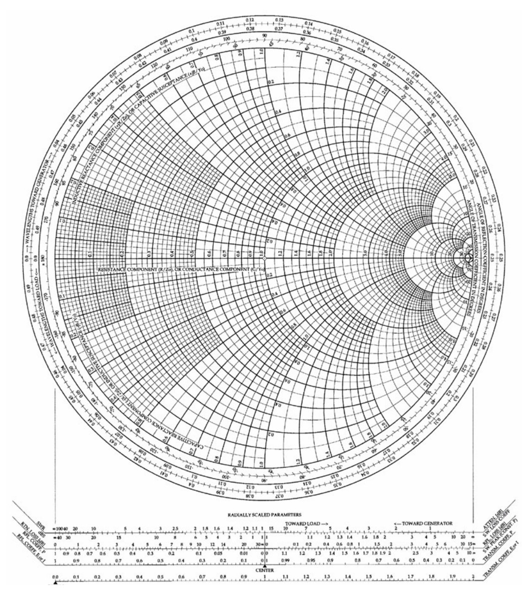
Introduction to Smith Charts

In other words:

Given a transmission-line that is terminated by <u>load impedance</u> $\boldsymbol{Z_R}$ (expressed in rectangular form), a **Smith Chart** may be utilized to graphically determine the <u>reflection coefficient of the load</u> $\boldsymbol{\Gamma_R}$ (expressed in polar form).

Note that the reverse is also true:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} \quad \Leftrightarrow \quad Z_R = R + jX$$



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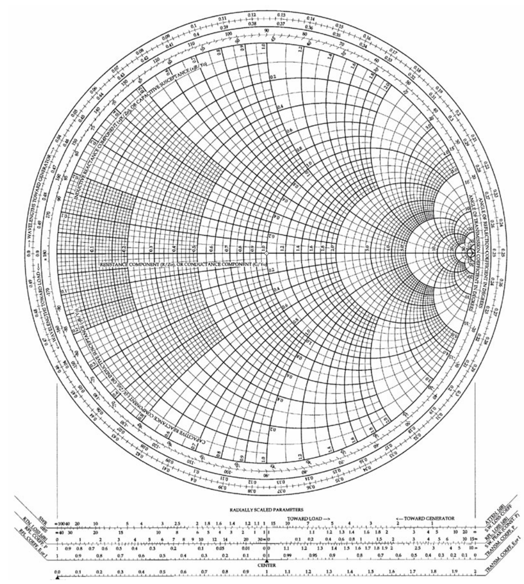
Introduction to Smith Charts

Understanding the layout of the Smith Chart’s complex number coordinate plane along with the steps required to both plot and read impedance values on a Smith Chart is an essential part of this process.

But this can be complicated by the amount of information displayed on and around the Smith Chart.



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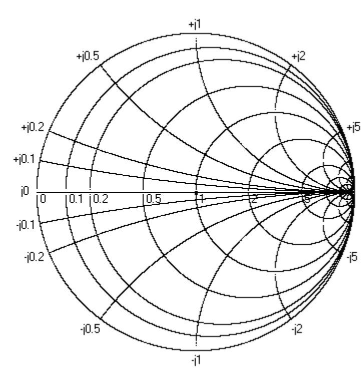
The Smith Chart Coordinate System

Shown to the right is a very simple version of only the impedance portion of a Smith Chart…

“simple” such that very few sets of points (circles and arcs) are displayed on the chart.

The **circles** and **arcs** provide a coordinate system that may be used to plot complex numbers expressed in rectangular form.

Basic Smith Chart



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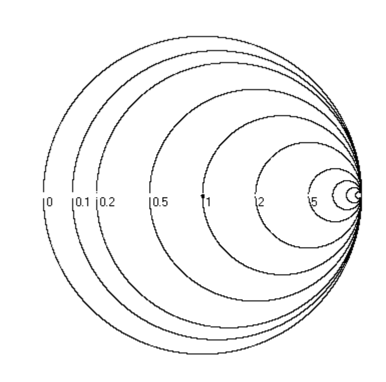
The Smith Chart Coordinate System

Sets of points with constant **real** values are defined by **circles** on the Smith Chart.

Note that only the circles relating to sets of points having non-negative real values are utilized since (passive) impedances are restricted to having non-negative resistances.

Real Values



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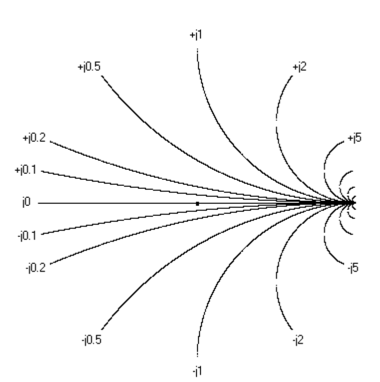
The Smith Chart Coordinate System

Sets of points with the same **imaginary** value are defined by **arcs** on the Smith Chart.

Note that the set of points defined by a specific arc are a subset of the points that define a circle on which the points have a constant imaginary value, but with the points further constrained to have only non-negative real values.

