ECET 3410

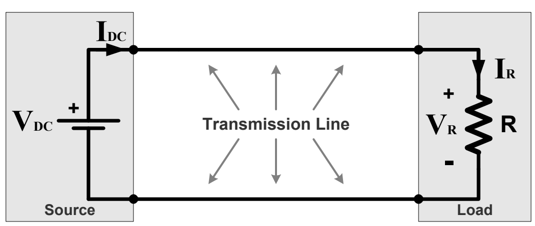
High Frequency Systems

Introduction to
Transmission Lines

1

Transmission Lines

In classic circuit theory, transmission lines are often considered to be ideal wires.



But, even if considered to be "ideal" (lossless), the concept of a transmission line also takes into account the travel velocity of the waveforms on the line.

A **transmission line** is a passive, conductor-based device that is used to transfer electric energy (voltage and current waveforms) from one location to another, such as from a source to a load.

A typical transmission line consists of two (or more) conductors that are separated by an insulating material.

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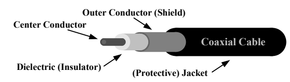
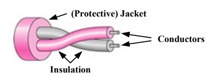
Transmission Lines

Transmission lines may appear in many different shapes and sizes.

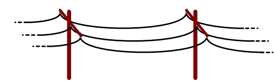
- **Coaxial Lines** (Cable/Satellite TV)



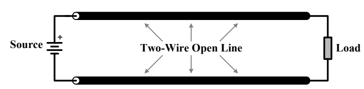
- **Twisted-Pair Lines** (Cat-5 Network Cable)



- **Three-Wire Open Lines** (Electric Power Lines)

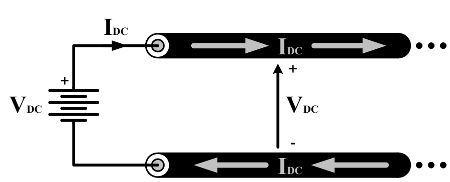


But, in order to minimize the effect of geometry, we will begin by considering the case of **Two-Wire Open Lines**.



3

Voltages & Currents on Transmission Lines



What does it mean if there is **current** flowing in the conductors of a transmission line, or if there is a **voltage** (potential difference) between the conductors of a transmission line?

It turns out that, although the concept of current (the flow of charge) is relatively straight forward, the concept of what must occur for a voltage to exist on a transmission line is less understood.

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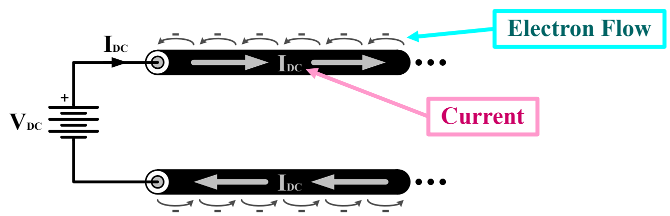
Current Flow in a Transmission Line



Note – even though it is negative charge (electrons) that flows within an electric circuit, **current** is traditionally defined based on a rate of **positive charge** flow.

Thus, if a current is defined within the conductors of a transmission line, then **electrons** are actually flowing through the conductors in the opposite direction of the defined current flow.

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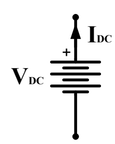
Voltage Sources

An **ideal voltage source** will maintain a **constant voltage**, **independent** of the amount of **current** that is actually flowing through the source.



Note that **steady-state current** will only flow if there is a **closed-loop path** that allows the current to flow externally from the source's positive terminal back to its negative terminal.

Voltage can be thought of as a measure of a potential force that tries to create (or oppose) the flow of current.

In an electric circuit, a **voltage source** is a device that provides a force that "**tries**" to **create the flow of current** within the circuit, **externally from its positive terminal to its negative terminal**, by developing an internal force that pushes the current from its negative to its positive terminal.

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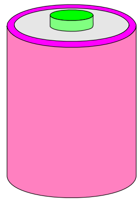
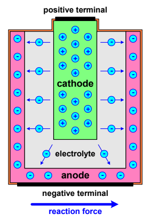
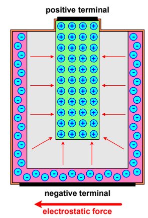
Voltage Sources – Batteries







In a **battery**, a chemical reaction removes electrons from the cathode (+ terminal) and deposits them on the anode (– terminal), resulting in a net charge-difference between the two regions.

As the charge-difference increases, an **electrostatic force** builds-up that tries to attract the electrons back towards the cathode (+).

Note that, although the **electrostatic force** is often represented by an **electric field** in the area between the regions, the **vector direction** of an **electric field** is actually based on the **force developed on positive charge**.

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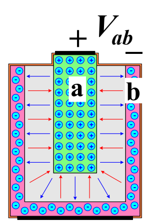
Voltage Sources – Batteries

The **voltage** (potential difference), V_{ab}, provided by the battery can be defined by the **integral** of the **electrostatic force** that built-up between the two charged regions.



As long as the reaction force is greater than the electrostatic force, the chemical reaction will continue to transport electrons from the cathode to the anode, in-turn increasing the charge-difference.

But the **reaction stops** when the **electrostatic force** is **equal but opposite** to the **reaction force** provided by the chemical reaction.

The **strength** of the **reaction force**, and in-turn the **electrostatic force** resulting from the build-up of charge, is determined by the materials (**chemical reactants**) chosen for the battery's anode and cathode.

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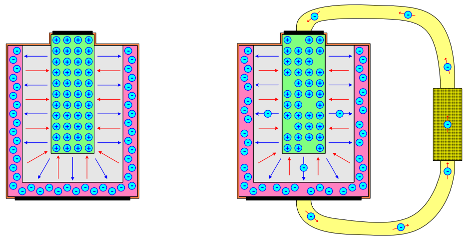
Voltage Sources – Batteries



When the **electrons flow externally** from anode to cathode, causing a reduction in the charge-difference between the regions, the **chemical reaction resumes** and begins replenishing the charge, thus allowing for the process to continue.
I.e. – **steady-state current flow**.

Although the negative charge is attracted back towards the cathode, the chemical reaction force prevents the electrons from traveling internally back from anode to cathode under normal conditions.

But, if an **external path** is provided from anode (–) to cathode (+), the electrostatic force will begin **pushing** the built-up **electrons out of the anode** while simultaneously **drawing electrons in to the cathode**, eventually resulting in closed-loop current flow.

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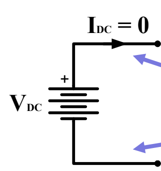
Voltage Sources



Note that the terminals of this source have been extended horizontally for illustration purposes only.
These extensions are not wires.
They are part of the ideal source and thus have no effect on the overall operation of either the source or the system to which it is connected.

Thus, given an ideal voltage source with voltage potential $\mathbf{V_{DC}}$:

If nothing is connected to the terminals of the source such that the terminals are open-circuited, then **no current** will flow externally from the source because an ideal “open circuit” provides as infinite barrier (oppositional force) to the flow of current.

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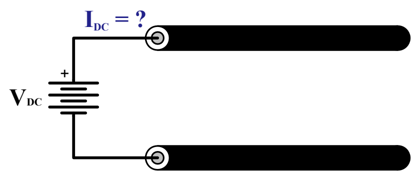
Voltage on a Transmission Line



But, what if the terminals of the voltage source are connected to one end of a **transmission line**, the other end of which remains "open-circuited"?

For purposes of discussion, assume that the transmission-line's conductors are ideal (lossless).

Although this assumption is not critical for this discussion, lossy conductors can introduce secondary effects that will be covered later in this presentation.

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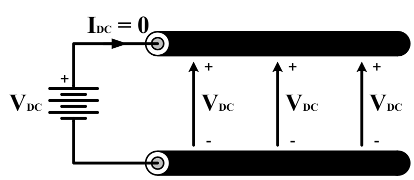
Voltage on a Transmission Line



According to **traditional circuit theory**:

Under **steady-state conditions**, **no current should flow** in the conductors due to the "open circuit" and the **source voltage should be present between the conductors** at all locations.

But what is the **mechanism** that results in a potential difference (voltage) to appear from one conductor to the other?

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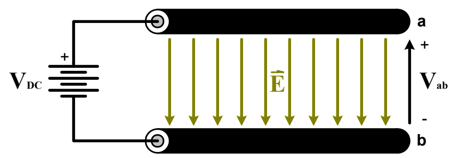
Voltage and Electric Fields



In order for a steady-state **voltage** to exist between the conductors, there must be a static **electric field** around the conductors that can be represented by a set of **electric field lines**, such that:

Remember that the **electric field** points in the **direction** of the **force** experienced by a **positive charge** located in that region.

$$V_{ab} = -\int_b^a \overline{E} \cdot dl$$

Since **voltage** defines a **difference in the potential** of **positive charge** at two locations, in order to increase its potential, the positive charge must be moved against the electric field.

