ECET 3000

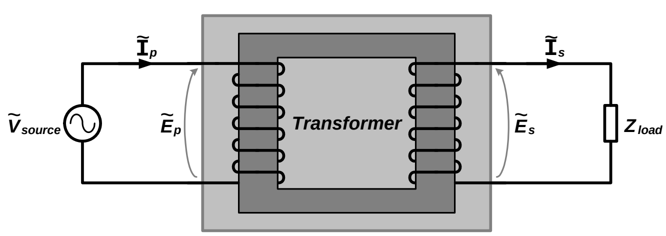
Electrical Principles

Ideal Transformers

Introduction to Ideal Transformers

An **ideal transformer** is a two-port device that receives an AC voltage waveform, $\tilde{E}_p$, at one magnitude and transforms it into a new AC voltage waveform, $\tilde{E}_s$, at a different magnitude.

The transformer's operation is based on the electro-magnetic interactions that occur between two mutually-linked coils that are magnetically coupled together by a **ferromagnetic core**.



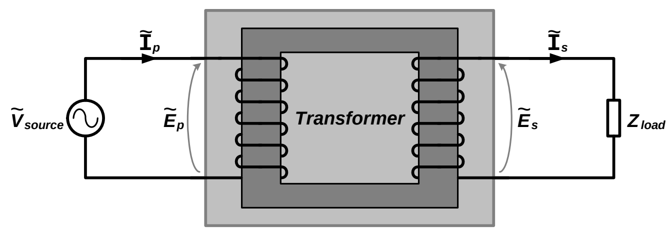
A **ferromagnetic core** is a high-permeability material, typically composed of iron or iron-based compounds, in which it is "easy" to induce a large magnetic field.

Ideal Transformer Definitions

Primary Winding ≡ the coil that creates the mutually-linked flux.
(I.e. – the sourced winding)

Secondary Winding ≡ the coil across which a voltage is induced.
(I.e. – the loaded winding)

Note – the primary & secondary winding designations can also be defined in terms of the power flow direction. (P_{IN} ➞ Primary, P_{OUT} ➞ Secondary)



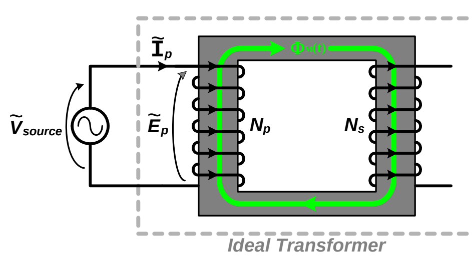
Sourced Primary Winding

If an AC voltage source is connected to the primary winding, a **magnetic flux**, Φ_M, will be created by that coil, the value of which may be determined from the Faraday's Law relationship:

$$\tilde{E}_p = N_p \cdot \frac{d\Phi_M(t)}{dt}$$

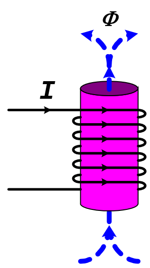
where: N_p is the number of turns of the sourced-coil.

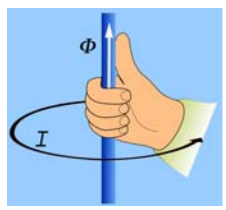
If the ferromagnetic core is assumed to be ideal (both lossless and infinitely permeable), then no current is initially required to create the mutually-linked flux.



Right-Hand-Rule
For Current-Carrying Coils

Right-Hand-Rule – Curl the fingers of your right hand in the direction that current flows around the coil. Your thumb will point in the direction that the field lines will pass through the center of the coil.