Imaginary Values



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Plotting an Impedance on a Smith Chart

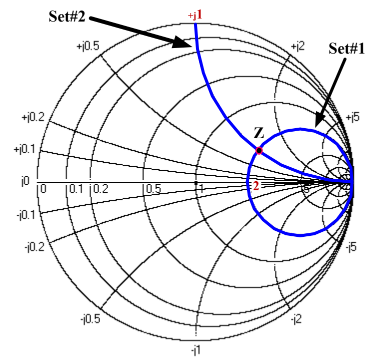
Let use go back and look at the impedance point: **Z = 2 + j1**.

This point, **Z**, can be thought of as the intersection of two sets of points:

- one having a **real** value of **2**, and
- the other having an **imaginary** value of **+1**,

similar the vertical and horizontal sets shown previously on the rectangular coordinate plot.

Real Circles and Imaginary Arcs



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Plotting an Impedance on a Smith Chart

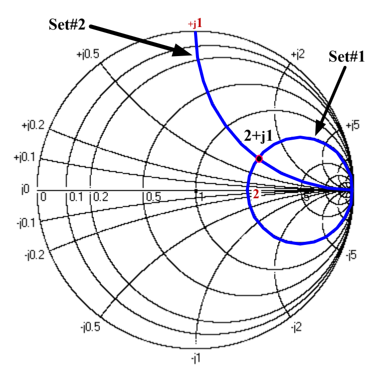
<u>But on the Smith Chart</u>:

Set#1 is defined by a circle on which every point has a real value of "**2**"

Set#2 is defined by an arc on which every point has an imaginary value of "**1**"

The intersection of which is the point **Z = 2 + j1**.

Real Circles and Imaginary Arcs



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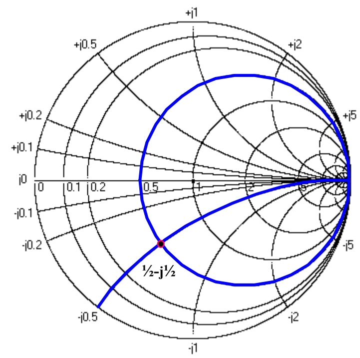
Plotting an Impedance on a Smith Chart

Similarly, the point **Z = ½ - j½** can be found at the intersection of the circle the has real values of "½" and the arc that has imaginary values of "**-½**".

Real Circles and Imaginary Arcs



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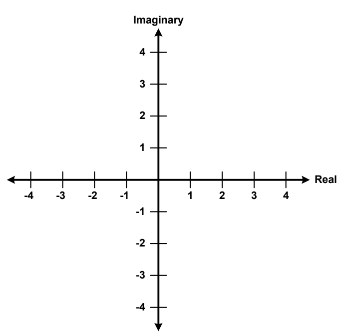
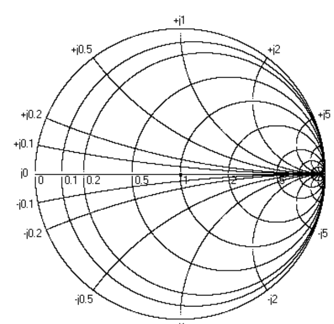
The Smith Chart Coordinate System

Smith Charts vs. Rectangular Coordinate Plots





In order to better understand the characteristics of a Smith Chart, let's begin by comparing and contrasting it to a rectangular coordinate plot.

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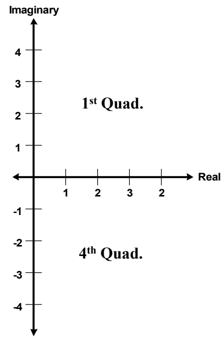
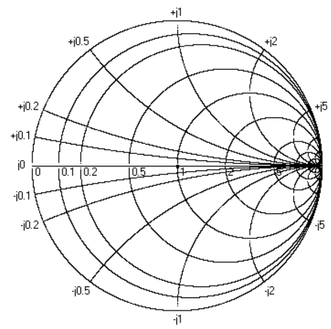
The Smith Chart Coordinate System

Smith Charts vs. Rectangular Coordinate Plots





Smith Charts only show **non-negative real-value** impedances, which is equivalent to displaying only the **1st and 4th quadrants of a rectangular coordinate plot**.

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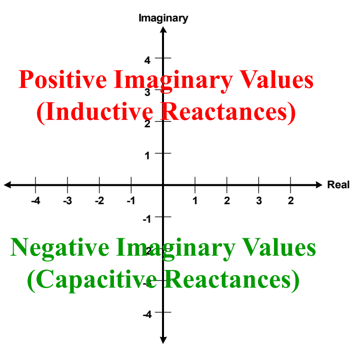
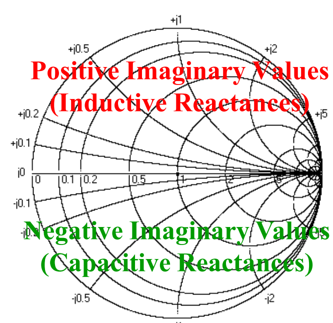
The Smith Chart Coordinate System

Smith Charts vs. Rectangular Coordinate Plots





Positive imaginary values appear on the **top half of both plots** and **negative imaginary values** appear on the **bottom half of both plots**.