Thus, the minus-sign in the equation.

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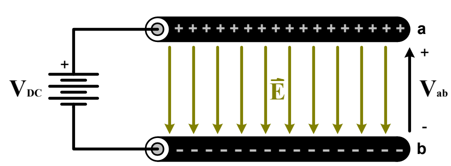
Electric Fields and Charge



But the mechanism that created an electrostatic force within a battery was the charge-difference between its anode and cathode.

It turns out that there is a similar requirement in order for a **voltage** to appear across the conductors; there must be a **charge difference** between the two conductors, resulting in an electrostatic force or electric field in the region around the two conductors.

But **how** do the conductors become **oppositely-charged**?

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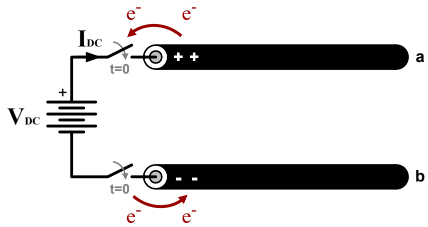
Charge and Current Flow

Switches have been added to the system to aid in the analysis.



Assuming that the conductors are uncharged before the switches close, then the charge-differential required in order to create the desired voltage must be supplied by the "voltage source" (battery).

Thus, when the switches close, electrons will begin to be pulled from the top conductor into the positive terminal of the battery and pushed out of the negative terminal of the battery into the bottom conductor.

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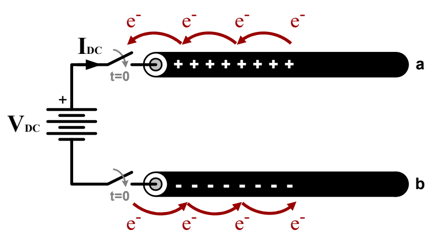
Charge and Current Flow



And, if the switches remain closed:

- The end of the top conductor will become positively-charged, in-turn causing electrons to be pulled from further down the line,
- The end of the bottom conductor will become negatively-charged, in-turn causing electrons to be pushed further down into the line.

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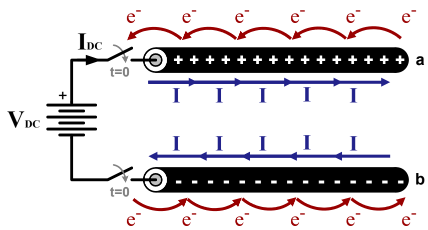
Charge and Current Flow



Note that, when the process begins to happen, it appears that current is flowing in the two conductors, but initially only in the ends of the two conductors that are closest to the source.

And then, the longer the switches remain closed, the **current** appears to **travel** progressively further and further **down the line** towards to the opposite end.

We will address what happens when the current reaches the open-circuit at the far end soon.

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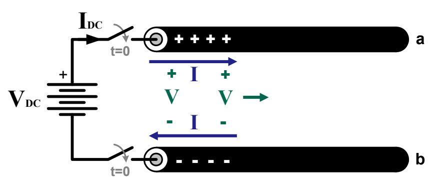
Traveling Current and Voltage Waveforms



Furthermore, as the current appears to begin flowing down the line from the source end, it develops a charge differential between the conductors, also beginning at the source end.

As since the charge differential results in an electric field and, in-turn, a potential difference across the conductors, there also appears to be a **voltage** that begins at the source end of the line and simultaneously **travels down the line** towards the other end.

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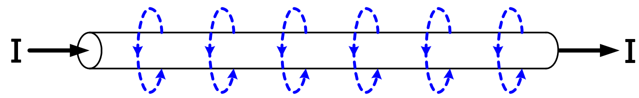
Current Flow and Magnetic Fields



$$I = \oint \overline{H} \cdot dl$$

Note that, whenever charge (current) flows through a conductor, a **magnetic field** will form around a conductor, the field-lines of which must form closed loops.

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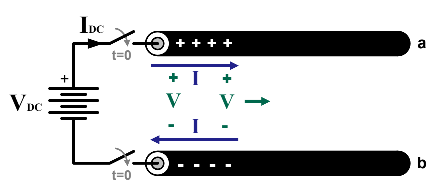
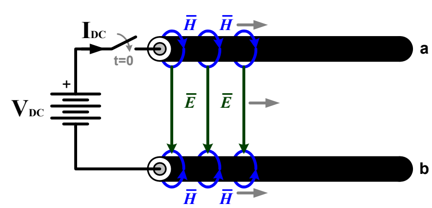
Voltage/Current ↔ Electric/Magnetic Fields





Thus, as the current begins to travel down the line, in-turn causing a voltage potential to develop between the conductors that also appears to travel down the line, there will also be an **Electric Field** and a **Magnetic Field** appear to simultaneously travel down the line along with the voltage and current waveforms.

Note that, during this entire process, the fact that the far end of the line was terminated by an "open-circuit" was never considered because the waveforms hadn't yet reached that end. Again, we will address this soon.

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Introduction to

Transmission Lines

Part II

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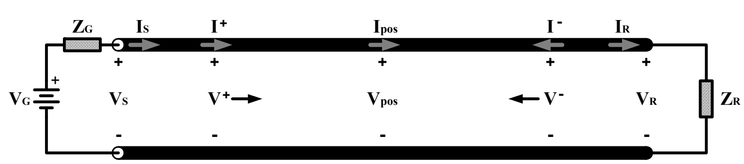
Transmission Line Definitions



Sending End – the end of the line that the source is connected or the end of the line into which power flows

Receiving End – the end of the line that a load is connected or the end of the line out of which power flows

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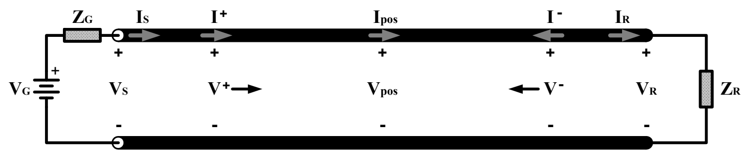
Transmission Line Definitions



Positive Voltage Rise – the rise in potential from the bottom conductor to the top conductor (as shown)

Positive Current – the current flowing in the direction from the sending end to the receiving end of the line

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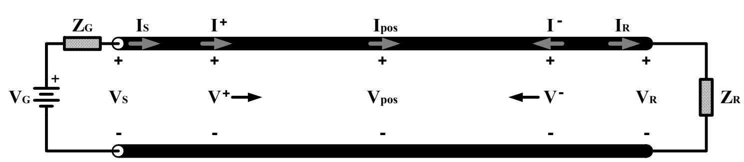
Transmission Line Definitions



Incident Wave – a wave traveling from the sending-end to the receiving-end of the line

(Note that incident voltages and currents are denoted by V^+ and I^+ respectively)

Reflected Wave – a wave traveling from the receiving-end to the sending-end of the line

(**Note that reflected voltages and currents are denoted by V^- and I^- respectively**)

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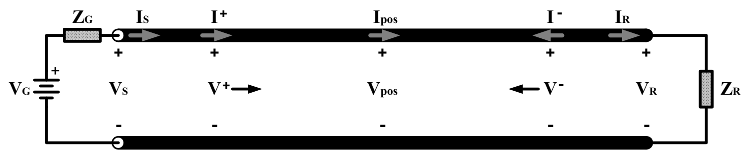
Transmission Line Definitions



The net (actual) **voltage** seen at any position on a transmission line will be equal to the **sum** of any incident and reflected voltages that have reached or passed-by that position:

$$V_x = V_x^+ + V_x^-$$

and the net **current** will be equal to the **difference** between any incident and/or reflected currents that have reached or passed-by that position:

$$I_x = I_x^+ - I_x^-$$

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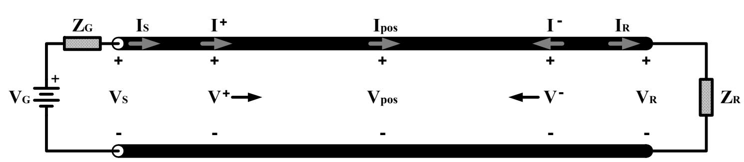
Transmission Line Definitions



Characteristic Impedance (*Z_o*) – characteristic impedance is the impedance (ratio of voltage over current) "seen" by an incident or a reflected waveform as it propagates on a transmission line.

$$Z_o = \frac{V^+}{I^+} = \frac{V^-}{I^-}$$

Characteristic impedance is also equal to the **input impedance** of an **infinitely long** or **matched transmission line**.

