ECET 3000
Electrical Principles

Electric Power Distribution
&
Three-Phase Systems

Overview
of the
Electric Power System

Electric Power Systems

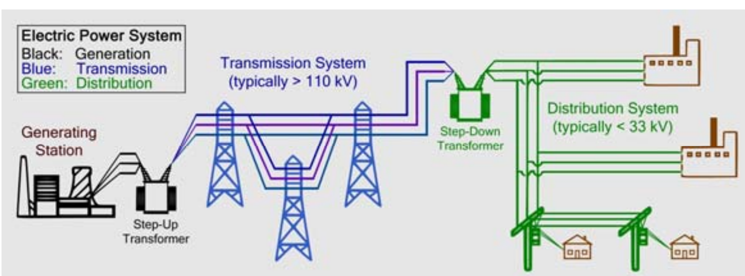
An <u>Electric Power System</u> is a complex network of electrical components used to reliably generate, transmit and distribute electric energy on a real-time or "as-needed" basis.

Within the United States, the primary method of distributing electric power is by means of a three-phase transmission and distribution system.

Electric Power Systems

In terms of its operation, an electric power system can be divided into three primary subsystems, each of which performs a key function:

- Generation
- Transmission
- Distribution



Electric Power Generation

In terms of its operation, an electric power system can be divided into three primary subsystems, each of which performs a key function:

- **Generation**
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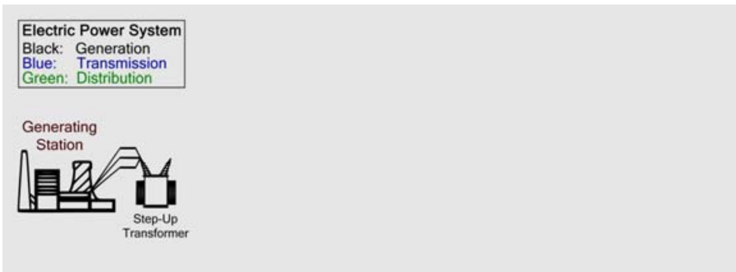


Electric Power Generation

Most of the electric energy that is transmitted/distributed by means of the electric power system is produced at <u>generating stations</u>, in which either fuel energy or hydraulic energy is converted into an electric form.

Gas-fired Combustion Turbine Generator
Sewell Creek Energy Facility – Oglethorpe Power

Electric Power Generation

The generating stations, or “power plants” as they are commonly called, are often located at great distances both from each other and from the end users of the electric energy that they produce.

All of the power plants and the electric loads are connected together by means of a complex wired (transmission/distribution) network, across which the electric energy can be transported from the various sources to the individual loads.

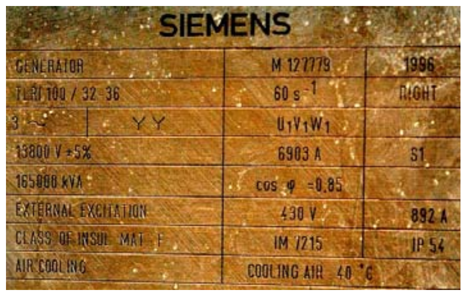
Electric Power Generation

It is important to note that the losses associated with transportation of electric energy across a “lossy” line are proportional to the square of the line current magnitude, making it <u>more efficient to transport the energy at a higher-voltage/lower-current level</u>.

Furthermore, since there is a limit to the amount of current that can be allowed to continuously flow through a practical conductor, more electric energy can be transported across a specific-sized line if it is transported at a higher-voltage/lower-current level.

Electric Power Generation

Although large modern generators typically produce electric energy at voltage levels ranging from 13.8kV to 24kV, higher voltages are required in order to efficiently transport that energy across large distances.



Nameplate from 165MVA, 13.8kV, 3Φ Generator

Electric Power Generation

Although large modern generators typically produce electric energy at voltage levels ranging from 13.8kV to 24kV, higher voltages are required in order to efficiently transport that energy across large distances.

For this reason, a <u>step-up transformer</u> is located at each power plant in order to raise the output voltage of the generator to <u>transmission levels</u>.

13.8kV – 230kV Step-Up Transformer

Electric Power Generation

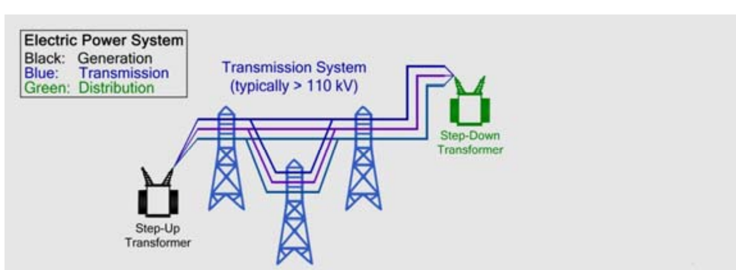
In terms of its operation, an electric power system can be divided into three primary subsystems, each of which performs a key function:

- **Generation**
- Transmission
- Distribution

Electric Power Transmission

In terms of its operation, an electric power system can be divided into three primary subsystems, each of which performs a key function:

- Generation
- **Transmission**
- Distribution



Electric Power Transmission

Electric Power Transmission is the bulk transfer of electric energy within an electric power system from the various generating stations to the "substations" that connect the transmission system to the distribution networks.

Note - An electric power substation is an assembly of equipment in an electric power system through which electric energy is passed for transmission, transformation, distribution, or switching purposes.

Electric Power Transmission

Electric Power Transmission is the bulk transfer of electric energy within an electric power system from the various generating stations to the "substations" that connect the transmission system to the distribution networks.

The help ensure that the electric energy is able to reach the end-users, even during times of equipment failure or other disruption, the transmission system is setup such that it provides multiple (redundant) paths for the energy to flow.

Electric Power Transmission

In terms of system design, it is uneconomical to connect all of the distribution substations to the high-voltage transmission lines that are used to transport large amounts of energy across long distances due the size and cost of the high-voltage equipment.

For this reason, the networks utilized for electric power transmission are divided into two categories based on their operating voltages:

Transmission:	typically 115kV – 765kV
Sub-transmission:	typically 34.5kV – 115kV

Electric Power Transmission

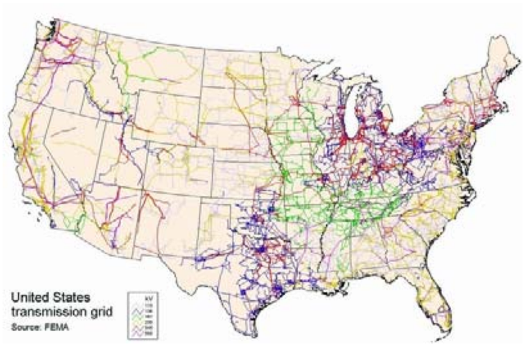
The Transmission Network or "Power Grid" consists of an interconnection of high-voltage transmission lines that allow large amounts of electric energy to flow from point to point across long distances.

Since the transmission network forms the backbone of the electric power system, interconnecting the generating stations to the various regional load centers, it must be able to deliver very large amounts of electric energy to the load centers and it must be able to accommodate any operational changes in the system.

Electric Power Transmission

The <u>standard operating voltages</u> for a transmission network are:

High Voltage (HV):	**115, 138 & 230kV**
Extra High Voltage (EHV):	**345, 500 & 765kV**



Electric Power Transmission

Unlike the transmission network that moves large amounts of power between regions, the <u>Sub-Transmission Network</u> provides power to a specific region.

For this reason, sub-transmission circuits are usually arranged in loops so that a single line failure does not cut-off power to a large amount of customers for more than a short time.

Sub-transmission networks operate at lower voltages than transmission networks, allowing for more economical connection to all of the distribution system substations.

Electric Power Transmission

Note that there is no fixed cutoff between transmission and sub-transmission networks.

As systems have evolved, the operating voltages of the sub-transmission networks have increased such that they overlap with those of the transmission networks, sometimes reaching up to 138kV.

Electric Power Distribution

In terms of its operation, an electric power system can be divided into three primary subsystems, each of which performs a key function:

- Generation
- Transmission
- **Distribution**

Electric Power Distribution

Electric Power Distribution is the final stage in the transfer of electric energy within an electric power system, during which the energy that was transferred from the transmission system to the distribution system is delivered to the customers.

The distribution system operates at medium-level voltages ranging from 4kV to 34.5kV, most commonly in the 11kV to 15kV range.

Although some large customers are fed directly from the distribution lines, most customers are supplied through a transformer that steps down the distribution voltage to a relatively low level for use by the equipment in the customer facility.

Electric Power Distribution

Distribution networks are typically configured as one of either two types:

- **Radial**
- **Interconnected**

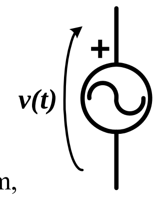
Radial networks serve their network area from a single substation, with no connection to any other supply.

Interconnected networks also serve their network area from a single substation, but they typically have multiple connections to other substations. These connections are normally open, but can be closed as needed during faults or times of maintenance.

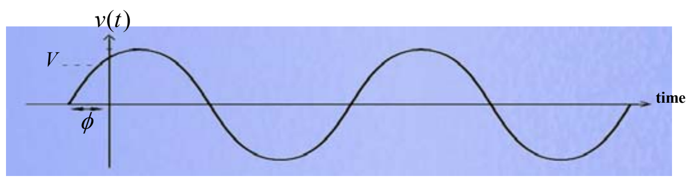
Single-Phase AC Voltage Sources

A (single-phase) **AC voltage source** is a source whose voltage varies sinusoidally, as defined by the function:

$$v(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi) \text{ volts}$$



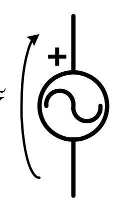
where: $V = \frac{V_{peak}}{\sqrt{2}}$ is the ***RMS*** or "***effective***" voltage magnitude of the AC waveform,
ω is the angular frequency ($2\pi f$) of the waveform, and
ϕ is the phase angle of the waveform.