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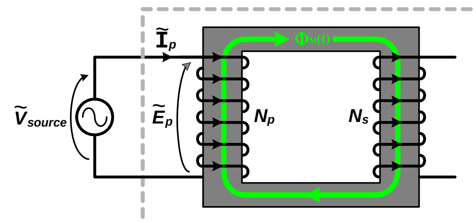


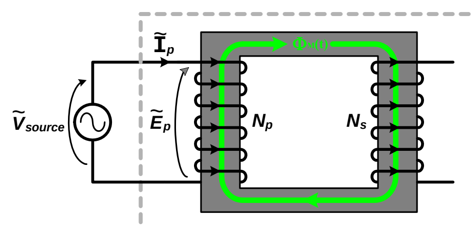
Ideal Transformer

Sourced Primary Winding

Thus, given the source voltage:

$$v(t) = \sqrt{2} \cdot V_{RMS} \cdot \cos(\omega \cdot t)$$

the flux created by the primary winding will be:



Ideal Transformer

$$\Phi_M(t) = \frac{1}{N_p} \cdot \int v(t)dt = \frac{1}{N_p} \cdot \int \sqrt{2} \cdot V_{RMS} \cdot \cos(\omega \cdot t)dt$$

$$= \sqrt{2} \cdot \frac{V_{RMS}}{\omega \cdot N_p} \cdot \sin(\omega \cdot t)$$

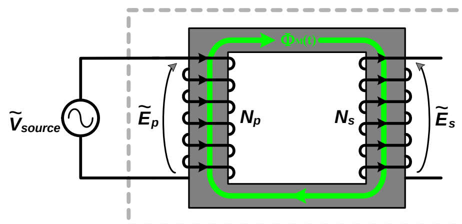
$$= \sqrt{2} \cdot \frac{V_{RMS}}{\omega \cdot N_p} \cdot \cos(\omega \cdot t - 90°)$$

Mutually-Linked Coils

But, if the ferromagnetic core is assumed to be ideal, then the total flux created by the sourced coil will pass through the second coil.

And, since the flux within the second coil is time-varying, a voltage will be induced across that coil, also defined by Faraday's Law:

$$\tilde{E}_s = N_s \cdot \frac{d\Phi_M(t)}{dt}$$



Ideal Transformer

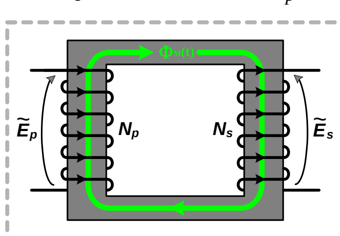
Mutually-Linked Coils

If the total flux passes through both coils, the rate of change, $\frac{d\Phi(t)}{dt}$, of the flux passing through both coils must be the equal.

The following relationship may be derived by solving for $\frac{d\Phi(t)}{dt}$ in both of the Faraday's Law equations and equating the results:

$$\frac{\tilde{E}_s}{N_s} = \frac{d\Phi_M(t)}{dt} = \frac{\tilde{E}_p}{N_p}$$

$$\tilde{E}_p = N_p \cdot \frac{d\Phi_M(t)}{dt}$$



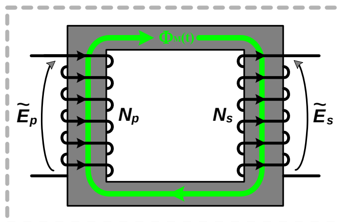
$$\tilde{E}_s = N_s \cdot \frac{d\Phi_M(t)}{dt}$$

Ideal Transformer

Voltage Relationship

If the voltage relationship between the two coils is expressed as a ratio of the voltages, it will equals to the ratio of the number of turns of the coils, and thus is referred to as the **turns ratio** (***a***) of the transformer.

$$\frac{\tilde{E}_p}{N_p} = \frac{\tilde{E}_s}{N_s} \quad \Rightarrow \quad \boxed{\frac{\tilde{E}_p}{\tilde{E}_s} = \frac{N_p}{N_s} = a}$$



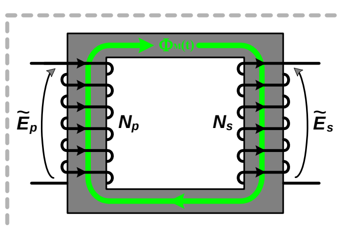
Ideal Transformer

Voltage Relationship

The **turns ratio**, ***a***, defines the basic operation of an ideal transformer in terms of its primary and secondary voltages.

$$a = \frac{N_p}{N_s} = \frac{\tilde{E}_p}{\tilde{E}_s}$$

Note that an ideal transformer only transforms the magnitude of the voltage; it does <u>not</u> change the phase angle of the voltage.