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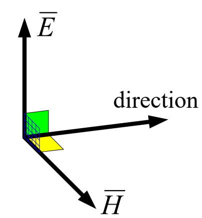
Transverse Electro-Magnetic (TEM)
Mode of Propagation

E ⊥ H ⊥ direction of propagation

Whenever voltage and current waveforms are propagating down a transmission line, there will be an associated **electric field** and a **magnetic field** that will also propagate down the line within the air (or other insulating material) that surrounds the conductors.

These electric and magnetic fields will travel in the **TEM mode** of propagation such that the **electric field**, the **magnetic field**, and the **direction of propagation** are all **<u>orthogonal</u>**.



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Velocity of Propagation

The **velocity**, ***v***, at which an electromagnetic wave propagates within a **lossless** medium is a function both the **permeability** (***μ***) and the **permittivity** (***ε***) of the material, such that:

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Permeability (***μ***) is a material property that is associated with the strength of a **magnetic field** that forms within that material.

Permittivity (***ε***) is a material property that is associated with the strength of an **electric field** that forms within that material.

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Velocity of Propagation in Air

The **permeability** of "**free space**" (**air**) is:

$$\mu_{air} = \mu_o = 4\pi x 10^{-7} \quad \frac{\text{H}}{\text{m}}$$

and the **permittivity** of "**free space**" (**air**) is:

$$\varepsilon_{air} = \varepsilon_o = \frac{1}{36\pi} x 10^{-9} \quad \frac{\text{F}}{\text{m}}$$

Thus, the **velocity** (***c***) of an electromagnetic wave **in air** is:

$$c = v_{air} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{1}{\sqrt{(4\pi x 10^{-7})(\frac{1}{36\pi} x 10^{-9})}} = 3x10^8 \ \frac{\text{m}}{\text{sec}}$$

***c* is the "speed of light" in a vacuum**

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Velocity in a Lossless Medium

Since free space (air) has both the smallest possible permeability and permittivity, the **actual permeability and permittivity** of any other material must be greater than or equal to those of air:

$$\mu_{material} \geq \mu_o \qquad \varepsilon_{material} \geq \varepsilon_o$$

To simplify things, a material is often characterized in terms of its **relative permeability** (***μ_r***) and a **relative permittivity** (***ε_r***) compared to those of are (***μ_o*** and ***ε_o***), such that:

$$\mu_r = \frac{\mu_{material}}{\mu_o} \qquad \varepsilon_r = \frac{\varepsilon_{material}}{\varepsilon_o} \qquad \text{or} \qquad \mu_{material} = \mu_r \mu_o \qquad \varepsilon_{material} = \varepsilon_r \varepsilon_o$$

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Velocity in a Lossless Medium

Thus, **velocity** can be expressed in terms of a material's relative permeability and permittivity, such that:

$$v_{material} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r\mu_o\varepsilon_r\varepsilon_o}} = \frac{1}{\sqrt{\mu_r\varepsilon_r}} \cdot \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c \cdot \frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

$$v_{material} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

Note that, for lossless **non-ferromagnetic materials** ($\mu_r = 1$), the **velocity of propagation** may be expressed as:

$$v_{material} = \frac{c}{\sqrt{\varepsilon_r}}$$

This expression is often utilized for relatively-lossless transmission-lines

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Velocity and Wavelength
as a Function of Relative Permittivity

Also note that the **wavelength** of a wave traveling within a material that has a **relative permittivity** greater than one ($\varepsilon_r > 1$) will be shorter than the wavelength of a wave having the same frequency that is traveling through air (free space).

Frequency 100 MHz	Free Space ($\varepsilon_r = 1$)	Polyethylene ($\varepsilon_r = 2.3$)
Velocity (m/s)	3×10^8	1.98×10^8
Wavelength (m)	3	1.98

$$v_{Polyethylene} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3x10^8}{\sqrt{2.3}} = 1.98x10^8 \text{ m/sec} \qquad \lambda = \frac{v}{f} = \frac{1.98x10^8}{100x10^6} = 1.98 \text{ m}$$

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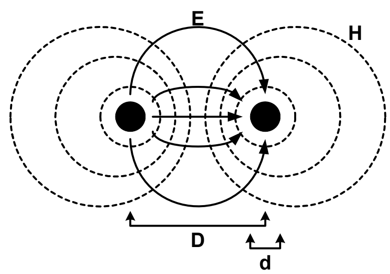
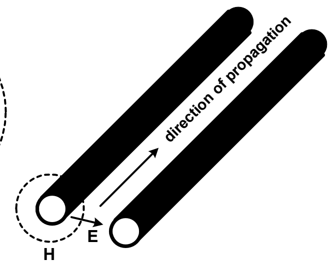
Two-Wire Open Lines

The **characteristic impedance** of a (lossless) **two-wire open line** can be defined in terms of the diameter of the conductors, ***d***, the separation distance between the conductors, ***D***, and the relative permittivity, ε_r, of the insulating material.

$$Z_o = \frac{120}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{2D}{d}\right)$$

Typically **200Ω – 600Ω**





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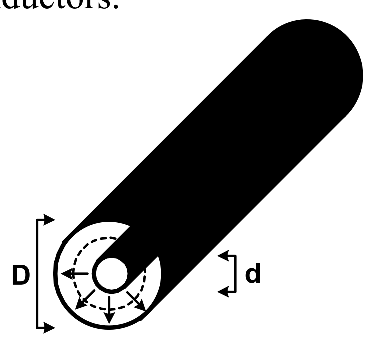
Coaxial Lines

Similarly, the **characteristic impedance** of a (lossless) **coaxial line** can be defined in terms of the diameter of the center conductor, ***d***, the inner diameter of the outer conductor or "shield", ***D***, and the relative permittivity, ε_r, of the insulating material between the conductors.



$$Z_o = \frac{60}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{D}{d}\right)$$

50Ω and **75Ω** coaxial lines are commonly utilized in communication systems.

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Traveling Waves on a Transmission Line

Several things can occur when a waveform is applied to the sending-end of a transmission line:

- The wave will begin **traveling** towards the receiving-end at a finite velocity
- The wave may **decay in magnitude** as it travels down the line
- A **reflection** may occur when the wave reaches the receiving-end of the line or any discontinuity on the line

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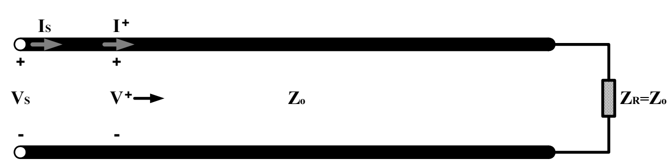
Lossless Transmission Lines



If a voltage (V_S) is applied to the sending-end of a line, then an **incident waveform** will immediately begin propagating down the line towards the receiving-end.

The **ratio** of **incident voltage** (V^+) over **incident current** (I^+) will be:

$$\frac{V^+}{I^+} = Z_o$$

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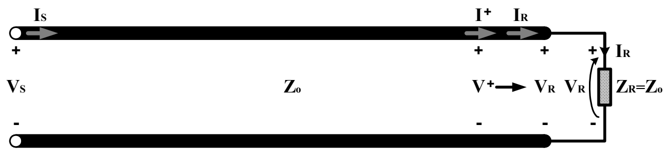
Lossless Transmission Lines



Eventually the incident waveform will reach the receiving-end of the line, at which point the actual **receiving-end voltage** (***V_R***) and **current** (***I_R***) will equal to the incident voltage and current:

$$V_R = V_R^+ \qquad I_R = I_R^+$$

It is at this point in time that the **load impedance**, ***Z_R***, is exposed to the applied voltage and current waveforms.

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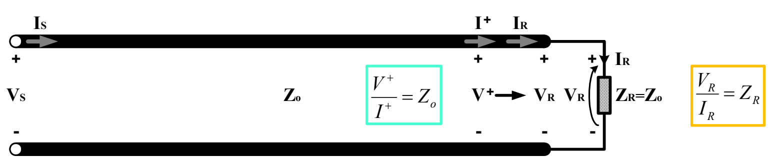
Matched Transmission Lines



But the **ratio** of **load voltage** (***V_R***) over **load current** (***I_R***) must also satisfy Ohm's Law:

$$\frac{V_R}{I_R} = Z_R$$

Thus, what happens to the incident waveform as it reaches the receiving-end of the line depends on the value of the load impedance (***Z_R***) compared to the characteristic impedance (***Z_o***).