Single-Phase AC Voltage Sources

The AC voltage:

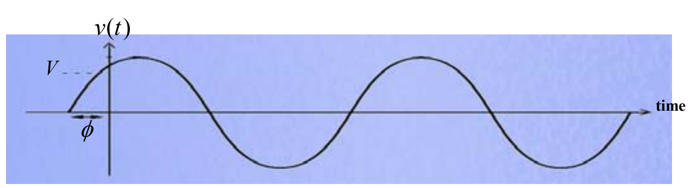
$$v(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi) \text{ volts}$$



may also be expressed in "***phasor***" form:

$$\tilde{V} = Ve^{j\phi} = V\angle\phi$$

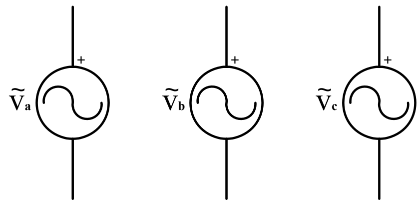
where: V is the RMS magnitude of the voltage, and
ϕ is the phase angle of the waveform.



Three-Phase AC Voltage Sources

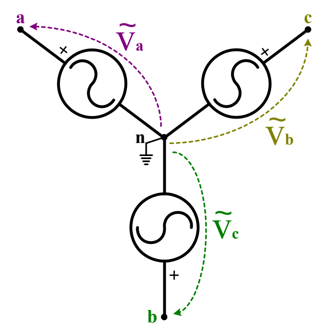
A ***three-phase*** (3Φ) ***AC voltage*** source is a composite source that can be modeled using three single-phase AC voltage sources that are connected together to function as one complete unit.

Note that the three single-phase AC voltage sources must be connected together in a symmetrical fashion.



Wye-connected Three-Phase Source

The three sources are typically connected together in a "***Wye***" (**Y**) format such that the reference terminals of the three supplies are tied to a common point of connection.



The common point of connection is referred to as the "***neutral point***".

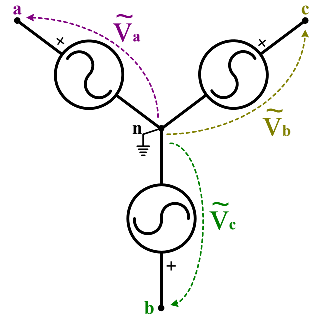
(node **n** in the figure)

Note that the neutral point is often grounded in order to provide a zero-volt reference for the source.

Wye-connected Three-Phase Source

If the remaining nodes are labeled **a**, **b**, and **c**, then:

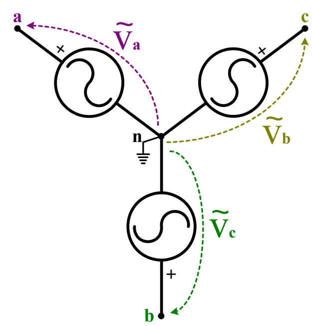
- Then the voltage $\tilde{V}_a$ can be defined as the voltage-rise from the neutral point **n** to node **a**.
- Similarly, voltages $\tilde{V}_b$ and $\tilde{V}_c$ can be defined as the rises from node **n** to **b** and node **n** to **c** respectively.



Phase Voltages

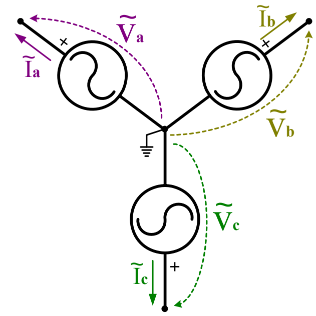
The voltages $\tilde{V}_a$, $\tilde{V}_b$, and $\tilde{V}_c$ are referred to as "***phase voltages***" because they correspond to the voltage across each individual phase of the wye-connected source.

The phase voltages are sometimes referred to as "***line-to-neutral voltages***", and as such may be expressed as $\tilde{V}_{an}$, $\tilde{V}_{bn}$, and $\tilde{V}_{cn}$.



Phase Currents

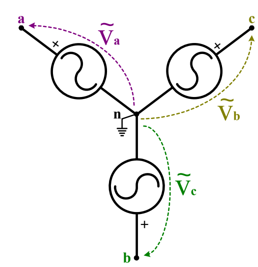
Similarly, the currents $\tilde{I}_a$, $\tilde{I}_b$, and $\tilde{I}_c$ are referred to as "***phase currents***" because they correspond to the current flowing through each individual phase of the wye-connected source.

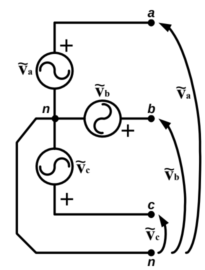


Wye-connected Three-Phase Source

Both of the figures shown to the right depict the same 3Φ source. The only differences are that the bottom figure has the three phases drawn in either a vertical or a horizontal orientation and a that wire has been connected to the neutral point to provide a forth point of connection.

Note that the phase voltages are also shown in the bottom figure, but this time with respect to the four points of connection, terminals **a**, **b**, **c**, and **n**.

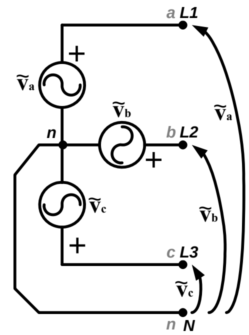




Wye-connected Source Terminals

The primary source terminals or connection points are nodes **a**, **b**, and **c**.

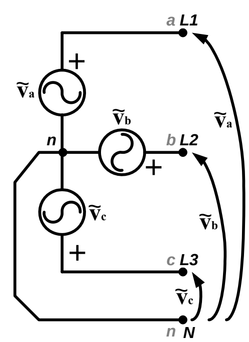
Nodes **a**, **b**, and **c** are sometimes defined as ***line terminals*** **L1**, **L2**, and **L3** because they are the terminals to which the three energized conductors of a 3Φ transmission line will be connected.



The line connected to the neutral-point is often referred to as the "***neutral line***" or the "***neutral conductor***".

Balanced Three-Phase Voltage Source

A "***balanced***" 3Φ source is a source whose phase voltages have <u>equal magnitudes</u> and <u>phase angles that are separated by 120º</u>.



Note that, despite slight magnitude differences that might exist between the three individual phases, most practical 3Φ sources are assumed to be balanced.

Thus, a balanced set of phase voltages can be defined as:

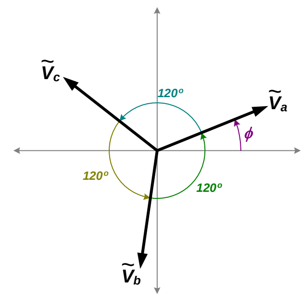
$$\tilde{V}_a = V\angle\phi$$

$$\tilde{V}_b = V\angle\phi - 120°$$

$$\tilde{V}_c = V\angle\phi - 240°$$

Balanced Phase Voltages

The figure below is a plot of the balanced set of phase voltages:

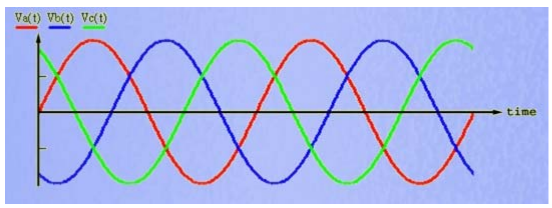


$$\tilde{V}_a = V\angle\phi$$

$$\tilde{V}_b = V\angle\phi - 120°$$

$$\tilde{V}_c = V\angle\phi - 240°$$

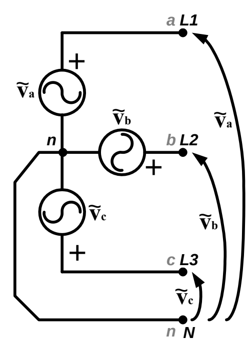
as a function of time, with ϕ = 0º as shown.



As defined, the voltages are considered to have a "***positive***" **phase sequence (a-b-c)** because phase **a** leads **b** and phase **b** leads **c**.

Balanced Phase Voltages Example

Given the phase voltage:



$$\tilde{V}_a = 120\angle 40°$$

determine the other phase voltages $\tilde{V}_b$ and $\tilde{V}_c$.

If $\tilde{V}_a = V\angle\phi$, then V =120 volts and ϕ = 40º, thus:

$$\tilde{V}_a = V\angle\phi = 120\angle 40°$$
$$\tilde{V}_b = V\angle\phi - 120° = 120\angle -80°$$
$$\tilde{V}_c = V\angle\phi - 240° = 120\angle -200°$$

→

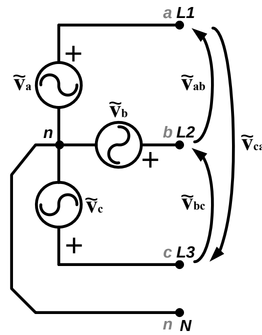
$$\tilde{V}_a = 120\angle 40°$$
$$\tilde{V}_b = 120\angle -80°$$
$$\tilde{V}_c = 120\angle -200°$$

Line Voltages

A second set of voltages can also be defined for the 3Φ source in terms of the voltage rise between each pair of terminals:

a-b, **b-c**, and **c-a**.