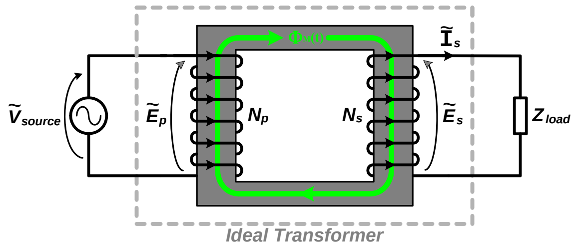
Ideal Transformer

Loaded Secondary Winding

But, what if a load is connected across the secondary winding?

The voltage induced across the secondary winding will cause a current to flow through the load.

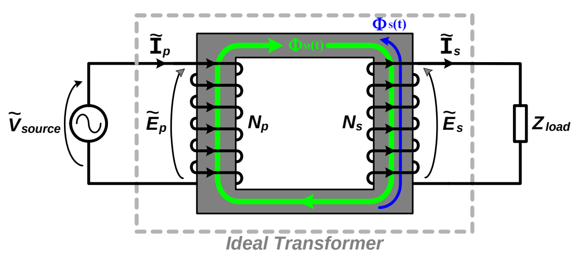
$$\tilde{I}_s = \frac{\tilde{E}_s}{Z_{load}}$$



Secondary Current Effects

Thus, secondary winding's current will induce a counter-flux, $\mathbf{\Phi_s}$, that opposes the original mutually-linked flux, $\mathbf{\Phi_M}$. But, in order for the Faraday's Law equations to remain true, the net flux passing through the coils cannot change.

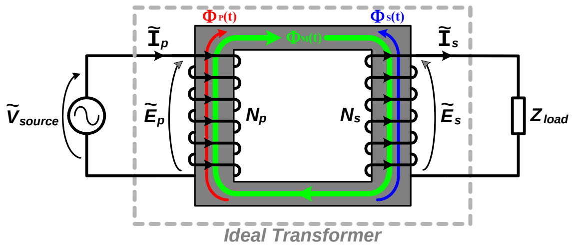
$$\Phi_{Net} = \Phi_M - \Phi_S$$



Secondary Current Effects

In order to maintain a constant net flux, when a current begins to flow in the secondary winding, a current will simultaneously be drawn into the primary winding in order to create and additional primary flux component, $\mathbf{\Phi_p}$, that counters the secondary flux.

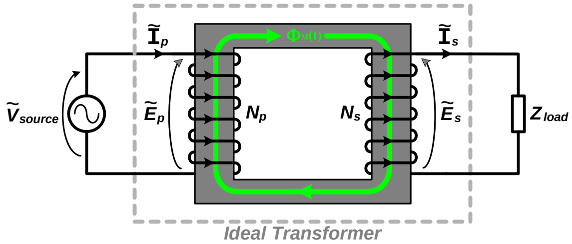
$$\Phi_{Net} = \Phi_M - \Phi_S + \Phi_P = \Phi_M$$



Energy Balance

Since energy can be neither created nor destroyed, only altered from one form to another, along with the assumption that the transformer is ideal, the energy (and power) flowing into the primary winding must equal to the energy (and power) flowing out of the secondary winding.

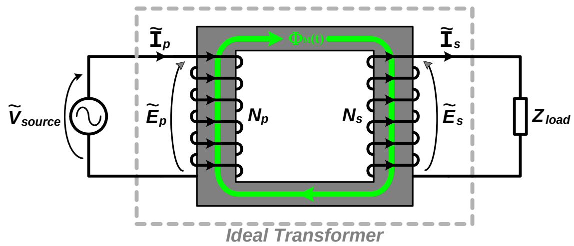
$$\tilde{E}_p \cdot \tilde{I}_p^* = \tilde{E}_s \cdot \tilde{I}_s^*$$



Ratio of Currents

In order to maintain the required energy balance, the magnitudes of the primary and secondary **currents** of the transformer must have an **inverse ratio** compared to that of the voltages:

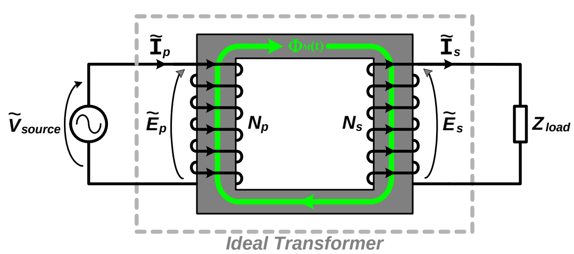
$$\frac{E_p}{E_s} = a \qquad \frac{I_p}{I_s} = \frac{1}{a}$$



Overall Operation of Ideal Transformer

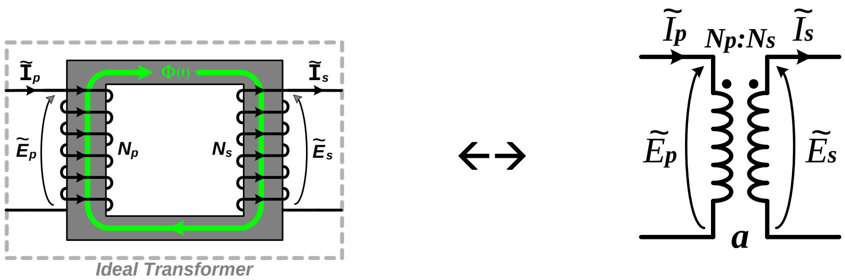
Thus, the overall operation of the ideal transformer that supplies a single load can be defined by the following set of equations:

$$\text{turns ratio} \quad a = \frac{N_p}{N_s} \qquad a = \frac{\tilde{E}_p}{\tilde{E}_s} \qquad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a} \qquad \tilde{I}_s = \frac{\tilde{E}_s}{Z_{load}}$$



Ideal Transformer Equivalent Circuit

The following circuit component will be used to represent an ideal transformer:



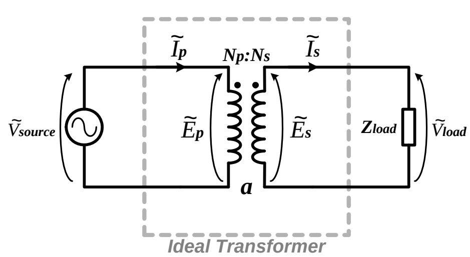
$$a = \frac{N_p}{N_s} \qquad \frac{\tilde{E}_p}{\tilde{E}_s} = a \qquad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a} \qquad a = \text{turns ratio}$$

Ideal Transformer Definitions

High-Voltage Winding ≡ the winding with the larger voltage magnitude.
(I.e. – the coil with the larger number of turns)

Low-Voltage Winding ≡ the winding with the smaller voltage magnitude.
(I.e. – the coil with the smaller number of turns)

Note – the high-voltage winding will have the larger number of turns while the low-voltage winding will have the smaller number of turns.

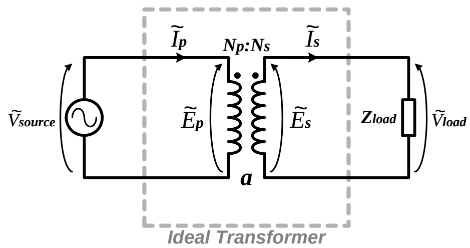


Ideal Transformer Definitions

Step-Up Transformer ≡ a transformer whose <u>voltage</u> increases from primary to secondary winding.

Step-Down Transformer ≡ a transformer whose <u>voltage</u> decreases from primary to secondary winding.

Notes: A step-up transformer's turns ratio will be less than one ($a < 1$).
A step-down transformer's turns ratio will be greater than one ($a > 1$).

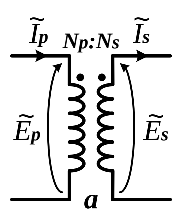


The "Dot Convention"

"Dots" are often included with the equivalent circuit to define the polarity relationship between the transformer windings.

1) An applied primary voltage whose voltage-rise points <u>toward</u> the primary winding's dot will induce a secondary voltage whose voltage-rise points <u>toward</u> the secondary winding's dot.