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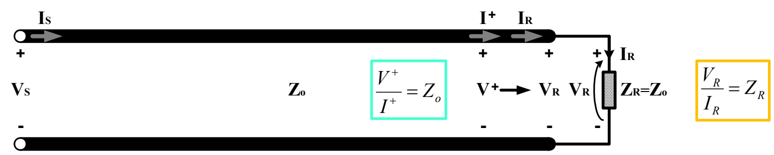
Matched Transmission Lines



If the load impedance is **matched** to the line impedance (Z_o=Z_R), then the Ohm's Law equation for the incident waveform and the Ohm's Law equation for the load are both satisfied since:

$$V_R = V_R^+ \qquad I_R = I_R^+$$

$$\frac{V^+}{I^+} = Z_o = Z_R = \frac{V_R}{I_R}$$

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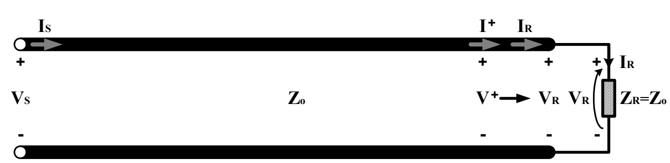
Matched Transmission Lines



Since both Ohm's Law equations are satisfied when the incident waveform reaches the receiving-end for a matched load (Z_o=Z_R), **no reflection will occur** → **steady-state operation** is achieved.

Thus, once the incident waveforms reach the matched load, the entire line will exhibit a **steady-state voltage** that is equal to the incident voltage and there will be a **steady-state current** flowing from the source to the load that is equal to the incident current.

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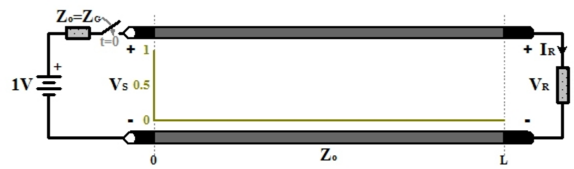
Matched Transmission Lines



$Z_R = Z_o$

Given a transmission line that is supplied by a **1V matched source*** and terminated by a **matched load**, determine what will happen when the switch closes at time t=0.

*** – A matched source is a practical source whose output impedance, Z_G, is equal to the characteristic impedance of the transmission line ($Z_G = Z_o$).**

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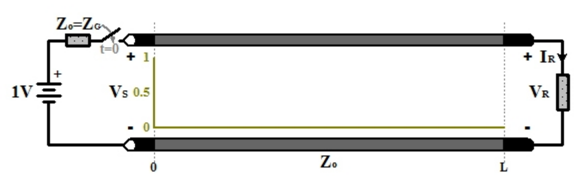
Matched Transmission Lines



$Z_R = Z_o$

Since the **characteristic impedance** of the line is the impedance experienced by the initially-applied transient waveform, the **incident voltage** applied to the sending-end of the line will equal:

$$V_S^+ = 1\text{V} \cdot \frac{Z_o}{Z_o + Z_G} = 1\text{V} \cdot \frac{Z_o}{2Z_o} = \frac{1}{2}\ \text{volt}$$

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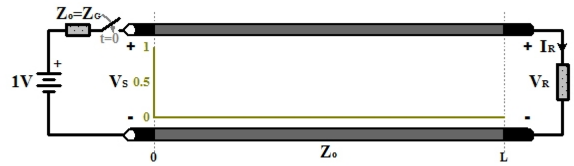
Matched Transmission Lines



$Z_R = Z_o$

The **½-volt incident voltage** (V^+) will travel down the line until it reaches the receiving-end, at which point it is applied across the **matched load** and steady-state conditions are achieved.

Since there is no reflection on the line, the **steady-state voltage** that will be present on the line is:

$$V_{Line} = V^+_{Line} + V^-_{Line} = \frac{1}{2} + 0 = \frac{1}{2} \text{ volt}$$

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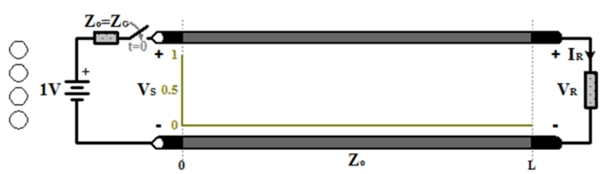
Matched Transmission Lines



$Z_R = Z_o$

The **½-volt incident voltage** (V^+) will travel down the line until it reaches the receiving-end, at which point it is applied across the **matched load** and steady-state conditions are achieved.

Since there is no reflection on the line, the **steady-state voltage** that will be present on the line is:

$$V_{Line} = V^+_{Line} + V^-_{Line} = \frac{1}{2} + 0 = \frac{1}{2} \text{ volt}$$

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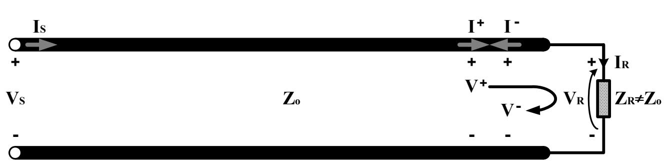
Introduction to

Transmission Lines

Part III

45

Mismatched Transmission Line



If the load impedance does **not match** the line impedance ($Z_R \neq Z_o$), then Ohm's Law is not satisfied at the load since:

$$\frac{V_R^+}{I_R^+} = Z_o \neq Z_R = \frac{V_R}{I_R}$$

causing **a reflection** to occur, such that the voltage and current present at the receiving-end of the line will be:

$$V_R = V_R^+ + V_R^- \qquad I_R = I_R^+ - I_R^-$$

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Mismatched Transmission Line

The **total voltage** (current) at any position on the line is the sum (difference) of the incident and reflected voltages (currents).

Therefore:

$$V = V^{+} + V^{-} \qquad I = I^{+} - I^{-}$$

Thus, if a reflection occurs at the load, then the **load voltage and current** will equal:

$$V_R = V_R^{\ +} + V_R^{\ -} \qquad I_R = I_R^{\ +} - I_R^{\ -}$$

and in turn:

$$\frac{V_R}{I_R} = Z_R$$

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Mismatched Transmission Line

The **load impedance Z_R** may be expressed in terms of both the incident and reflected voltage waveforms and the characteristic impedance Z_o as follows:

$$Z_R = \frac{V_R}{I_R} = \frac{V_R^{\ +} + V_R^{\ -}}{I_R^{\ +} - I_R^{\ -}}$$

$$= \frac{V_R^{\ +} + V_R^{\ -}}{\dfrac{V_R^{\ +}}{Z_o} - \dfrac{V_R^{\ -}}{Z_o}}$$

$$= Z_o \cdot \frac{V_R^{\ +} + V_R^{\ -}}{V_R^{\ +} - V_R^{\ -}}$$

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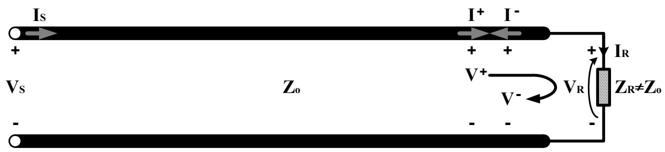
Mismatched Transmission Line



Based on the equation:

$$Z_R = Z_o \cdot \frac{V_R^+ + V_R^-}{V_R^+ - V_R^-}$$

the **ratio** of the **reflected and incident voltages** required to satisfy Ohm's Law for the load impedance is:

$$\frac{V_R^-}{V_R^+} = \frac{Z_R - Z_o}{Z_R + Z_o}$$

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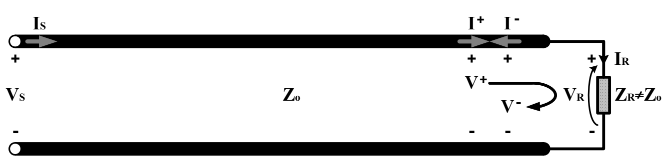
Reflection Coefficient



Reflection Coefficient is defined as the ratio of a reflected and an incident voltage:

$$\Gamma = \frac{V^-}{V^+}$$

Thus, the reflection coefficient due to the mismatched load can be defined in terms of the characteristic and load impedances as:

$$\Gamma_R = \frac{V_R^-}{V_R^+} = \frac{Z_R - Z_o}{Z_R + Z_o}$$

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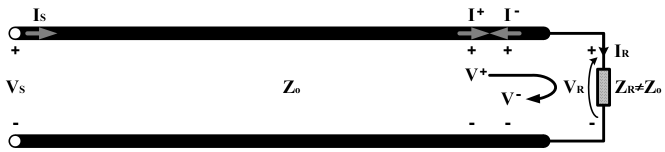
Reflection Coefficient



Given the values of both the incident voltage that reaches the load and the reflection coefficient due to the mismatched load, the value of the **reflected voltage** seen at the receiving-end of the line will be:

$$V_R^- = \Gamma_R \cdot V_R^+ \qquad \text{where:} \qquad \Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

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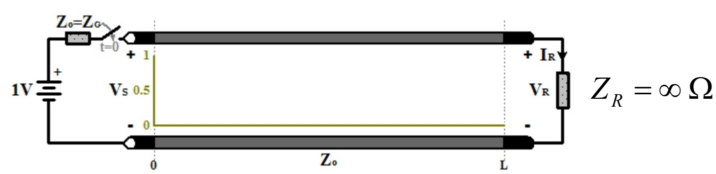
Mismatched Transmission Lines



Given a transmission line that is supplied by a **1V matched source** and terminated by an ideal **"open circuit"** ($Z_R = \infty\,\Omega$), determine what will happen when the switch closes at time t=0.