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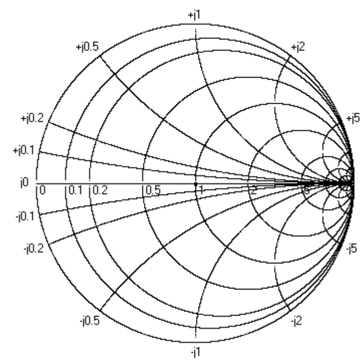
The Smith Chart Coordinate System

Smith Charts vs. Rectangular Coordinate Plots

On a rectangular coordinate plot, magnitudes of both the real and imaginary parts from 0 → 1 take up much less physical space than magnitudes ranging from 1 → ∞.

But the Smith Chart tends to expand or spread-out impedances whose magnitudes range from 0 → 1 and compress those whose magnitudes range from 1 → ∞.

For this reason, it is often easier to accurately plot/read small impedances on a Smith Chart.



Converting Impedances to Admittances

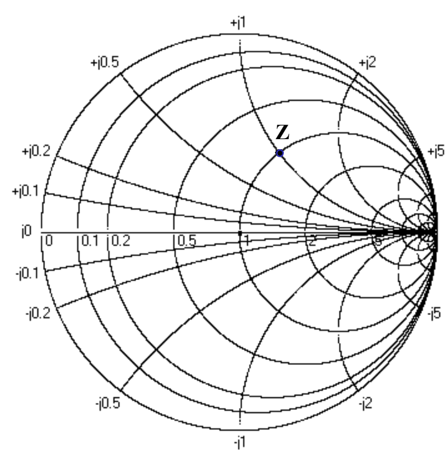
<u>**Utilizing a Smith Chart**</u>**:**

A simple method may by applied using a Smith Chart to invert a complex number.

For example, given the impedance:

$$\mathbf{Z = 1 + j1\ \Omega}$$

the admittance value relating to this impedance is **Y = 1/Z**.

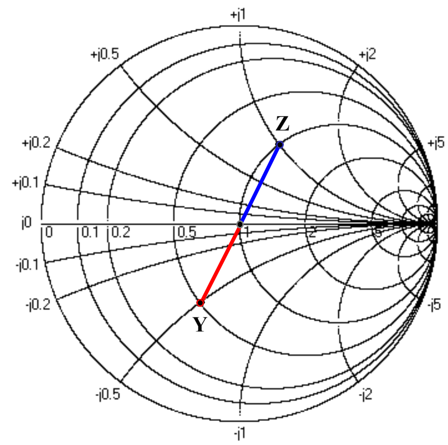


Converting Impedances to Admittances

<u>To invert a complex impedance Z</u>:

Plot the impedance **Z** on the Smith Chart and then find the point on the chart that is ***equidistant from the origin but on the exact opposite side of the chart***.

Given the impedance **Z = 1 + j1 Ω**, the admittance determined by using this method is **Y = ½ - j½.**



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Impedance ⇔ Reflection Coefficient Mapping

A Smith Chart provides a **mapping** of:

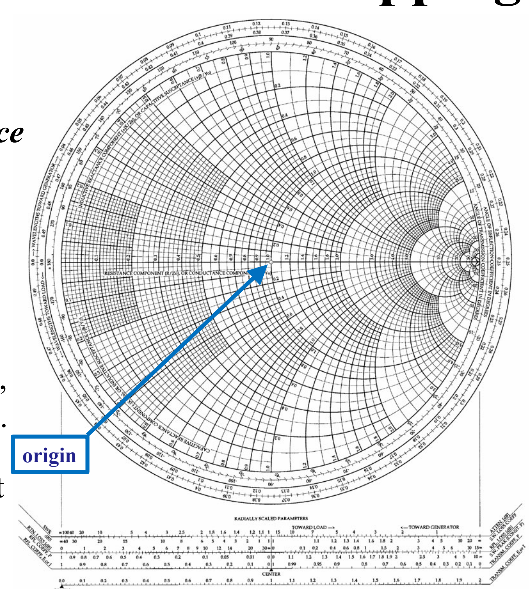
Reflection Coefficient to ***Load Impedance***

based on the equation:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

provided that the impedance plane is scaled such that its <u>origin</u> is equal to Z_o, the characteristic impedance of the line.

But, this would require a new Smith Chart for each type of line with a different characteristic impedance.



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Normalized Smith Charts

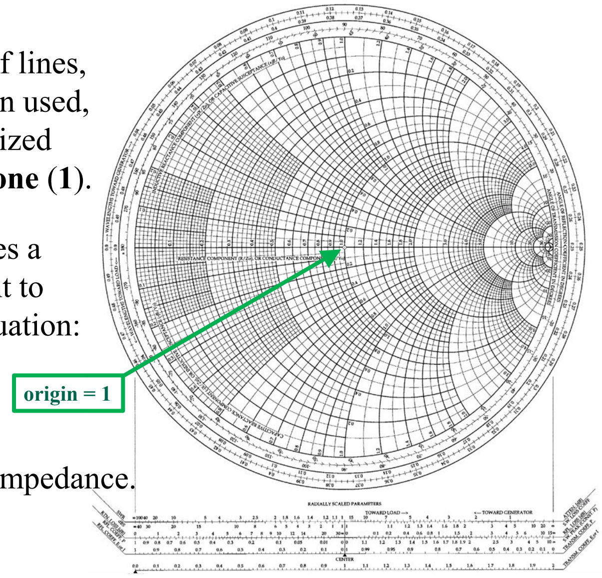
In order to accommodate a variety of lines, a **normalized** Smith Chart is often used, such that the origin of the normalized chart has an impedance value of **one** (**1**).