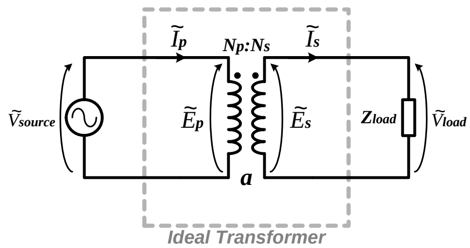
2) A current will be drawn <u>into</u> the dot side of the primary winding when a current flows <u>out of</u> the dot side of the secondary winding.



Turns-Ratio Consideration

It is important to note that the **turns ratio**, ***a***, will change depending on which of the two windings are utilized as the primary winding.

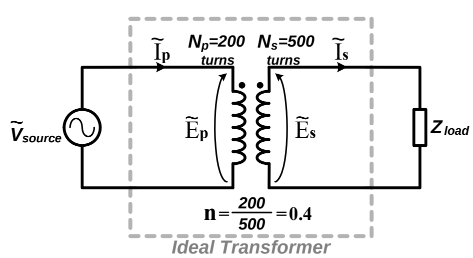
$$a = \frac{N_p}{N_s} \qquad \frac{\tilde{E}_p}{\tilde{E}_s} = a \qquad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a} \qquad a = \text{turns ratio}$$



Turns-Ratio Consideration

For Example – Given a transformer with a **200-turn** winding and a **500-turn** winding:

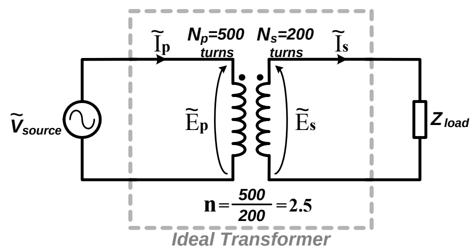
The transformer will have a turns ratio ***a*=0.4** if the **200-turn** winding is used as the **primary**, or



Turns-Ratio Consideration

For Example – Given a transformer with a **200-turn** winding and a **500-turn** winding:

The transformer will have a turns ratio ***a*=2.5** if the **500-turn** winding is used as the **primary**.



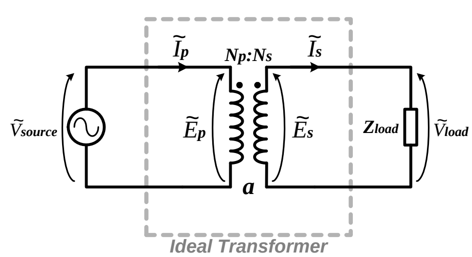
Turns-Ratio Consideration

Based on the voltage ratio:

$$\frac{\tilde{E}_p}{\tilde{E}_s} = \frac{N_p}{N_s} = a$$

it can be seen that:

- $\boldsymbol{E_p > E_s}$ when $\boldsymbol{a > 1}$ (**step-down** operation), and
- $\boldsymbol{E_s > E_p}$ when $\boldsymbol{a < 1}$ (**step-up** operation).

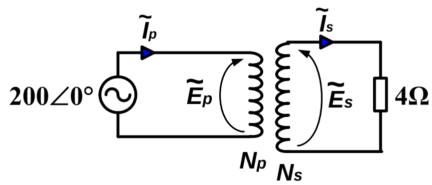


Ideal Transformer Example Problem

Given a transformer that contains windings having **50-turns** and **500-turns**:

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**,

Determine:
- The **load voltage**,
- The **load current**,
- The **real power** consumed by the **load**,
- The **source current**, and
- The **real power** produced by the **source**.