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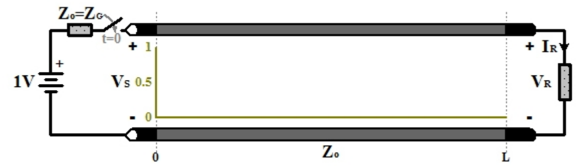
Mismatched Transmission Lines



$Z_R = \infty\ \Omega$

Since the **characteristic impedance** of the line is the impedance "seen" by the initially-applied transient waveform, the incident waveform applied to the sending-end of the line will equal:

$$V_S^+ = 1\text{V} \cdot \frac{Z_o}{Z_o + Z_G} = 1\text{V} \cdot \frac{Z_o}{2Z_o} = \frac{1}{2}\ \text{volt}$$

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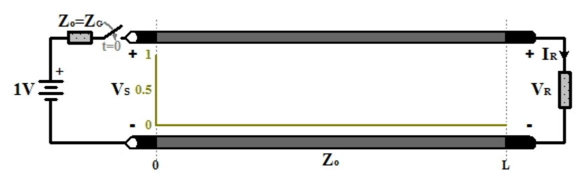
Mismatched Transmission Lines



$Z_R = \infty\ \Omega$

The **½-volt incident voltage** (V^+) will travel down the line until it reaches the receiving-end, at which point it is applied across the ideal "**open-circuited**" load.

But this time the load impedance does **not match** the line impedance, resulting in a non-zero reflection coefficient; thus, a **reflection will occur**:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} = \frac{\infty - Z_o}{\infty + Z_o} = 1$$

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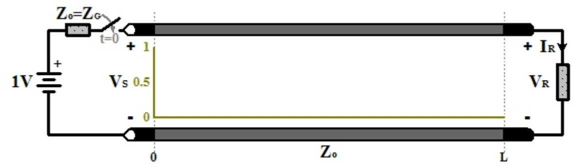
Mismatched Transmission Lines



$Z_R = \infty\ \Omega$

Since the **reflection coefficient**, Γ_R, for an open-circuited load is equal to one, the **reflected voltage** will be equal to:

$$V_R^- = \Gamma_R \cdot V_R^+ = 1 \cdot \frac{1}{2} = \frac{1}{2} \text{ volt}$$

Thus, a **½V reflected voltage** waveform will be created by the load at the receiving-end of the line, and it will travel back down the line until it reaches the **matched source**, at which point steady-state operation occurs.

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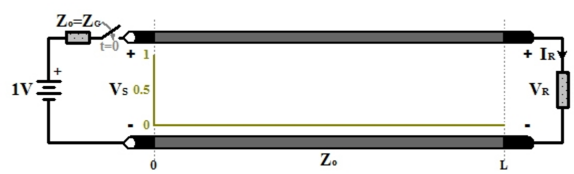
Mismatched Transmission Lines



$Z_R = \infty\ \Omega$

Once **steady-state operation** occurs, the voltage present on the line will be the **sum** of the **incident** and the **reflected waveforms**.

Thus:

$$V_{Line} = V_{Line}^+ + V_{Line}^- = \frac{1}{2} + \frac{1}{2} = 1 \text{ volt}$$

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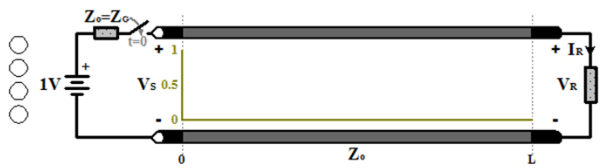
Mismatched Transmission Lines



$Z_R = \infty\ \Omega$

Once **steady-state operation** occurs, the voltage present on the line will be the **sum** of the **incident** and the **reflected waveforms**.

Thus:

$$V_{Line} = V^{+}_{Line} + V^{-}_{Line} = \frac{1}{2} + \frac{1}{2} = 1 \text{ volt}$$

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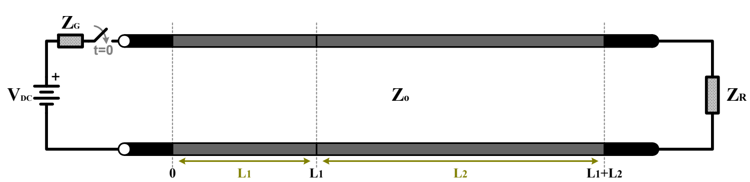
Transient Example Problem



Given the system shown above, plot the **voltage** at the **sending-end** of the line (position **x=0**), the **voltage** at **position x=L$_1$**, and the **voltage** at the **receiving-end** of the line (position **x=L$_1$+L$_2$**), all as a **function of time** if the switch closes at time ***t=0***.

Note – assume that the line is **lossless** (i.e. – assume that the waves do not attenuate as they propagate on the line)

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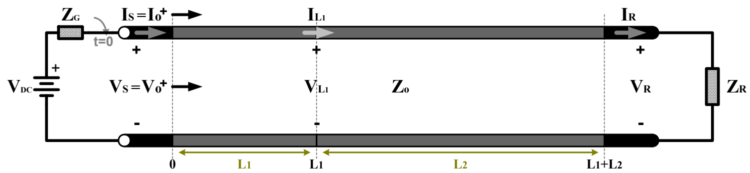
Transient Example Problem



At time ***t=0***, the switch will close and **incident voltage & current** waveforms will be applied to the sending-end of the line, such that:

$$V_S = V_{applied} = V_{DC} \cdot \left(\frac{Z_o}{Z_o + Z_G} \right) = V_0^+$$

The applied **waveforms** will immediately **begin propagating towards the receiving end** of the line.

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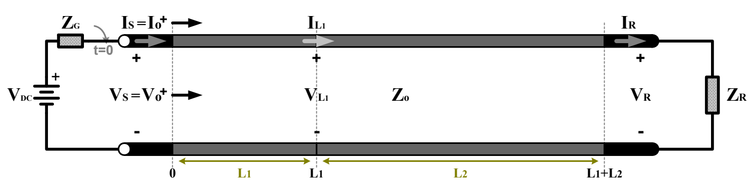
Transient Example Problem



If the source impedance **matches** the line impedance ($Z_G = Z_o$) then the magnitude of the **incident voltage** applied to the **sending-end of the line** will be:

$$V_S = V_{applied} = V_{DC} \cdot \left(\frac{Z_o}{Z_o + Z_G} \right) = V_{DC} \cdot \left(\frac{Z_o}{Z_o + Z_0} \right) = \frac{V_{DC}}{2} = V_0^+$$

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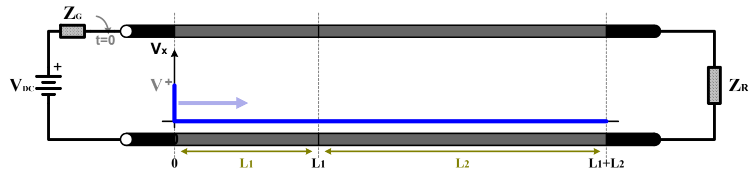
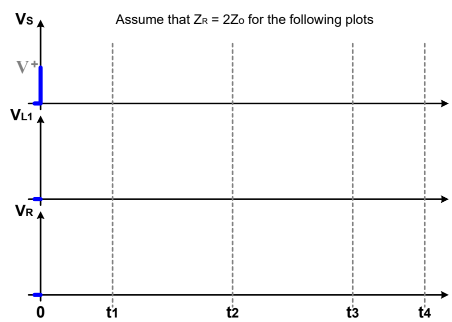
Transient Example Problem



Note that, at time ***t=0***, the **voltage** everywhere else on the line will still equal to zero since no time has passed for the incident wave to propagate down the line.

$$V_X = V_X^{\ +} + V_X^{\ -}$$



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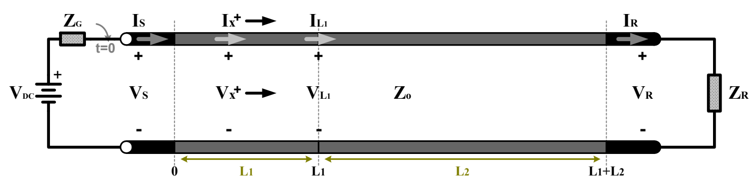
Transient Example Problem



If the **incident waveform** propagates down the line with **velocity *v***, then the waveform will reach **position *L₁*** at **time *t₁***, where:

$$t_1 = \frac{L_1}{v}$$

at which point in time the **voltage** at **position *L₁*** be equal to the incident voltage.