The normalized Smith Chart provides a mapping of Reflection Coefficient to Load Impedance based on the equation:

$$\Gamma_R = \frac{\overline{Z}_R - 1}{\overline{Z}_R + 1}$$

where $\overline{Z}_R$ is the normalized load impedance.

$$\overline{Z}_R = \frac{Z_R}{Z_o}$$



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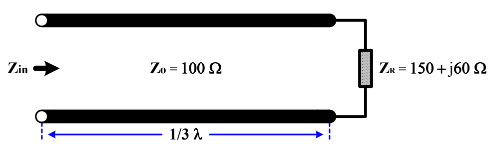
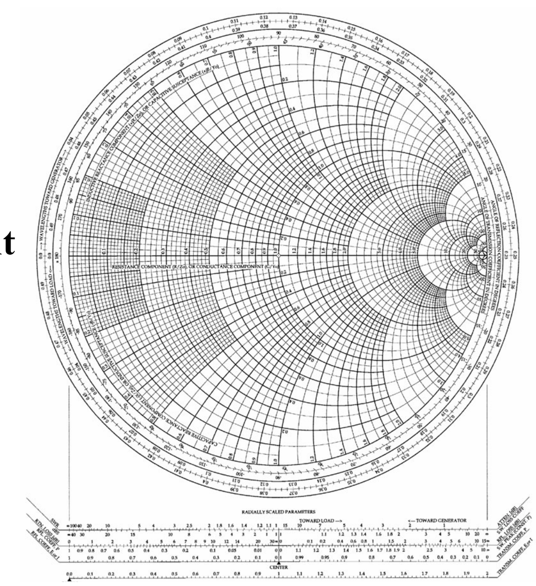
Impedance ⇔ Reflection Coefficient Mapping



Determine the **reflection coefficient** of a $\mathbf{Z_R = (150 + j60)\Omega}$ load that terminates a $\mathbf{Z_o = 100\Omega}$ line:



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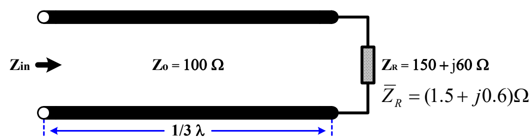
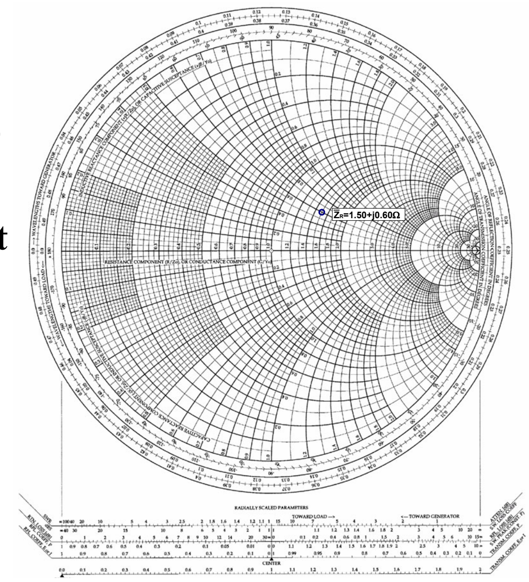
Impedance ⇔ Reflection Coefficient Mapping



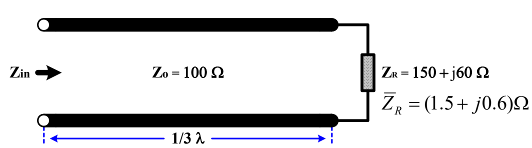


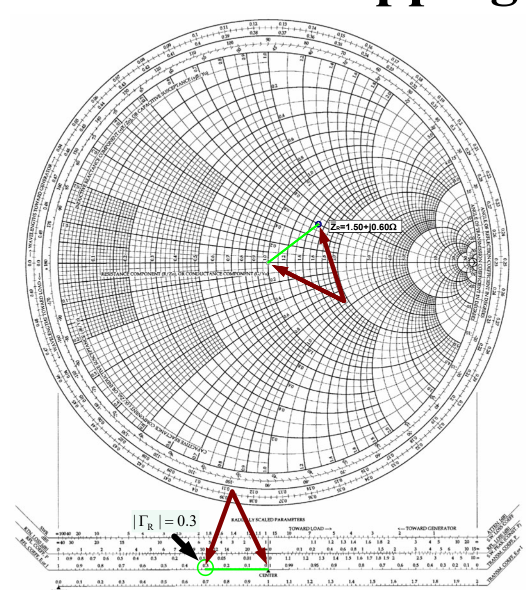
Determine the **reflection coefficient** of a $\mathbf{Z_R = (150 + j60)\Omega}$ load that terminates a $\mathbf{Z_o = 100\Omega}$ line:

Step 1: Normalize and plot the load impedance.

$$\overline{Z}_R = \frac{Z_R}{Z_o} = \frac{150 + j60}{100} = (1.5 + j0.6)\Omega$$

Impedance ⇔ Reflection Coefficient Mapping

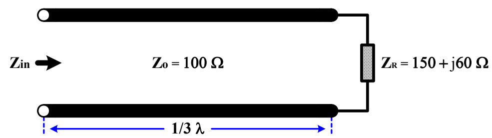




Step 2: Determine the $|\Gamma_R|$ by measuring the distance from $\mathbf{Z_R}$ to the **origin** and then reading the value of that distance to the <u>left of the center point</u> on the **3rd scale (RFL COEFF E or I)** at the bottom of the Smith Chart.