The voltages $\widetilde{V}_{ab}$, $\widetilde{V}_{bc}$ and $\widetilde{V}_{ca}$ are referred to as "***line voltages***" because they are the voltages between any pair of line terminals.



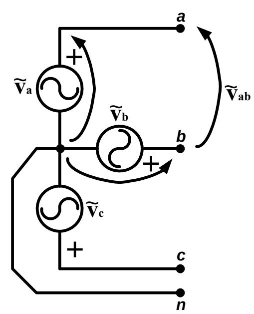
Line Voltages

The ***line voltages*** for a balanced 3Φ source are closely related to the source's phase voltages.

The line voltage $\widetilde{V}_{ab}$ defines the voltage rise from terminal **b** to terminal **a**, and can be expressed in terms of the phase voltages:

$$\widetilde{V}_{ab} = -\widetilde{V}_b + \widetilde{V}_a = \widetilde{V}_a - \widetilde{V}_b$$

The same logic can be used to express all three line voltages in terms of their respective phase voltages:



$$\widetilde{V}_{ab} = \widetilde{V}_a - \widetilde{V}_b$$

$$\widetilde{V}_{bc} = \widetilde{V}_b - \widetilde{V}_c$$

$$\widetilde{V}_{ca} = \widetilde{V}_c - \widetilde{V}_a$$

Line Voltages

It turns out that, given a balanced 3Φ source with phase voltage:

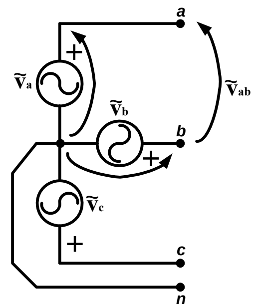
$$\tilde{V}_a = V\angle\phi°$$

the **line voltage** $\tilde{V}_{ab}$ for that source can be determined as follows:

$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b$$

$$= V\angle\phi° \; - \; V\angle\phi - 120°$$

$$= \sqrt{3}\cdot V\angle\phi + 30°$$



$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b$$

$$\tilde{V}_{bc} = \tilde{V}_b - \tilde{V}_c$$

$$\tilde{V}_{ca} = \tilde{V}_c - \tilde{V}_a$$

Line Voltages

Similarly, a complete analysis of a 3Φ source having the phase voltages:

$$\tilde{V}_a = V\angle\phi°$$

$$\tilde{V}_b = V\angle\phi - 120°$$

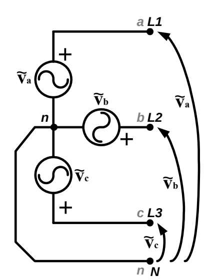
$$\tilde{V}_c = V\angle\phi - 240°$$

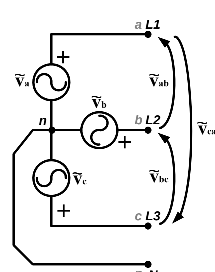
will result in the following set of line voltages:

$$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$$

$$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$$

$$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$$

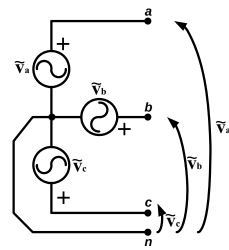


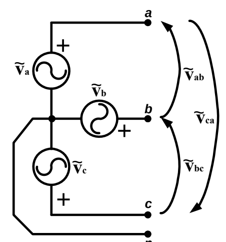


Balanced Line Voltages

Note that the line voltages have equal magnitudes and a 120º phase separation between each pair;

Thus, the line voltages maintain the same **balanced** relationship as the phase voltages:





Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

Phase ↔ Line Voltage Relationships

A comparison of the phase and line voltages:

$$\tilde{V}_a = V\angle\phi° \quad \tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$$

reveals that the line voltages are:

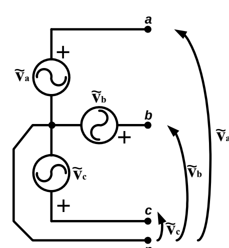
- $\sqrt{3}$x greater in magnitude, and
- 30° greater in phase angle

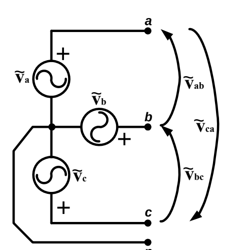
compared to the phase voltages.

$$\tilde{V}_{ab} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_a$$

$$\tilde{V}_{bc} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_b$$

$$\tilde{V}_{ca} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_c$$



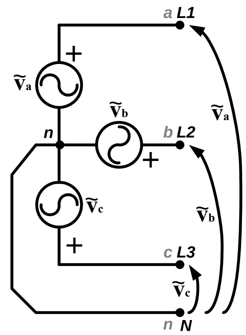


Line Voltage Example

Given a 3Φ source with phase voltages:

$$\tilde{V}_a = 120\angle 40^\circ$$
$$\tilde{V}_b = 120\angle -80^\circ$$
$$\tilde{V}_c = 120\angle -200^\circ$$

determine the line voltages $\tilde{V}_{ab}$, $\tilde{V}_{bc}$ and $\tilde{V}_{ca}$.



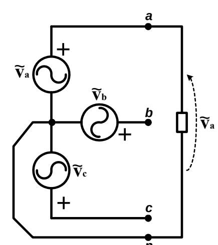
$$\tilde{V}_{ab} = (\sqrt{3}\angle 30^\circ)\cdot\tilde{V}_a = (\sqrt{3}\angle 30^\circ)\cdot 120\angle 40^\circ = 208\angle 70^\circ$$
$$\tilde{V}_{bc} = (\sqrt{3}\angle 30^\circ)\cdot\tilde{V}_b = (\sqrt{3}\angle 30^\circ)\cdot 120\angle -80^\circ = 208\angle -50^\circ$$
$$\tilde{V}_{ca} = (\sqrt{3}\angle 30^\circ)\cdot\tilde{V}_c = (\sqrt{3}\angle 30^\circ)\cdot 120\angle -200^\circ = 208\angle -170^\circ$$

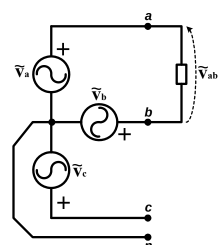
1Φ Voltages Available from 3Φ Source

A **single-phase load** may be supplied from a three-phase source if the load is connected across two of the source's terminals.

If the load is connected between a <u>line terminal</u> and the <u>neutral terminal</u>, then a ***phase voltage*** will appear across the load.

If the load is connected between two <u>line terminals</u>, then a ***line voltage*** will appear across the load.





Note – if the neutral terminal is not available, then only the ***line voltages*** can be utilized from the supply and <u>not the phase voltages</u>.

Balanced Three-Phase Loads

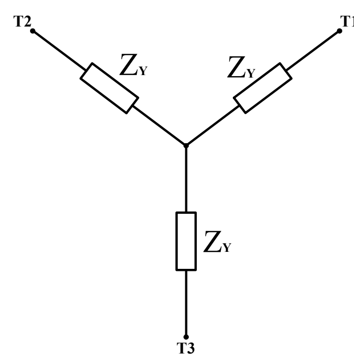
A **three-phase load** consists of three individual loads that are connected together in a symmetrical fashion, either **Wye** (**Y**) or **Delta** (**Δ**), to form a composite load that can be supplied by a 3Φ source.

A **balanced 3Φ load** is constructed using three loads that all have the same impedance value.

When a balanced 3Φ load is connected to a balanced 3Φ source, the resultant currents will also maintain a balanced relationship similar to that of the phase and line voltages.

Wye-connected Three-Phase Loads

A **wye-connected**, three-phase load is constructed by connecting one end of the three individual loads to form a common (neutral) node.

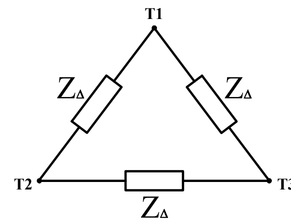


The opposite end of the three individual loads provide the terminals for connection to a 3Φ system.

These terminals are often defined as load terminals **T1**, **T2**, and **T3**, because they will be connected to source terminals **L1**, **L2**, and **L3** respectively.

Delta-connected Three-Phase Loads

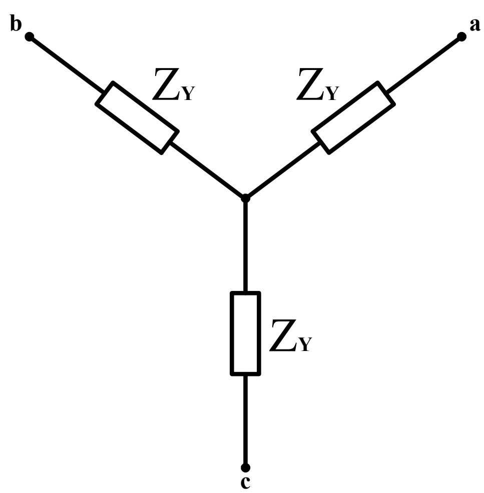
A **delta-connected**, three-phase load is constructed by connecting three impedances together as shown to the right.



The three nodes that connect each pair of impedances together provide the terminals for connection to a 3Φ system.

These terminals may also be defined as load terminals **T1**, **T2**, and **T3**, because they will also be connected to source terminals **L1**, **L2**, and **L3** respectively.