$$V_{L1} = V_{L1}^{+} = \frac{V_{DC}}{2}$$

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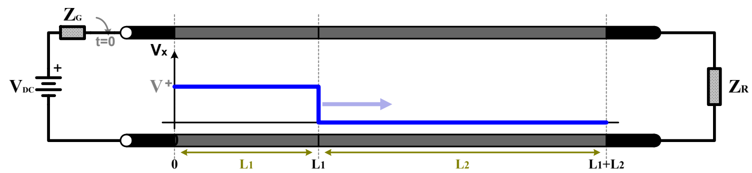
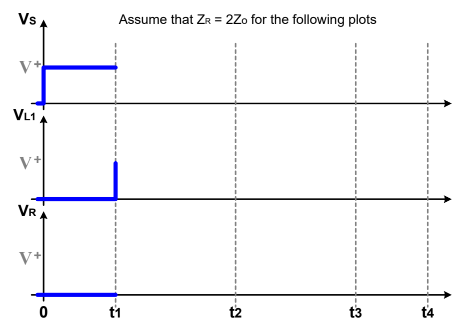
Transient Example Problem



Note that, at **time t=t_1**:

- the **voltage** from the **sending-end to position L_1** will equal to the **incident voltage**
- the **voltage** from **position L_1 to the receiving-end** will still equal **zero**.



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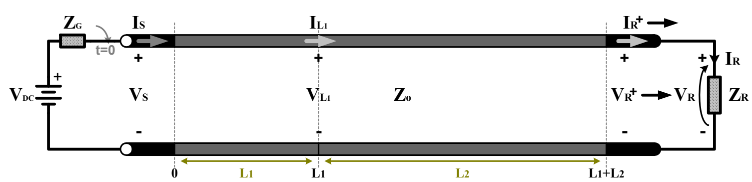
Transient Example Problem



After a **total time t_2** has passed, where:

$$t_2 = \frac{L_1 + L_2}{v}$$

the **incident wave** will **reach** the **receiving-end** of the line, and if the load impedance **matches** the line impedance (Z_R=Z_o), then **no reflection** will occur, and **steady-state operation** will begin:

$$V_R = V_R^+ = \frac{V_{DC}}{2}$$

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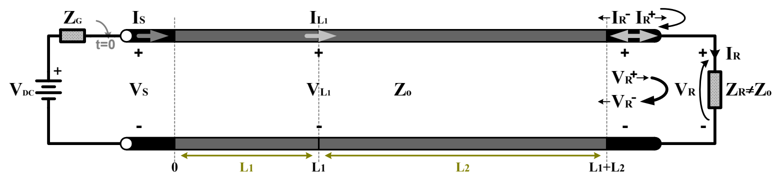
Transient Example Problem



If the load impedance does **not match** the line impedance ($Z_R \neq Z_o$), then a **reflection** must occur in order to satisfy Ohm's Law at the load, such that:

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

The **reflected waveform** will then begin **propagating** back **towards the sending-end** of the line.

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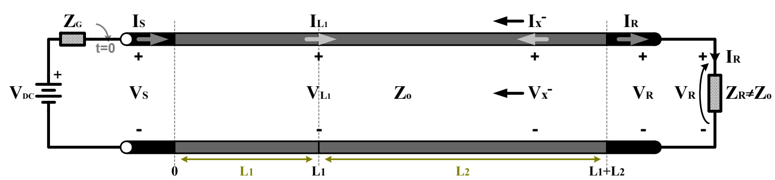
Transient Example Problem



The value of the **reflected waveform** will be:

$$V_R^- = \Gamma_R \cdot V_R^+$$

Since the reflection occurs immediately when the incident wave reaches the load, the **actual receiving-end voltage** will equal the **sum** of the **incident and reflected waves** at time $\boldsymbol{t_2}$:

$$V_R = V_R^+ + V_R^- = V_R^+ + \Gamma_R \cdot V_R^+ = V_R^+ \cdot (1 + \Gamma_R)$$

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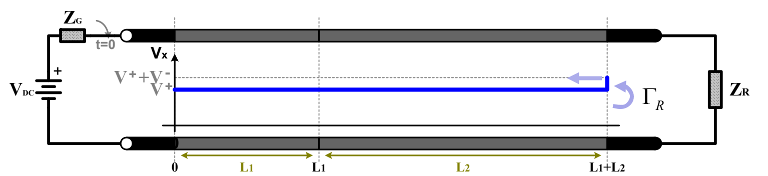
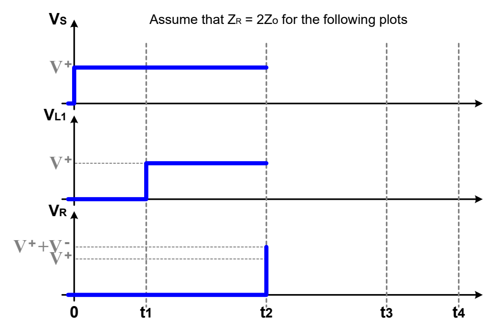
Transient Example Problem



Note that, at time $t=t_2$:

- the **voltage everywhere** on the line will equal to the **incident voltage**, except
- the **voltage** for at the **receiving-end** will equal to the **sum** of the **incident and reflected voltages**.



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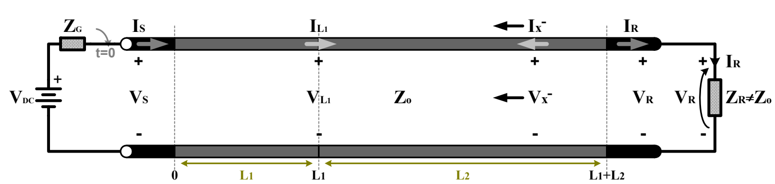
Transient Example Problem



After **total time t_3** has passed, where:

$$t_3 = \frac{L_1 + 2L_2}{v}$$

the **reflected wave** will **reach** the **position L_1**, at which point the **voltage** at that position will also be equal to the **sum** of the **incident and reflected waves**:

$$V_{L1} = V_{L1}^{+} + V_{L1}^{-} = V_{L1}^{+} + \Gamma_R \cdot V_{L1}^{+} = V_{L1}^{+} \cdot (1 + \Gamma_R)$$

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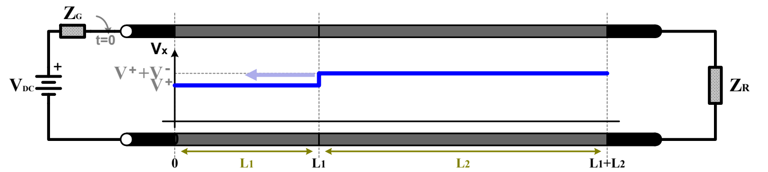
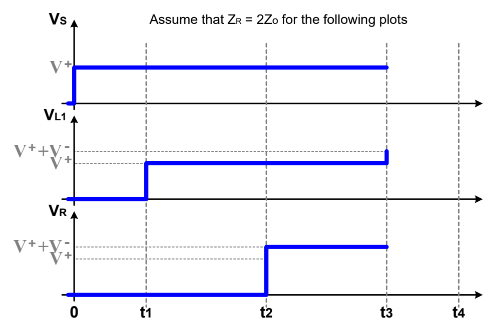
Transient Example Problem



Thus, at **time $t=t_3$**, the **voltages** at the **various positions** on the lines are shown to the right.



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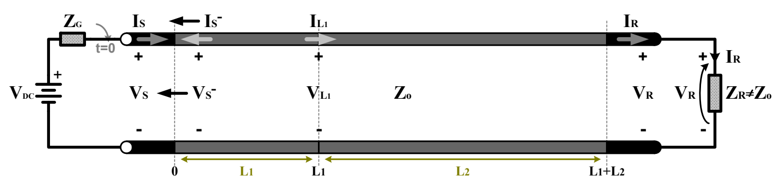
Transient Example Problem



Finally, after **total time t_4** has passed, where:

$$t_4 = \frac{2L_1 + 2L_2}{v} = \frac{2 \cdot length}{velocity}$$

the **reflected wave** will **reach** the **sending-end** of the line, at which point the **voltage** at that position will also be equal to the **sum** of the **incident and reflected waves**:

$$V_S = V_S^+ + V_S^- = V_S^+ + \Gamma_R \cdot V_S^+ = V_S^+ \cdot (1 + \Gamma_R)$$

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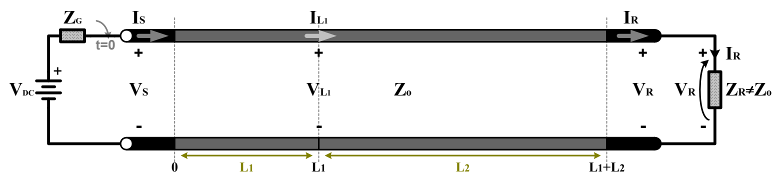
Transient Example Problem



If the source impedance **matches** the line impedance, then **no further reflections will occur** when the reflected waveform reaches the sending-end… and **steady-state operation** occurs.