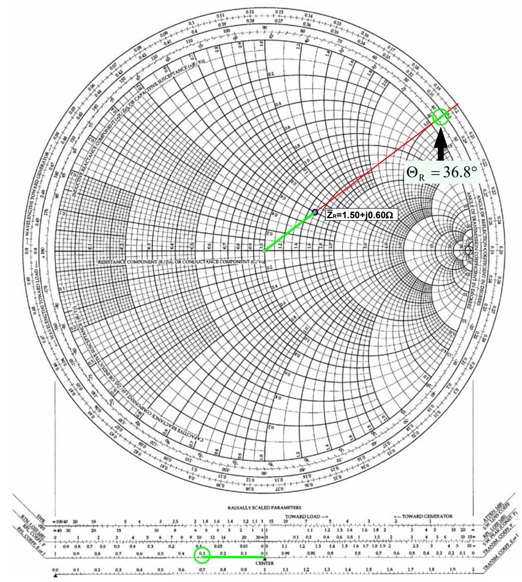
$$|\Gamma_R| = 0.3$$

Impedance ⇔ Reflection Coefficient Mapping



Step 3: Determine the $\angle\Gamma_R$ by reading the position that a line beginning at the origin and passing through **Z_R** crosses the **3rd band (Angle of Reflection Coefficient in Degrees)** around the outer portion of the chart.

$$\Theta_R = 36.8°$$



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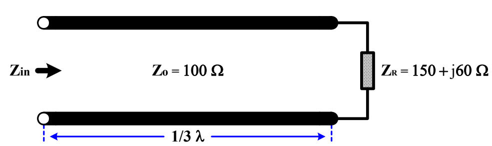
Impedance ⇔ Reflection Coefficient Mapping

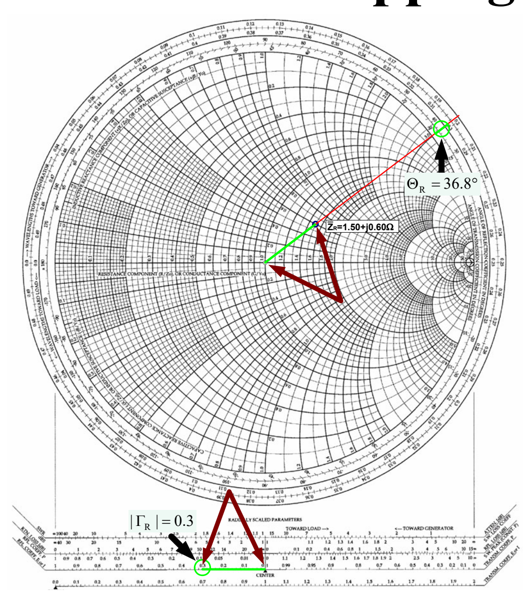


Thus, Γ_R is:

$$\Gamma_R = 0.3\angle 36.8°$$

Checking the result:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} = \frac{50 + j60}{250 + j60} = 0.304\angle 36.7°$$



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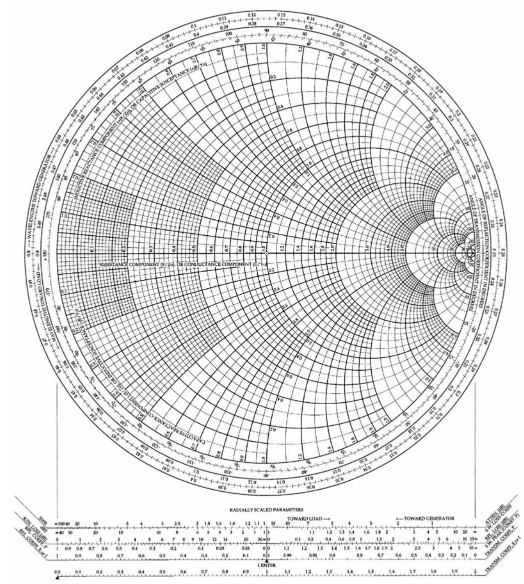
Transmission Line Analysis using Smith Charts

Now that we have explored the Smith Chart's complex impedance plane and its mapping of reflection coefficient to impedance, we are ready to begin a more detailed analysis of **Smith Charts** and their application to the **solution of transmission line problems**.



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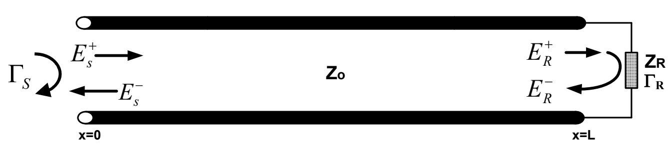
Smith Charts & Reflection Coefficients



Since the expression relating Z_R and Γ_R is in the same form as the expression relating Z_{in} and Γ_{in}:

$$Z_R = Z_o \cdot \frac{1+\Gamma_R}{1-\Gamma_R} \qquad Z_{in} = Z_o \cdot \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$$

the mapping between impedance and reflection coefficient on a Smith Chart can also be applied to the input of a line.