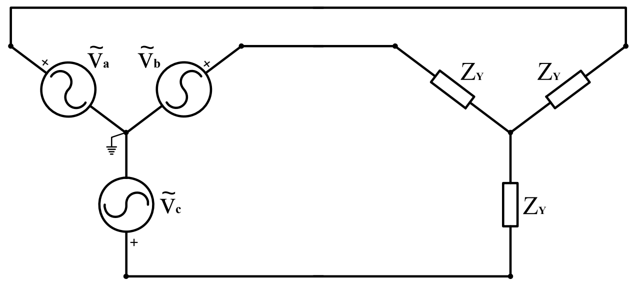
Wye-connected Three-Phase Loads



Wye-connected Loads in 3Φ Systems

The simple 3Φ system shown below consists of a ***wye-connected source*** and a ***wye-connected load***.

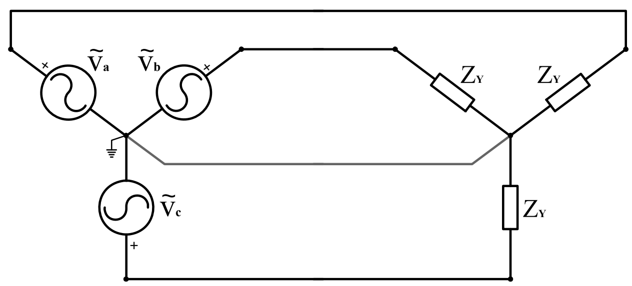
The neutral-point of the source is grounded to provide a zero-volt reference for the system.



Wye-connected Loads in 3Φ Systems

Three wires or "***lines***" are used to connect the source terminals to the terminals of the Y-connected load.

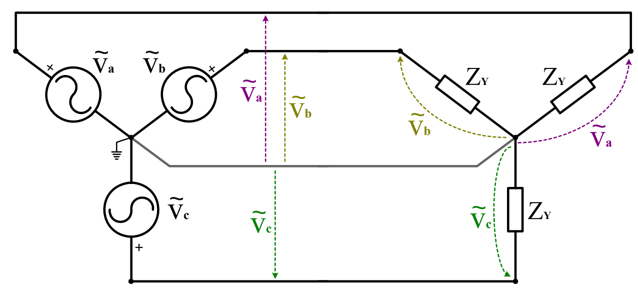
A "***neutral wire***" can be added to connect the grounded neutral-point of the source to the center-point of the load, holding both neutral points at a zero-volt potential.



Wye-connected Loads in 3Φ Systems

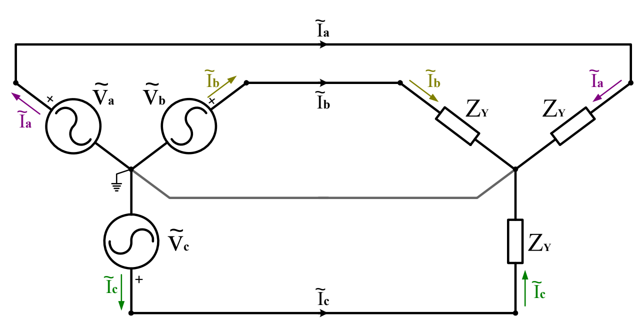
Note that the voltage potential present on each line (w.r.t. the neutral wire) is equal to the ***phase voltage*** of the source's phase to which the line is connected.

Thus, the four-wire connection results in the presence of a ***phase voltage*** across each phase of the load.



Wye-connected Load Currents

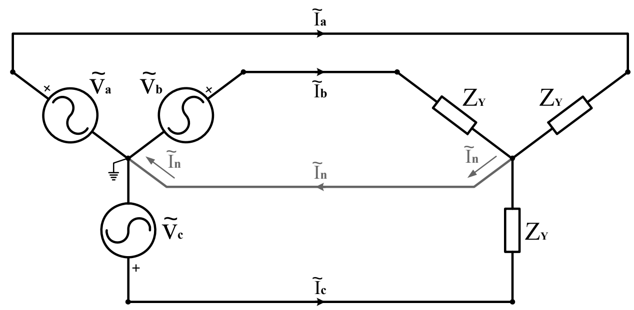
A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) can be defined that flow from each phase of the source, down the lines and into the individual phases of the load.



Wye-connected Load Currents

A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) can be defined that flow from each phase of the source, down the lines and into the individual phases of the load.

A ***neutral current*** ($\tilde{I}_n$) can also be defined that flows in the neutral wire from the load back to the source.

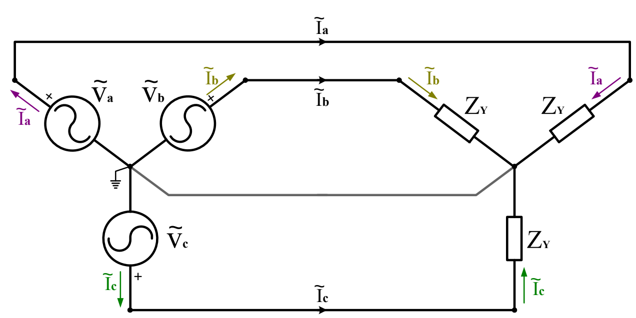


Wye-connected Load Currents

Note that the ***line currents*** may also be referred to as

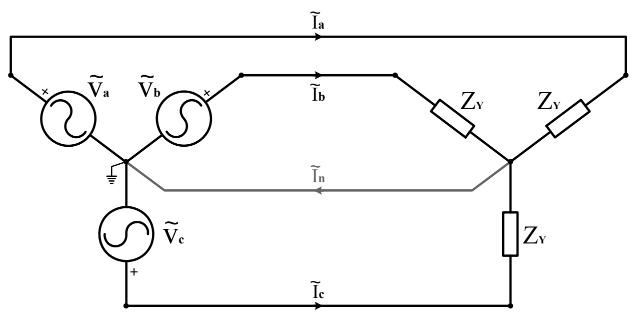
- ***phase currents of the source***, or
- ***phase currents of the load***

because they flow through the individual phases of both the source and the load.



Wye-connected Load Currents

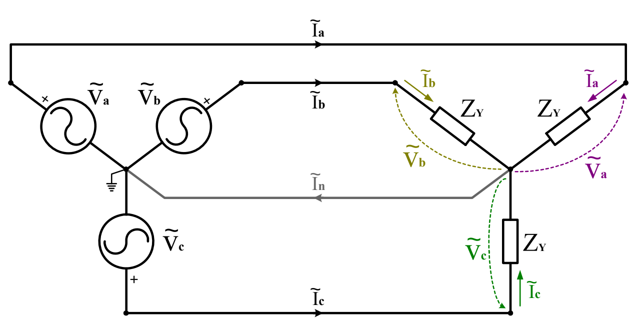
If the source voltages and load impedances are all known, then the ***line currents*** and the ***neutral current*** can all be determined using basic circuit theory.



Wye-connected Load Currents

Since the phase voltages of the load and source are equal, the line currents can each be solved independently by applying Ohm's Law at each load.

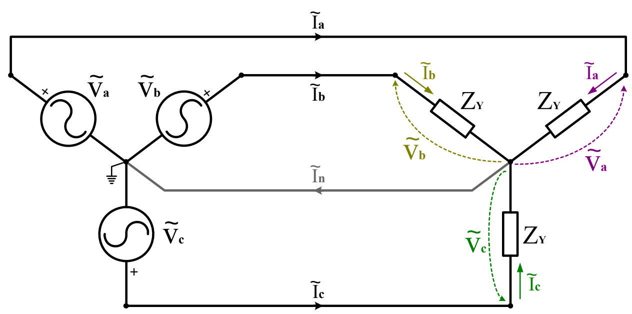
$$\tilde{I}_a = \frac{\tilde{V}_a}{Z_Y} \qquad \tilde{I}_b = \frac{\tilde{V}_b}{Z_Y} \qquad \tilde{I}_c = \frac{\tilde{V}_c}{Z_Y}$$



Wye-connected Load Currents

Furthermore, if the source voltages are balanced and the load impedances are all equal, then the line currents will also be balanced.

$$\tilde{I}_a = I\angle\delta° \qquad \tilde{I}_b = I\angle\delta - 120° \qquad \tilde{I}_c = I\angle\delta - 240°$$



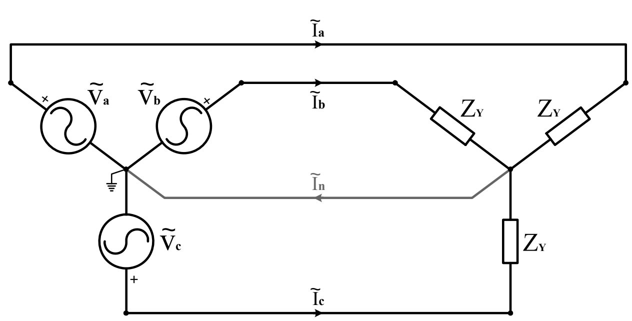
Neutral Current in 3Φ Systems

The neutral current $\tilde{I}_n$ can be determined from the node equation:

$$\tilde{I}_n = \tilde{I}_a + \tilde{I}_b + \tilde{I}_c$$

In a balanced system, the neutral current will be:

$$\tilde{I}_n = \tilde{I}_a + \tilde{I}_b + \tilde{I}_c = I\angle\delta + I\angle(\delta\text{-}120°) + I\angle(\delta\text{-}240°) = 0 \text{ amps}$$

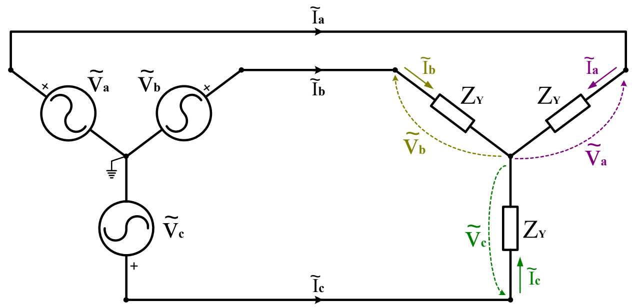


Neutral Wires in 3Φ Systems

In a **balanced system**, the line currents will always sum to zero…

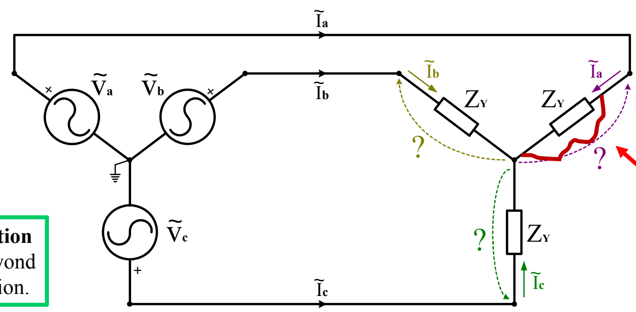
→ **<u>no current will flow in the neutral wire</u>**

Thus, removal of the neutral wire will theoretically have no effect on the system under normal operating conditions.