If the source impedance does **not match** the line impedance, then the wave will **reflect off the source** and create a **new incident waveform**, after which the process will repeat.

(Note that multiple reflections will are covered in this presentation.)

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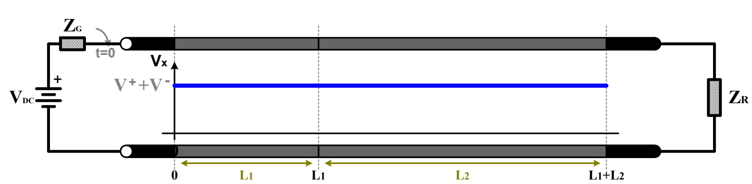
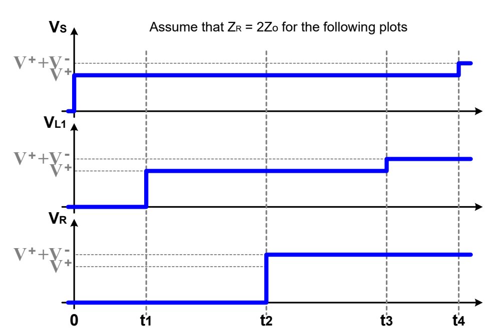
Transient Example Problem



Thus, at **time $t=t_4$**, the **voltages** at the **various positions** on the line are shown to the right.

(Both the incident and the reflected waveforms have traveled the entire length of the line.)



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Introduction to

Transmission Lines

Part IV

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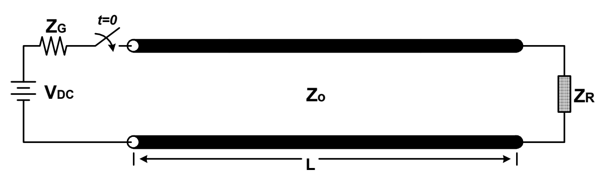
Time Domain Reflectometry



Time Domain Reflectometry is a technique utilized to investigate the characteristics of a transmission line, to locate or characterize any discontinuities on the line, and/or to locate or characterize the load that terminates the transmission line, by **measuring any reflections that result from an incident wave** that is applied to the sending-end of the transmission line.

Note that this technique only requires access to one end of the transmission line.

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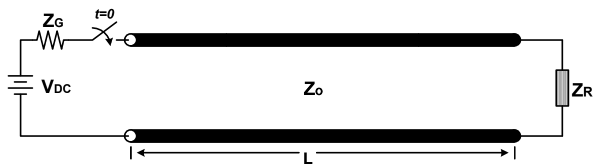
Time Domain Reflectometry



Thus, a **Time Domain Reflectometer** (**TDR**) is a device that is able to **apply an incident waveform**, typically a step-function, to the sending end of the line, and then **measure any reflections** that result from the applied incident waveform.

Although some simple TDRs may only measure the **time delay** until the reflection arrives back at the sending end of the line, others will provide a **time-plot** of the **sending-end voltage**.

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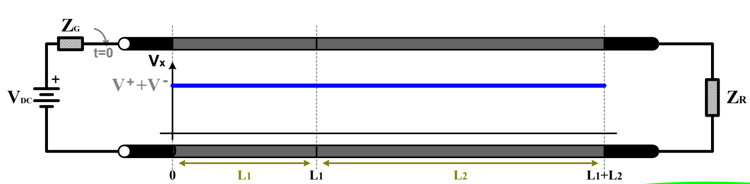
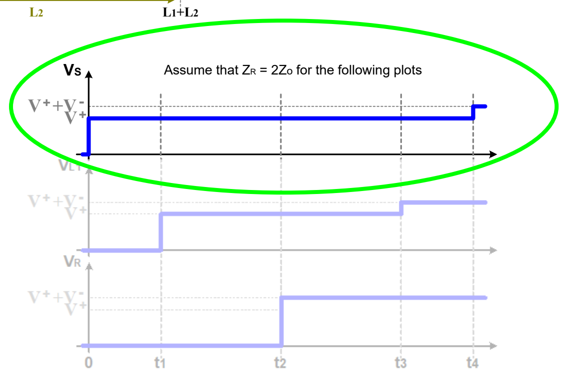
Transient Example Problem



The plot of V_S, the **sending-end voltage**, as a function of time in this example is **equivalent to a TDR plot** of the voltage that would be recorded by a fully-functional TDR.



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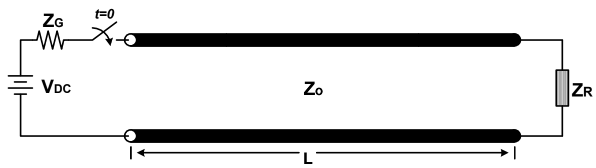
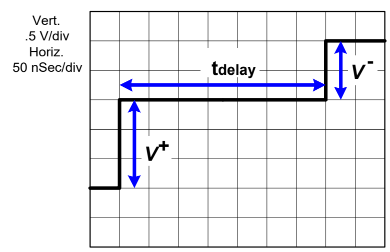
TDR Plots



The **TDR plot** shown to the right is representative of the results that would be displayed by a TDR if it was connected to a line that is terminated by a **resistive load** that is **larger** than the **characteristic impedance** of the line.



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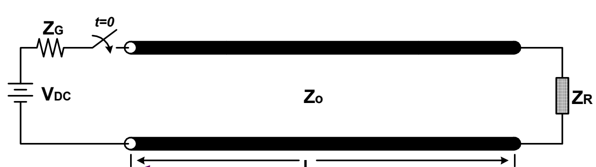
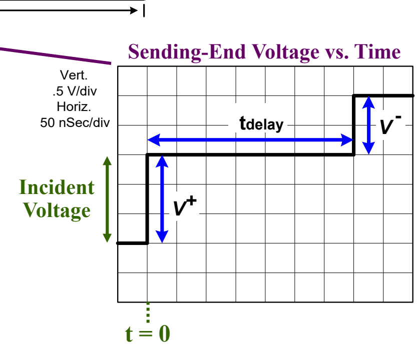
TDR Plots



The **TDR plot** displays the **voltage** present at the **sending-end** of the line as a function of time is an incident voltage (V^{+}) is applied to the line at **time t=0**.



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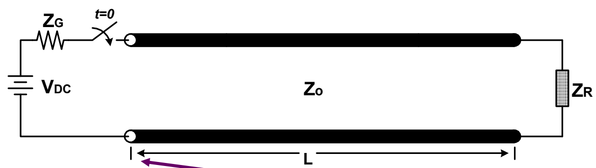
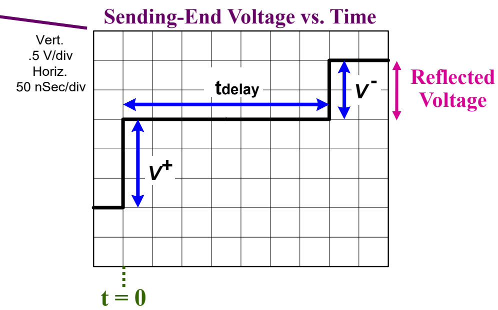
TDR Plots



Since the load is **not matched** to the line, a **reflection occurs** when the incident voltage reaches the load.

That **reflection** will travel back to the sending-end of the line, resulting in a **change in the sending-end voltage**, since: $V_S = V_S^{+} + V_S^{-}$



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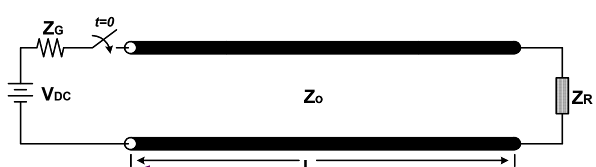
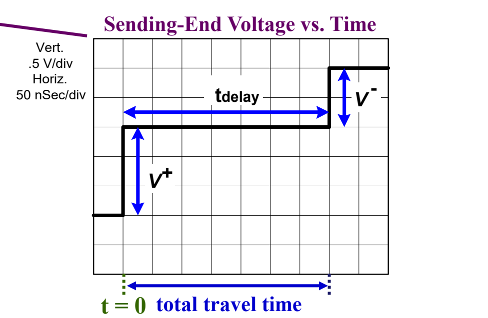
TDR Plots



The **total travel time** required for the **incident wave to reach the load** and the **reflected wave to return to the sending-end** is the time delay, t_{delay}, between the appearance of the incident and reflected waves in the plot.