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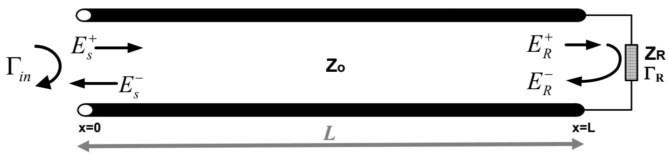
Smith Charts & Reflection Coefficients



But, the key to preforming a Smith Chart analysis of a transmission line problem is the relationship between the reflection coefficient due to a load, Γ_R, and the reflection coefficient seen at the input of a line, Γ_{in}:

$$\Gamma_{in} = \Gamma_R \cdot e^{-2\cdot\gamma\cdot L}$$

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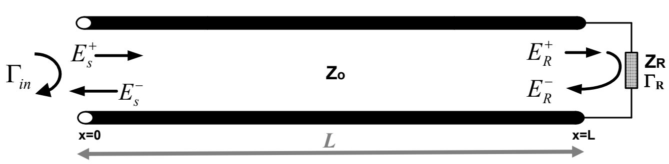
Investigating the Relationship $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$



The relationship:

$$\Gamma_{in} = \Gamma_R \cdot e^{-2\cdot\gamma\cdot L}$$

can be expanded by substituting $\gamma = \alpha + j\beta$, such that:

$$\Gamma_{in} = \Gamma_R \cdot e^{-2\cdot\gamma\cdot L} = \Gamma_R \cdot e^{-2\cdot(\alpha + j\beta)\cdot L} = \Gamma_R \cdot (e^{-2\cdot\alpha\cdot L} \cdot e^{-j2\cdot\beta\cdot L}) = \Gamma_R \cdot (e^{-2\cdot\alpha\cdot L} \angle -2\beta L)$$

where: α is the **attenuation constant** for the line (Np/m), and
β is the **phase constant** for the line (rad/m).

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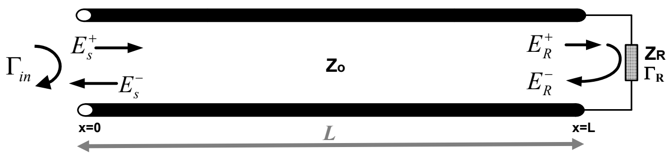
Investigating the Relationship $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$



If the reflection coefficients are expressed in polar form:

$$\Gamma_{in} = |\Gamma_{in}| \angle \theta_{in} \qquad \Gamma_R = |\Gamma_R| \angle \theta_R$$

then the reflection coefficient relationship can be rewritten as:

$$\Gamma_{in} = \left(|\Gamma_R| \angle \theta_R \right) \cdot (e^{-2 \cdot \alpha \cdot L} \angle -2\beta L) = |\Gamma_R| e^{-2 \cdot \alpha \cdot L} \angle (\theta_R - 2\beta L)$$

where: $|\Gamma_{in}| = |\Gamma_R| e^{-2 \cdot \alpha \cdot L}$ and $\theta_{in} = \theta_R - 2\beta L$

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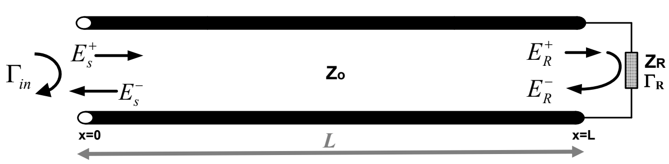
Investigating the Relationship $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$



Given the relationship:

$$|\Gamma_{in}| = |\Gamma_R| e^{-2 \cdot \alpha \cdot L}$$

*** – the set of points that are all equidistant from the origin of the Smith Chart forms a circle that is centered at the origin.**

if $\boldsymbol{\alpha > 0}$, then the magnitude of the input reflection coefficient will decrease exponentially as the length of the line, ***L***, increases.

But, if $\boldsymbol{\alpha = 0}$ (lossless line), then $|\Gamma_{in}| = |\Gamma_R|$. When this occurs, both points will be **equidistant from the origin** of the Smith Chart*.

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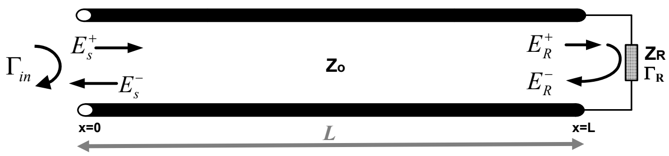
Investigating the Relationship $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$



Furthermore, given the relationship:

$$\theta_{in} = \theta_R - 2\beta L$$

*** – when expressed in polar form, complex numbers repeat periodically with every ± 360° change in angle. A∠θ = A∠θ ± 360°**

since $\mathbf{\beta > 0}$, the angle of the input reflection coefficient will decrease linearly as the length of the line, $\boldsymbol{L}$, increases.

But, on a polar plot, a decrease (on increase) in angle by 360° (or 2π radians) results in one complete rotation around the plot*.