Neutral Wires in 3Φ Systems

Note that, although removal of the neutral wire will not affect the normal operation of a balanced system, the importance of the neutral wire comes into play during times of abnormal operation (i.e. –unbalanced operation and/or fault conditions) during which it's existence can greatly affect the system's operation.



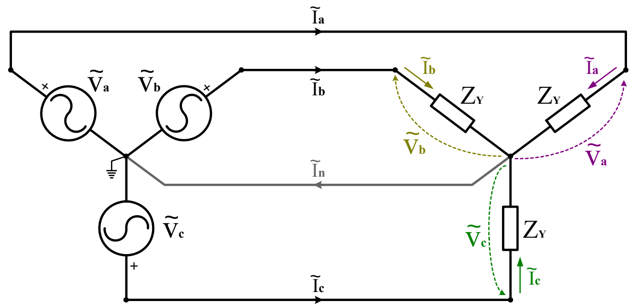
Both **unbalanced operation** and system **faults** are beyond the scope of this discussion.

For example, when a line→neutral fault (short-circuit) occurs, the existence (or not) of the neutral wire will greatly affect the system.

Complex Power in 3Φ Systems

The total complex power produced or consumed by a 3Φ source or load is equal to the sum of the complex powers produced or consumed by each of the source's or load's three individual phases.

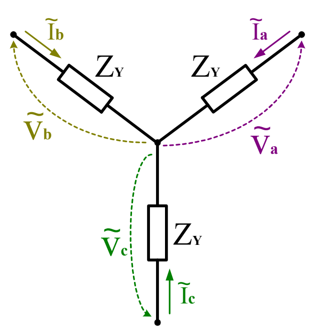
$$S_{3\Phi} = S_a + S_b + S_c$$



Complex Power in Y-connected Loads

In the case of a 3Φ, Y-connected load, the complex powers consumed by each of the load's three individual phases are:

$$S_a = \tilde{V}_a \cdot \tilde{I}_a^* \qquad S_b = \tilde{V}_b \cdot \tilde{I}_b^* \qquad S_c = \tilde{V}_c \cdot \tilde{I}_c^*$$



Complex Power in Y-connected Loads

If the system is balanced, with voltages and currents:

$$\tilde{V}_a = V\angle\phi° \qquad \tilde{I}_a = I\angle\delta°$$
$$\tilde{V}_b = V\angle\phi - 120° \qquad \tilde{I}_b = I\angle\delta - 120°$$
$$\tilde{V}_c = V\angle\phi - 240° \qquad \tilde{I}_c = I\angle\delta - 240°$$

then:

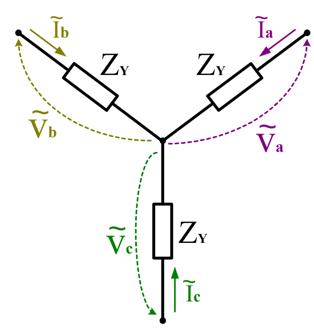
$$S_a = \tilde{V}_a \cdot \tilde{I}_a^* = [V\angle\phi]\cdot[I\angle-(\delta)] \Rightarrow \boxed{S_a = V\cdot I\angle\phi-\delta}$$
$$S_b = \tilde{V}_b \cdot \tilde{I}_b^* = [V\angle\phi-120°]\cdot[I\angle-(\delta-120°)] \Rightarrow \boxed{S_b = V\cdot I\angle\phi-\delta}$$
$$S_c = \tilde{V}_c \cdot \tilde{I}_c^* = [V\angle\phi-240°]\cdot[I\angle-(\delta-240°)] \Rightarrow \boxed{S_c = V\cdot I\angle\phi-\delta}$$

(all three phases consume the same complex power)

Complex Power in Y-connected Loads

Thus, the total complex power consumed by a balanced, 3Φ, Y-connected load will be equal to **3x** the power consumed by any individual phase:

$$S_{3\Phi} = S_a + S_b + S_c = 3\cdot S_{1\Phi}$$



allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3\cdot\tilde{V}_a\cdot\tilde{I}_a^* = 3\cdot V\cdot I\angle\phi-\delta$$

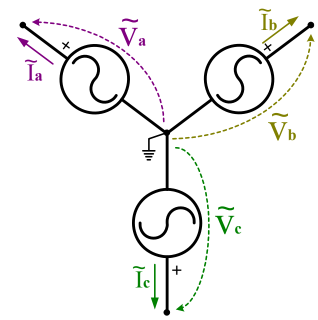
where: $\tilde{V}_a = V\angle\phi$ volts

$\tilde{I}_a = I\angle\delta$ amps

Complex Power in Y-connected Sources

Similarly, the total complex power produced by a <u>balanced</u>, 3Φ, Y-connected <u>source</u> will be equal to 3x the power produced by any individual phase:

$$S_{3\Phi} = S_a + S_b + S_c = 3 \cdot S_{1\Phi}$$



allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_a \cdot \tilde{I}_a^* = 3 \cdot V \cdot I\angle\phi - \delta$$

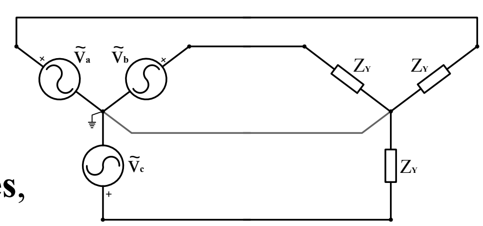
where: $\tilde{V}_a = V\angle\phi$ volts

$\tilde{I}_a = I\angle\delta$ amps

3Φ Wye-connected Load Example

Given a **480V**, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Y-connected, balanced load having individual per-phase impedances:

$$\boldsymbol{Z_Y = 80 + j60\ \Omega},$$



For, this system, determine:

a) all of the **phase and line voltages**,

b) all of the **line currents**, and

c) the **total complex power** supplied by the source to the Y-load.

$\tilde{V}_a$

3Φ Wye-connected Load Example

Since the source is a Y-connected, positive-sequence, balanced source, the phase and line voltages will adhere to the following relationships:

Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

The values of $\boldsymbol{V}$ and $\boldsymbol{\phi}$ can be determined from the information provided in the problem statement.

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$

Standard: if a single voltage magnitude is specified for a 3Φ source, then the value specified is the source's **line-voltage** magnitude.

Thus, given a balanced 480V source, the magnitudes of the line and phase voltage can all be specified as:

$$V_{line} = \sqrt{3}\cdot V = 480 \text{ volts} \quad \Rightarrow \quad V_{phase} = V = \frac{480}{\sqrt{3}} = 277 \text{ volts}$$

If the source is Y-connected with an accessible neutral point, then the line and phase voltage magnitudes are often specified for convenience:
I.e. – **480/277V**

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle\phi$	$\tilde{V}_{ab} = 480\angle\phi + 30°$
$\tilde{V}_b = 277\angle\phi - 120°$	$\tilde{V}_{bc} = 480\angle\phi - 90°$
$\tilde{V}_c = 277\angle\phi - 240°$	$\tilde{V}_{ca} = 480\angle\phi - 210°$

As with any steady-state AC circuit solution, the <u>first phase angle</u> in a 3Φ circuit may be chosen arbitrarily, after which all other phase angles (voltage and current) must be calculated based to the initial choice.

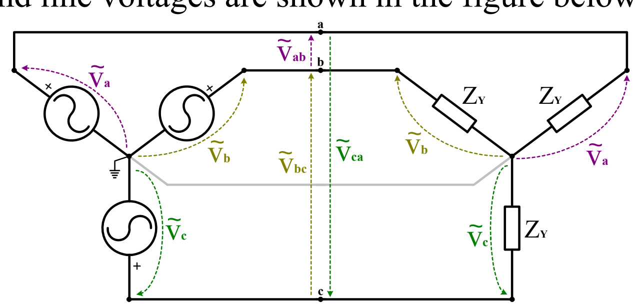
For convenience, the first angle is often chosen to be 0°. Thus, for this example, the angle of the phase voltage $\tilde{V}_a$ will be set to 0°.

$$\phi = 0°$$

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle + 30°$
$\tilde{V}_b = 277\angle - 120°$	$\tilde{V}_{bc} = 480\angle - 90°$
$\tilde{V}_c = 277\angle - 240°$	$\tilde{V}_{ca} = 480\angle - 210°$

The phase and line voltages are shown in the figure below:



3Φ Wye-connected Load Example

Phase Voltages	**Line Voltages**
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

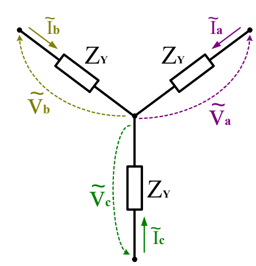
Now that all of the voltages have been specified in the system, the next step is to solve for all of the line currents that will flow in the 3Φ system from the source to the load.