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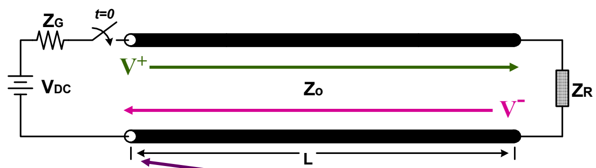
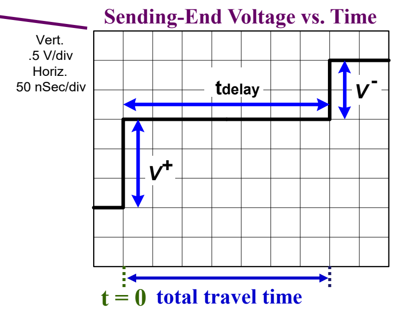
Line Length from a TDR Plot



During the time t_{delay}, the total travel distance of the incident and reflective waves is twice the length of the line.

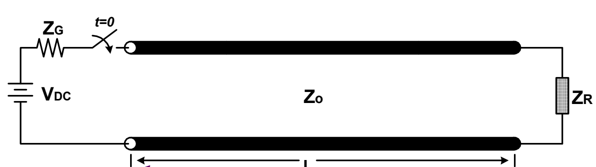


If the **travel velocity** on the line is known, then the **length of the line** can be determined from the physics equation:

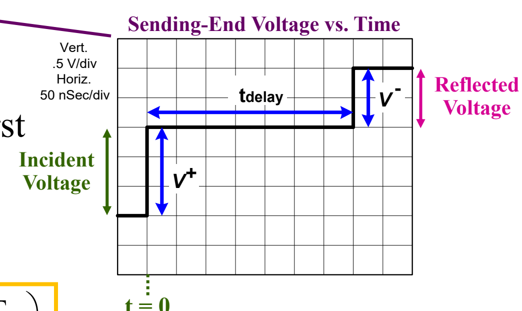
$$t_{delay} = \frac{\text{distance}}{\text{velocity}} = \frac{2 \cdot L}{\text{velocity}}$$

$$\rightarrow \quad L = \frac{\text{velocity} \cdot t_{delay}}{2}$$

Load Resistance from a TDR Plot



The magnitude of the incident and reflected voltages is represented by the change in the sending-end voltage.

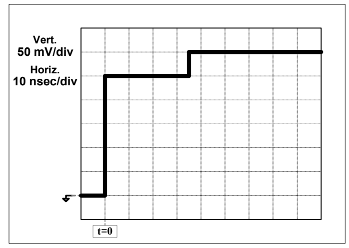


If the **characteristic impedance** of the line is known, then the **load resistance** can be determined by first solving for **reflection coefficient** based on the incident and reflected voltage values:

$$\Gamma_R = \frac{V^-}{V^+} = \frac{Z_R - Z_o}{Z_R + Z_o} \quad \rightarrow \quad Z_R = Z_o \cdot \left(\frac{1 + \Gamma_R}{1 - \Gamma_R} \right)$$

TDR Example

Given the TDR plot shown to the right:



The measurement was performed on a line that had a **300Ω** characteristic impedance, and the velocity of travel on the line was **3x10⁸ m/sec**.

Determine the **length** of the line upon which the test was performed, and the value of the **load resistance** that was terminating the line.

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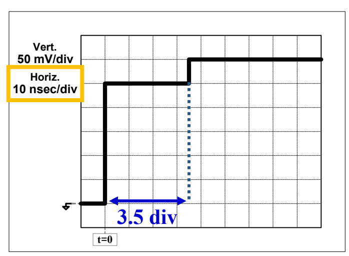
TDR Example

Line Length:



Based on the plot, the **time delay** between the incident and reflected waveforms is:

$$t_{delay} = \left(10\ {}^{\text{nsec}}\!/_{\text{div}}\right)\cdot\left(3.5\ \text{div}\right) = 35\ \text{nsec} = 35x10^{-9}\ \text{sec}$$

And if the velocity is $3x10^{8}$ m/sec, then the **length** of the line is:

$$L = \frac{\text{velocity}\cdot t_{delay}}{2} = \frac{3x10^{8}\cdot 35x10^{-9}}{2} = \boxed{5.25\ \text{meters}}$$

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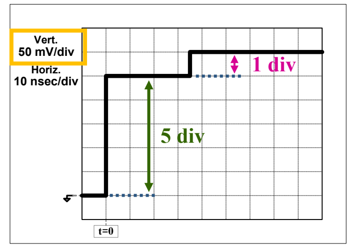
TDR Example

Load Resistance:



Based on the plot, values of the **incident and reflected waveforms** are:

$$V^{+} = \left(50\ ^{\text{mV}}\!/_{\text{div}}\right)\cdot\left(5\ \text{div}\right) = 250\ \text{mV}$$

$$V^{-} = \left(50\ ^{\text{mV}}\!/_{\text{div}}\right)\cdot\left(1\ \text{div}\right) = 50\ \text{mV}$$

And if the characteristic impedance is **300Ω**, then the **load resistance** is:

$$\Gamma_R = \frac{V^-}{V^+} = \frac{50\text{mV}}{250\text{mV}} = 0.2 \qquad Z_R = Z_o\cdot\left(\frac{1+\Gamma_R}{1-\Gamma_R}\right) = 300\,\Omega\cdot\frac{1+0.2}{1-0.2} = \boxed{450\,\Omega}$$

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Characteristics of Coaxial Lines

(From Table 1-3 in the Textbook)

The following table shows some of the characteristics of several standard types of coaxial cable.

The characteristics upon which we will primarily focus are **Nominal Impedance**, **Nominal Velocity of Propagation** and **Nominal Attenuation**.

Coaxial Cables

RG #	AWG Material	Insulation	# Shields	Jacket	Nom. O.D. (inch)	Nom. Imp. (Ohms)	Nom. Vel. Of Prop.	Nom. Cap. (pF/ft.)	Nom. Attenuation per 100' MHz	Nom. Attenuation per 100' dB	Standard Spool Lengths
14/U	20 Copper	Poly-ethylene	1	Black Vinyl	.420	95	66%	16.0	100 200 400	3.0 4.5 6.0	100, 500
14A/U	20 Copper	Poly-ethylene	1	Black Vinyl	.420	92	66%	16.0	100 200 400	3.5 5.0 7.0	100, 500
16A/U	18 Copper	Cellular Poly-ethylene	1	Black Vinyl	.195	50	78%	30.8	100 200 400	5.0 7.0 9.5	100,500 1000

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Characteristics of Coaxial Lines

(Table 1-3)

The **Nominal Impedance** values shown in the table are the expected **Characteristic Impedances** for the listed standard cable types.

$$Z_o \equiv \text{Nominal Impedance}$$

Coaxial Cables

RG #	AWG Material	Insulation	# Shields	Jacket	Nom. O.D. (inch)	Nom. Imp. (Ohms)	Nom. Vel. Of Prop.	Nom. Cap. (pF/ft.)	Nom. Attenuation per 100' MHz	dB	Standard Spool Lengths
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16A/U	18 Copper	Cellular Poly-ethylene	1	Black Vinyl	.195	50	78%	30.8	100 200 400	5.0 7.0 9.5	100,500 1000

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Characteristics of Coaxial Lines

(Table 1-3)

The **Nominal Velocity of Propagation** provides the propagation velocity for a wave on the cable in terms of a percentage of the "speed of light" in a vacuum, such that:

$$v_{actual} = \frac{(\text{Nom. Vel. of Prop.})}{100} \cdot c = \frac{(\text{Nom. Vel. of Prop.})}{100} \cdot 3x10^8 \ \frac{\text{meters}}{\text{second}}$$

Coaxial Cables

RG #	AWG Material	Insulation	# Shields	Jacket	Nom. O.D. (inch)	Nom. Imp. (Ohms)	Nom. Vel. Of Prop.	Nom. Cap. (pF/ft.)	Nom. Attenuation per 100' MHz	dB	Standard Spool Lengths
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Characteristics of Coaxial Lines

(Table 1-3)

The **Nominal Attenuation** provides a measure of the rate at which the magnitude of a wave propagating on the line will decay due to the loss characteristics of the line.

Note that the Nominal Attenuation increases with increasing frequency of the applied waveform.

Coaxial Cables

RG #	AWG Material	Insulation	# Shields	Jacket	Nom. O.D. (inch)	Nom. Imp. (Ohms)	Nom. Vel. Of Prop.	Nom. Cap. (pF/ft.)	Nom. Attenuation per 100' MHz	dB	Standard Spool Lengths
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16A/U	18 Copper	Cellular Poly-ethylene	1	Black Vinyl	.195	50	78%	30.8	100 200 400	5.0 7.0 9.5	100,500 1000

We will neglect attenuation until we cover AC-sourced transmission lines.

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