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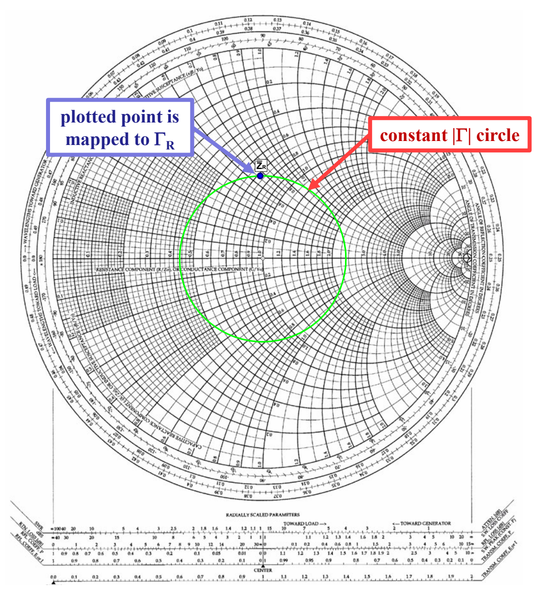
Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossless Line

Thus, on a **lossless line** (**α = 0**), since:

$$|\Gamma_{in}| = |\Gamma_R|$$

if Γ_R is located on a Smith Chart, then Γ_{in} must fall on a **circle** that is **centered about the origin** and passes through the point Γ_R.

Note that Γ_R can be located by plotting the normalized load impedance $\bar{Z}_R$.



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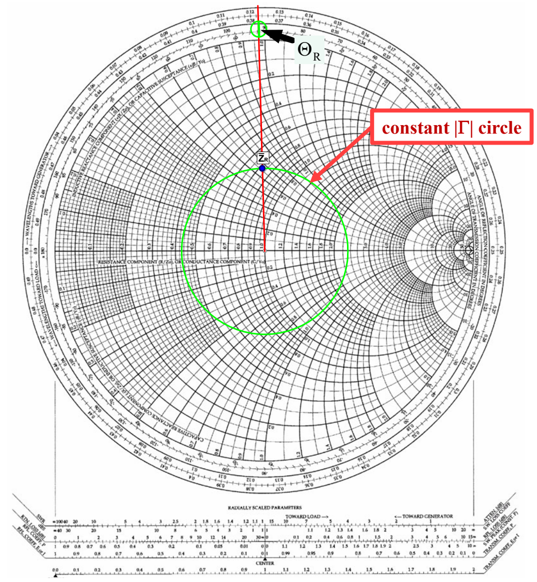
Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossless Line

If the angle θ_R is determined from the plot of the normalized load impedance, then the angle θ_{in} can be determined from:

$$\theta_{in} = \theta_R - 2\beta L$$

and the **position of Γ_{in}** will be located wherever an **angle-line relating to the angle θ_{in} crosses the constant $|\Gamma|$ circle**.



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Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossless Line

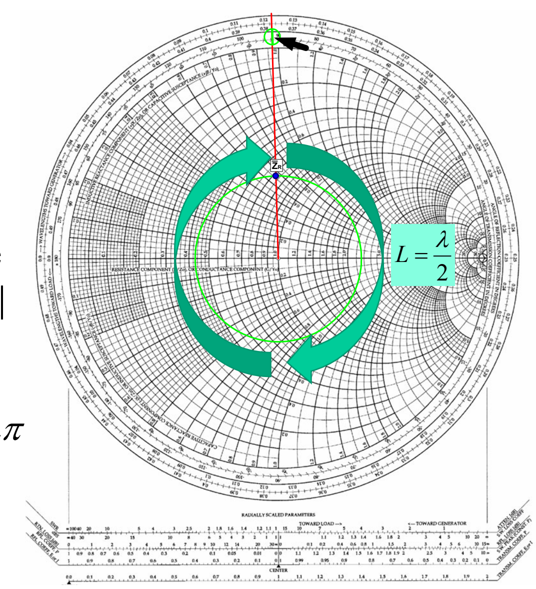
Given the relationship:

$$\theta_{in} = \theta_R - 2\beta L$$

whenever $2\beta L = 2\pi$ radians, the location of Γ_{in} will rotate **CW** one revolution around the constant $|\Gamma|$ circle beginning at Γ_R.

Note that this occurs when $2\beta L = 2\pi$ or when:

$$L = \frac{2\pi}{2\beta} = \frac{\pi}{\beta} = \frac{\pi}{2\pi/\lambda} = \frac{\lambda}{2}$$



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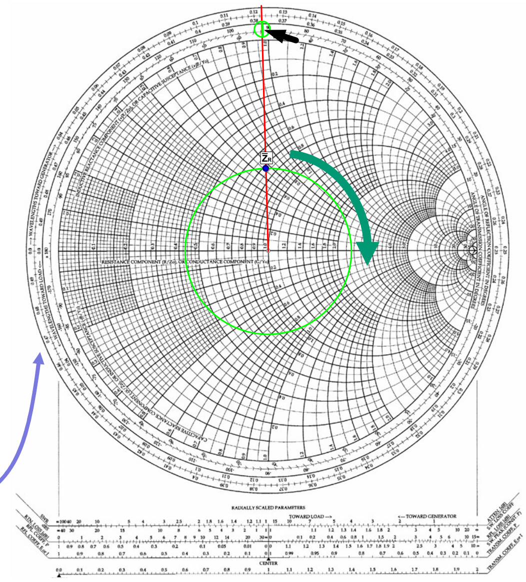
Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossless Line

And, since the relationship:

$$\theta_{in} = \theta_R - 2\beta L$$

provides a linear decrease in the angle θ_{in} as a function of line length $\boldsymbol{L}$, Γ_{in} will rotate **CW** around the constant $|\Gamma|$ circle proportional to one revolution per ½-wavelength of line length.