Since the system is balanced, the resultant line currents will be balanced. Thus, the complete set of line currents may be determined by first solving for one of the currents and then utilizing the balanced relationship in order to specify the remaining currents.

3Φ Wye-connected Load Example

Phase Voltages	**Line Voltages**
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Applying Ohm's Law to "***phase a***" of the load results the line current:



$$\tilde{I}_a = \frac{\tilde{V}_a}{Z_Y} = \frac{277\angle 0°}{80 + j60} = 2.77\angle -36.9°$$

from which the remaining line currents can be solved.

3Φ Wye-connected Load Example

Phase Voltages	**Line Voltages**
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Given: $\tilde{I}_a = 2.77\angle -36.9°$ → $I = 2.77$ $\delta = -36.9°$

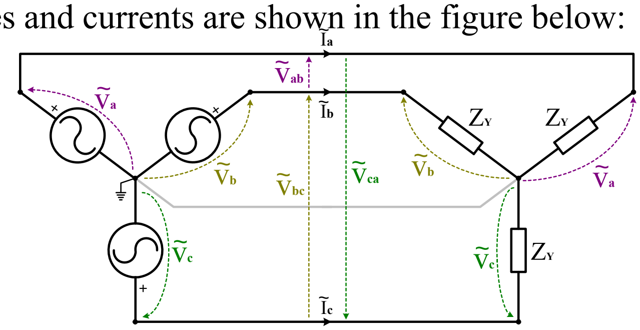
The remaining line currents can be determined from:

Balanced Relationships		**Line Currents**
$\tilde{I}_a = I\angle\delta$		$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{I}_b = I\angle\delta - 120°$	→	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{I}_c = I\angle\delta - 240°$		$\tilde{I}_c = 2.77\angle -276.9°$

3Φ Wye-connected Load Example

Phase Voltages	**Line Voltages**	**Line Currents**
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

The voltages and currents are shown in the figure below:



3Φ Wye-connected Load Example

Phase Voltages	Line Voltages	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

Since the total complex power produced/consumed in a balanced, 3Φ system is equal to 3x the complex power produced/consumed in a any individual phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_a \cdot \tilde{I}_a^* = 3 \cdot [277\angle 0°] \cdot [2.77\angle -(-36.9°)]$$
$$= 3 \cdot [614.4 + j460.8] = 1843.2 + j1382.4$$

3Φ Wye-connected Load Example

Phase Voltages	Line Voltages	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_a = 2.77\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_b = 2.77\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_c = 2.77\angle -276.9°$

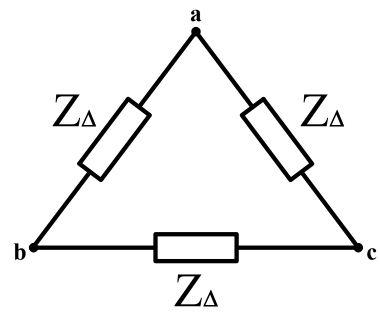
If desired, the complex power result:

$$S_{3\Phi} = 1843.2 + j1382.4$$

can be broken down into its real and reactive power components:

$$P_{3\Phi} = 1843.2\, Watts \qquad Q_{3\Phi} = 1382.4\, Vars$$

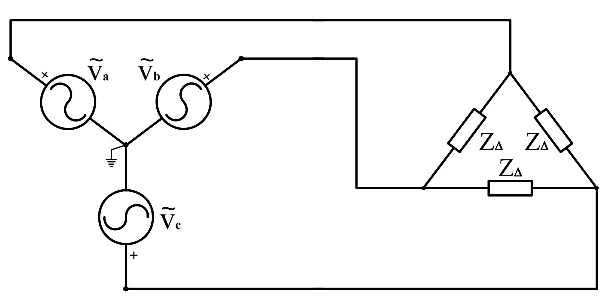
Delta-connected Three-Phase Loads



Delta-connected Loads in 3Φ Systems

The simple 3Φ system shown below consists of a ***wye-connected source*** and a ***delta-connected load***.

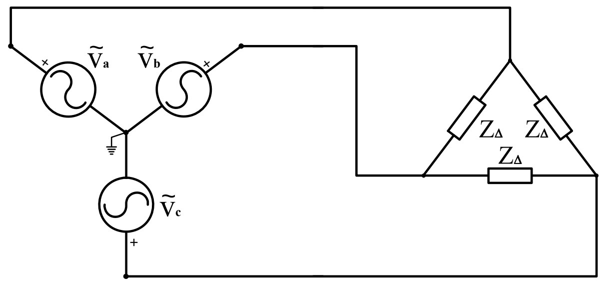
The neutral-point of the source is still grounded to provide a zero-volt reference for the system.



Delta-connected Loads in 3Φ Systems

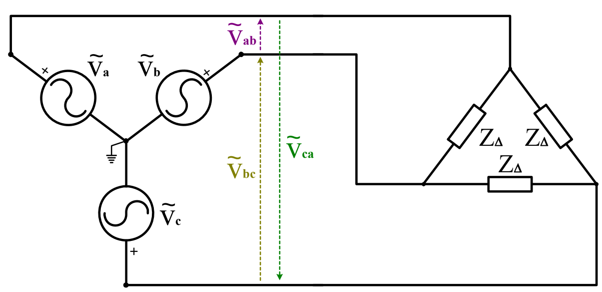
Three "***lines***" are also used to connect the source terminals to the terminals of the Δ-connected load.

Note that no neutral wire can be connected to the load because the Δ-connected load has no central node to which the wire can be symmetrically connected.



Delta-connected Load Voltages

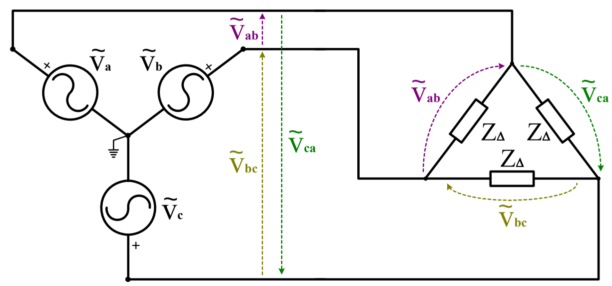
The voltage potential between each pair of lines is equal to the ***line voltage*** of the source.



Delta-connected Load Voltages

The voltage potential between each pair of lines is equal to the line voltage of the source.

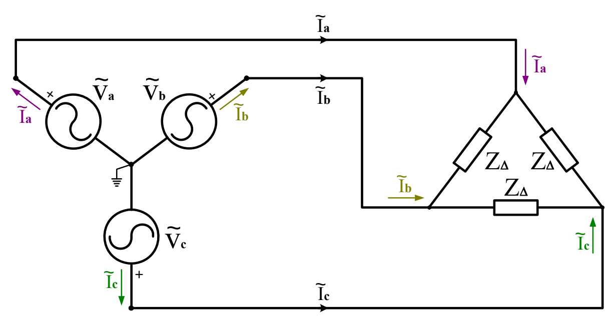
Since each phase of the Δ-connected load connects across a pair of lines, the three-wire connection provides a ***line voltage*** across each phase of the load.



Delta-connected Load Currents

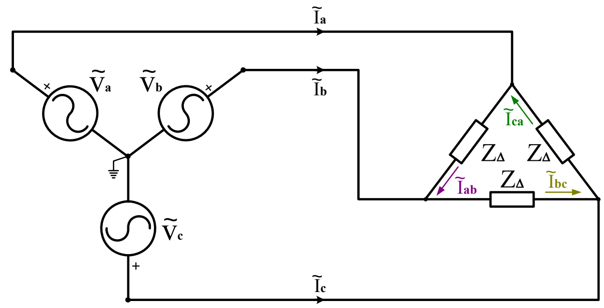
A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) was defined to flow in the lines from the source to the load.

Although the line currents flow through each phase of the source, they do **not** flow through the individual phases of the Δ-load.



Delta-connected Load Currents

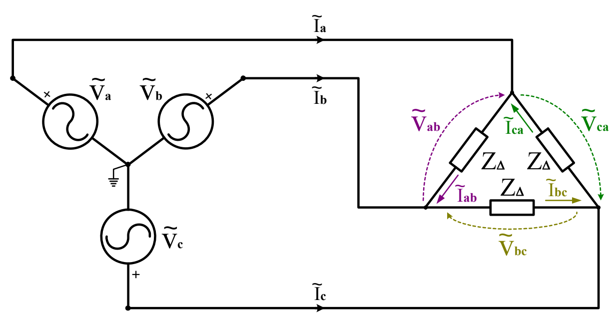
In order to fully characterize the Δ-connected load's operation, a set of ***phase currents*** ($\tilde{I}_{ab}$, $\tilde{I}_{bc}$ and $\tilde{I}_{ca}$) that flow through the individual phases of the load must also be defined.