Note that a linear, ½–wavelength scale is included in the outer bands of the Smith Chart in order to facilitate the required rotation.

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Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossy Line

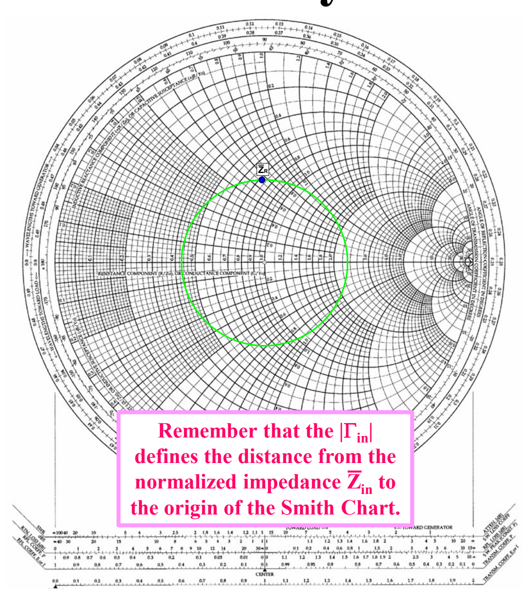
What if the line is **lossy** (**$\alpha > 0$**)?

On a **Lossy Line**:

$$|\Gamma_{in}| = |\Gamma_R| e^{-2\cdot\alpha\cdot L}$$

Since $e^{-2\cdot\alpha\cdot L}$ is an exponential decay function, the $|\Gamma_{in}|$ will decrease as the length of the line increases.

Thus, the impedance $\bar{Z}_{in}$ will be closer to the origin than $\bar{Z}_R$.



Remember that the $|\Gamma_{in}|$ defines the distance from the normalized impedance $\bar{Z}_{in}$ to the origin of the Smith Chart.

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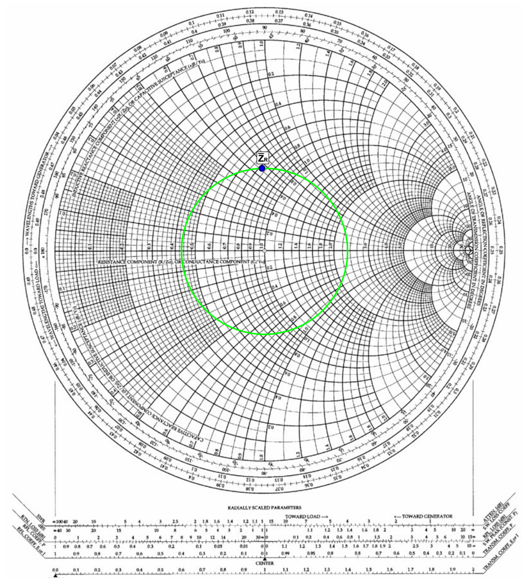
Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossy Line

To find $|\Gamma_{in}|$ on a lossy line:

- Plot $\bar{Z}_R$ and measure the distance from $\bar{Z}_R$ to the origin.
- Use the 3rd scale to determine the $|\Gamma_R|$.
- Calculate $|\Gamma_{in}|$ from:

$$\left|\Gamma_{in}\right| = \left|\Gamma_R\right| e^{-2\cdot\alpha\cdot L}$$

(Use the 3rd scale again to determine the distance of $\bar{Z}_{in}$ from the origin)



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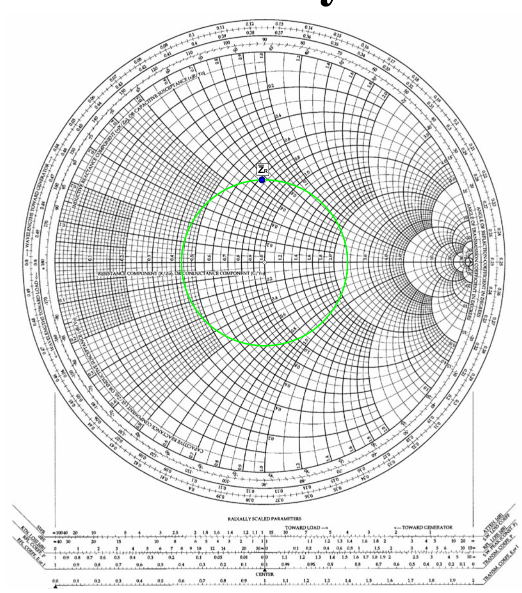
Investigating $\Gamma_{in} = \Gamma_R \cdot e^{-2\gamma L}$ on a Lossy Line

Note that the magnitude change due to the attenuation constant $\boldsymbol{\alpha}$ is <u>independent</u> of the angle change due to the phase constant $\boldsymbol{\beta}$.

$$\left|\Gamma_{in}\right| = \left|\Gamma_R\right| e^{-2\cdot\alpha\cdot L}$$

$$\theta_{in} = \theta_R - 2\beta L$$

This allows the two effects to be accounted for independently.



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