Delta-connected Load Currents

If the line voltages and load impedances are known, then the ***phase currents of the load*** can each be solved independently by applying Ohm's Law for each phase:

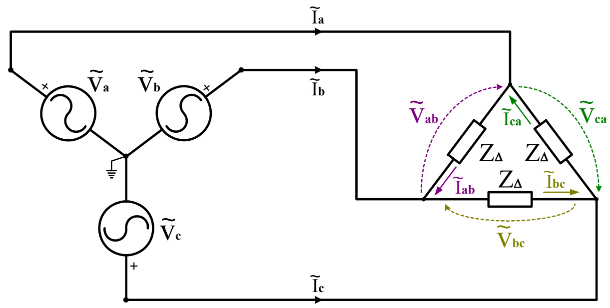
$$\tilde{I}_{ab} = \frac{\tilde{V}_{ab}}{Z_\Delta} \qquad \tilde{I}_{bc} = \frac{\tilde{V}_{bc}}{Z_\Delta} \qquad \tilde{I}_{ca} = \frac{\tilde{V}_{ca}}{Z_\Delta}$$



Delta-connected Load Currents

Furthermore, if the source voltages are balanced and the load impedances are all equal, then the phase currents of the load will also be balanced.

$$\tilde{I}_{ab} = I_\Delta \angle \beta \qquad \tilde{I}_{bc} = I_\Delta \angle \beta - 120° \qquad \tilde{I}_{ca} = I_\Delta \angle \beta - 240°$$



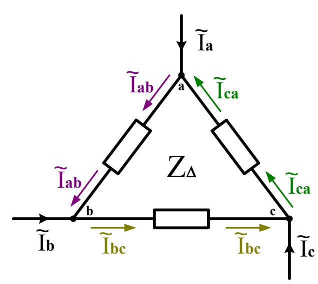
Delta-connected Load Currents

Once the phase currents of the load have been determined, the ***line currents*** flowing into the load may also be determined by solving a node equation for each of the three connection points to the load.

Node a: $\tilde{I}_a = \tilde{I}_{ab} - \tilde{I}_{ca}$

Node b: $\tilde{I}_b = \tilde{I}_{bc} - \tilde{I}_{ab}$

Node c: $\tilde{I}_c = \tilde{I}_{ca} - \tilde{I}_{bc}$



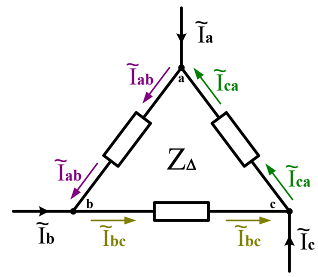
Delta-connected Load Currents

Given the balanced set of phase currents:

$$\tilde{I}_{ab} = I_{\Delta}\angle\beta \qquad \tilde{I}_{bc} = I_{\Delta}\angle\beta - 120° \qquad \tilde{I}_{ca} = I_{\Delta}\angle\beta - 240°$$

the line current $\tilde{I}_a$ can be determined as follows:

$$\begin{aligned}\tilde{I}_a &= \tilde{I}_{ab} - \tilde{I}_{ca} \\ &= I_{\Delta}\angle\beta - I_{\Delta}\angle(\beta - 240°) \\ &= \sqrt{3}\cdot I_{\Delta}\angle(\beta - 30°) \\ &= (\sqrt{3}\angle -30°)\cdot \tilde{I}_{ab}\end{aligned}$$



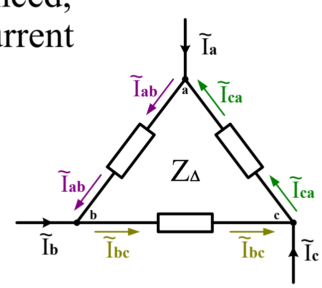
Delta-connected Load Currents

Since the phase currents are balanced:

$$\tilde{I}_{ab} = I_{\Delta}\angle\beta \qquad \tilde{I}_{bc} = I_{\Delta}\angle\beta - 120° \qquad \tilde{I}_{ca} = I_{\Delta}\angle\beta - 240°$$

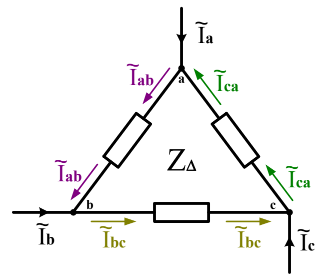
the resultant line currents will also be balanced, allowing a complete set of phase-to-line current relationships to be defined:

$$\begin{aligned}\tilde{I}_a &= (\sqrt{3}\angle -30°)\cdot \tilde{I}_{ab} \\ \tilde{I}_b &= (\sqrt{3}\angle -30°)\cdot \tilde{I}_{bc} \\ \tilde{I}_c &= (\sqrt{3}\angle -30°)\cdot \tilde{I}_{ca}\end{aligned}$$



Phase ↔ Line Current Relationship

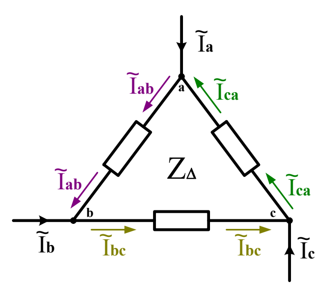
The phase-to-line current relationships can then be used to specify a complete set of currents for a balanced, Δ-connected load as follows:



Phase Currents	Line Currents
$\tilde{I}_{ab} = I_\Delta \angle \beta$	$\tilde{I}_a = \sqrt{3} \cdot I_\Delta \angle \beta - 30°$
$\tilde{I}_{bc} = I_\Delta \angle \beta - 120°$	$\tilde{I}_b = \sqrt{3} \cdot I_\Delta \angle \beta - 150°$
$\tilde{I}_{ca} = I_\Delta \angle \beta - 240°$	$\tilde{I}_c = \sqrt{3} \cdot I_\Delta \angle \beta - 270°$

Phase ↔ Line Current Relationship

Note – to correspond with the line-currents defined for the Y-connected load, the phase and line current expressions can be rewritten such that:



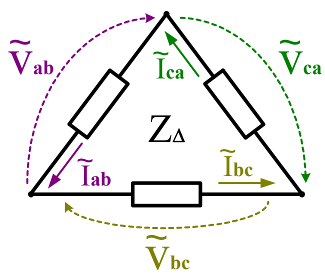
$$I = \sqrt{3} \cdot I_\Delta \qquad \delta = \beta - 30°$$

Line Currents	Phase Currents
$\tilde{I}_a = I \angle \delta$	$\tilde{I}_{ab} = \frac{I}{\sqrt{3}} \angle \delta + 30°$
$\tilde{I}_b = I \angle \delta - 120°$	$\tilde{I}_{bc} = \frac{I}{\sqrt{3}} \angle \delta - 90°$
$\tilde{I}_c = I \angle \delta - 240°$	$\tilde{I}_{ca} = \frac{I}{\sqrt{3}} \angle \delta - 210°$

Complex Power in Δ-connected Loads

In the case of a 3Φ, Δ-connected load, the complex power consumed by each of the load's three individual phases are:

$$S_{ab} = \widetilde{V}_{ab} \cdot \widetilde{I}_{ab}^{*} \qquad S_{bc} = \widetilde{V}_{bc} \cdot \widetilde{I}_{bc}^{*} \qquad S_{ca} = \widetilde{V}_{ca} \cdot \widetilde{I}_{ca}^{*}$$



Complex Power in Δ-connected Loads

If the system is balanced, with voltages and currents defined as:

$$\widetilde{V}_{ab} = \sqrt{3} \cdot V \angle \phi + 30° \qquad \widetilde{I}_{ab} = \frac{I}{\sqrt{3}} \angle \delta + 30°$$

$$\widetilde{V}_{bc} = \sqrt{3} \cdot V \angle \phi - 90° \qquad \widetilde{I}_{bc} = \frac{I}{\sqrt{3}} \angle \delta - 90°$$

$$\widetilde{V}_{ca} = \sqrt{3} \cdot V \angle \phi - 210° \qquad \widetilde{I}_{ca} = \frac{I}{\sqrt{3}} \angle \delta - 210°$$

then:

$$S_{ab} = \widetilde{V}_{ab} \cdot \widetilde{I}_{ab}^{*} = \left[\sqrt{3} \cdot V \angle \phi + 30°\right] \cdot \left[\frac{I}{\sqrt{3}} \angle -(\delta + 30°)\right] = V \cdot I \angle \phi - \delta$$

$$S_{bc} = \widetilde{V}_{bc} \cdot \widetilde{I}_{bc}^{*} = \left[\sqrt{3} \cdot V \angle \phi - 90°\right] \cdot \left[\frac{I}{\sqrt{3}} \angle -(\delta - 90°)\right] = V \cdot I \angle \phi - \delta$$

$$S_{ca} = \widetilde{V}_{ca} \cdot \widetilde{I}_{ca}^{*} = \left[\sqrt{3} \cdot V \angle \phi - 210°\right] \cdot \left[\frac{I}{\sqrt{3}} \angle -(\delta - 210°)\right] = V \cdot I \angle \phi - \delta$$

Complex Power in Δ-connected Loads

If the system is balanced, with voltages and currents defined as:

$$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30° \qquad \tilde{I}_{ab} = \frac{I}{\sqrt{3}}\angle\delta + 30°$$

$$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90° \qquad \tilde{I}_{bc} = \frac{I}{\sqrt{3}}\angle\delta - 90°$$

$$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210° \qquad \tilde{I}_{ca} = \frac{I}{\sqrt{3}}\angle\delta - 210°$$

then:

$$S_{ab} = \tilde{V}_{ab}\cdot\tilde{I}_{ab}^{*} = V\cdot I\angle\phi - \delta$$

$$S_{bc} = \tilde{V}_{bc}\cdot\tilde{I}_{bc}^{*} = V\cdot I\angle\phi - \delta$$

$$S_{ca} = \tilde{V}_{ca}\cdot\tilde{I}_{ca}^{*} = V\cdot I\angle\phi - \delta$$

all three phases will consume equal complex power.

Complex Power in Δ-connected Loads

Thus, the total complex power consumed by a balanced, 3Φ, Δ-connected load will be equal to 3x the power consumed by any individual phase:

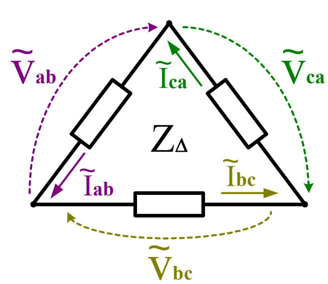
$$S_{3\Phi} = S_{ab} + S_{bc} + S_{ca} = 3\cdot S_{1\Phi}$$

allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3\cdot\tilde{V}_{ab}\cdot\tilde{I}_{ab}^{*} = 3\cdot V\cdot I\angle\phi - \delta$$

where: $\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$

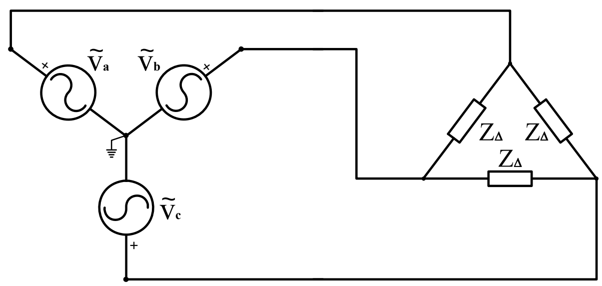
$\tilde{I}_{ab} = \frac{I}{\sqrt{3}}\angle\delta + 30°$



3Φ Delta-connected Load Example

Given a 480V, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Δ-connected, balanced load with individual phase impedances

$$Z_{\Delta} = 80 + j60\ \Omega,$$



3Φ Delta-connected Load Example

Given a 480V, 3Φ, Y-connected, positive-sequence, balanced source that is supplying a Δ-connected, balanced load with individual phase impedances

$$Z_{\Delta} = 80 + j60\ \Omega,$$

Determine:

a) all of the phase and line voltages in the system,

b) all of the phase and line currents in the system, and

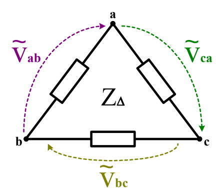
c) the total complex power provided by the source to the Δ-connected load.

Note – choose the angle of the phase voltage $\tilde{V}_a$ to be the 0° reference angle for the system.

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

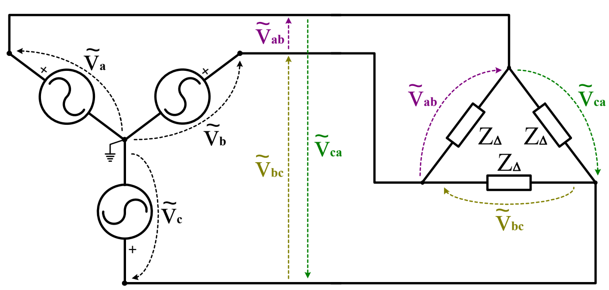
Since the source defined in this example is the same as that in the Y-connected load example, the phase and line voltages shown above are provided without the logic required to obtain those values.



3Φ Delta-connected Load Example

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

The phase and line voltages are shown in the figure below:



3Φ Delta-connected Load Example

Line Voltages

$$\tilde{V}_{ab} = 480\angle +30^\circ$$

$$\tilde{V}_{bc} = 480\angle -90^\circ$$

$$\tilde{V}_{ca} = 480\angle -210^\circ$$

Note that although the phase and line voltages both exist at the Y-connected source, only the line voltages appear at the Δ-connected load due to the absence of a neutral point.

3Φ Delta-connected Load Example

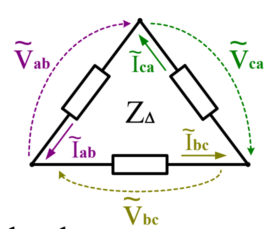
Line Voltages

$$\tilde{V}_{ab} = 480\angle +30^\circ$$

$$\tilde{V}_{bc} = 480\angle -90^\circ$$

$$\tilde{V}_{ca} = 480\angle -210^\circ$$

By applying Ohm's Law to the load connected across nodes **a** and **b**, the phase current can be determined:



$$\tilde{I}_{ab} = \frac{\tilde{V}_{ab}}{Z_\Delta} = \frac{480\angle 30^\circ}{80 + j60} = 4.8\angle -6.9^\circ$$

from which the remaining phase currents can then be solved.

3Φ Delta-connected Load Example

Line Voltages

$$\tilde{V}_{ab} = 480\angle +30°$$
$$\tilde{V}_{bc} = 480\angle -90°$$
$$\tilde{V}_{ca} = 480\angle -210°$$

Given: $\tilde{I}_{ab} = 4.8\angle -6.9°$ → $\frac{I}{\sqrt{3}} = 4.8 \quad \delta + 30° = -6.9°$

The remaining phase currents can be determined from:

Balanced Relationships

$$\tilde{I}_{ab} = \frac{I}{\sqrt{3}}\angle \delta + 30°$$
$$\tilde{I}_{bc} = \frac{I}{\sqrt{3}}\angle \delta - 90°$$
$$\tilde{I}_{ca} = \frac{I}{\sqrt{3}}\angle \delta - 210°$$

→

Phase Currents

$$\tilde{I}_{ab} = 4.8\angle -6.9°$$
$$\tilde{I}_{bc} = 4.8\angle -126.9°$$
$$\tilde{I}_{ca} = 4.8\angle -246.9°$$

3Φ Delta-connected Load Example

Line Voltages	Phase Currents
$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$
$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$
$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$

Additionally: $\frac{I}{\sqrt{3}} = 4.8 \quad \delta + 30° = -6.9°$ → $I = 8.31 \quad \delta = -36.9°$

The line currents can be determined from:

Balanced Relationships

$$\tilde{I}_{a} = I\angle \delta$$
$$\tilde{I}_{b} = I\angle \delta - 120°$$
$$\tilde{I}_{c} = I\angle \delta - 240°$$

→

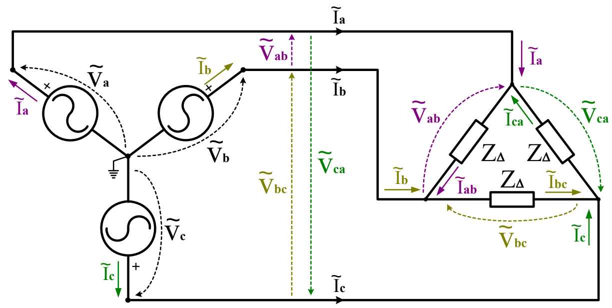
Line Currents

$$\tilde{I}_{a} = 8.31\angle -36.9°$$
$$\tilde{I}_{b} = 8.31\angle -156.9°$$
$$\tilde{I}_{c} = 8.31\angle -276.9°$$

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

The voltages and currents are shown in the figure below:



3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

Now that all of the voltages and currents have been specified in the system, the next step is to solve for the total complex power that will be provided by the 3Φ source to the 3Φ load.

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

Since the total complex power consumed by a balanced Δ-connected load is equal to 3x the complex power consumed by each individual phase of the load:

$$S_{3\Phi} = 3 \cdot \tilde{V}_{ab} \cdot \tilde{I}_{ab}^{*} = 3 \cdot [480\angle 30°] \cdot [4.8\angle -(-6.9°)]$$
$$= 3 \cdot [1843.2 + j1382.4] = 5529.6 + j4147.2$$

3Φ Delta-connected Load Example

Phase Voltages	Line Voltages	Phase Currents	Line Currents
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$	$\tilde{I}_{ab} = 4.8\angle -6.9°$	$\tilde{I}_a = 8.31\angle -36.9°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$	$\tilde{I}_{bc} = 4.8\angle -126.9°$	$\tilde{I}_b = 8.31\angle -156.9°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$	$\tilde{I}_{ca} = 4.8\angle -246.9°$	$\tilde{I}_c = 8.31\angle -276.9°$

If desired, the complex power result:

$$S_{3\Phi} = 5529.6 + j4147.2$$

can be broken down into its real and reactive power components:

$$P_{3\Phi} = 5529.6\,Watts \qquad Q_{3\Phi} = 4147.2\,Vars$$

Y ↔ Δ Load Comparison

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

Based on the results of the previous examples:

If a balanced 3Φ source is supplying both a Y-connected load and a Δ-connected load, each having the same per-phase impedances:

$$Z_\Delta = Z_Y$$

then the Δ-connected load will consume **3x** more power than the Y-connected load.

Y ↔ Δ Load Comparison

Phase Voltages	Line Voltages
$\tilde{V}_a = 277\angle 0°$	$\tilde{V}_{ab} = 480\angle +30°$
$\tilde{V}_b = 277\angle -120°$	$\tilde{V}_{bc} = 480\angle -90°$
$\tilde{V}_c = 277\angle -240°$	$\tilde{V}_{ca} = 480\angle -210°$

It can also be proven that:

If a balanced 3Φ source is supplying both a Y-connected load and a Δ-connected load, but the per-phase Δ-impedances are **3x** larger than the per-phase Y-impedances:

$$Z_\Delta = 3 \cdot Z_Y$$

then the Δ-connected load and the Y-connected load will consume the **same** amount of power.