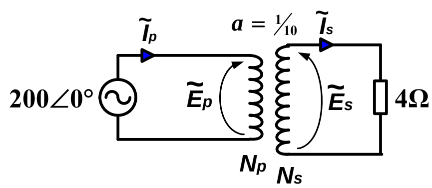


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **turns-ratio** for the transformer (as connected) is:

$$a = \frac{N_p}{N_s} = \frac{50}{500} = \frac{1}{10}$$



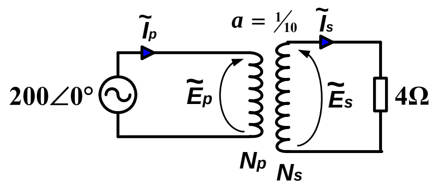
Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

Since the source is directly connected to the primary winding, the **primary voltage $\tilde{E}_p$** is equal to the source voltage, thus:

$$\tilde{E}_p = 200\angle 0°$$

And, since the load is connected directly to the secondary winding, the load voltage and current are equal to $\tilde{E}_s$ and $\tilde{I}_s$ respectively.

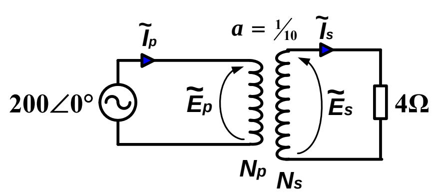


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **secondary** (load) **voltage $\tilde{E}_s$** can be determined from the equation:

$$\tilde{V}_{load} = \tilde{E}_s = \frac{\tilde{E}_p}{a} = \frac{200\angle 0°}{\frac{1}{10}} = 2{,}000\angle 0° \text{ volts}$$

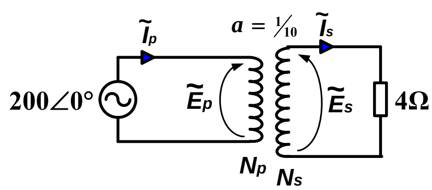


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and
a **4Ω load** is connected across the **500-turn winding**…

The resultant **secondary** (load) **current** $\tilde{I}_S$ will be:

$$\tilde{I}_{load} = \tilde{I}_s = \frac{\tilde{V}_{load}}{Z_{load}} = \frac{2{,}000\angle 0°}{4} = 500\angle 0° \text{ amps}$$



Ideal Transformer Example Problem

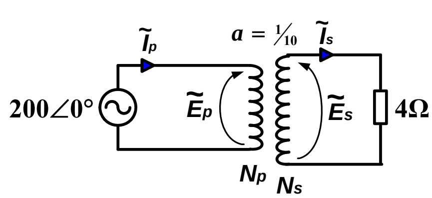
If a **200∠0° volt source** is connected across the **50-turn winding** and
a **4Ω load** is connected across the **500-turn winding**…

The **complex power**, S_{load}, consumed by the load will be:

$$S_{load} = \tilde{V}_{load} \cdot \tilde{I}^*_{load} = (2{,}000\angle 0°)\cdot(500\angle 0°) = \boxed{1{,}000{,}000} + j0$$

from which the **load's real power** can be determined:

$$P_{load} = 1{,}000{,}000 \text{ watts}$$

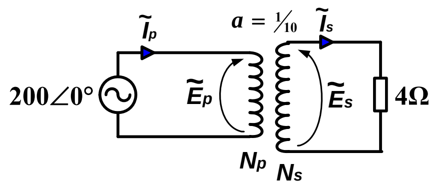


Ideal Transformer Example Problem

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

The **primary** (source) **current** $\tilde{I}_p$ can be determined from the equation:

$$\tilde{I}_{source} = \tilde{I}_p = \frac{\tilde{I}_s}{a} = \frac{500\angle 0^\circ}{\frac{1}{10}} = 5{,}000\angle 0^\circ \text{ amps}$$



Ideal Transformer Example Problem

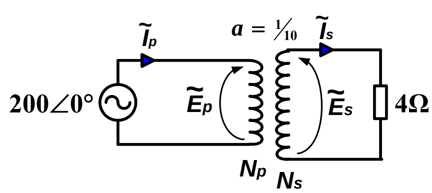
If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

Finally, the **complex power**, S_{source}, produced by the source will be:

$$S_{source} = \tilde{V}_{source} \cdot \tilde{I}^*_{source} = (200\angle 0^\circ)\cdot(5{,}000\angle 0^\circ) = \boxed{1{,}000{,}000} + j0$$

from which the **source's real power** can be determined:

$$P_{source} = 1{,}000{,}000 \text{ watts}$$

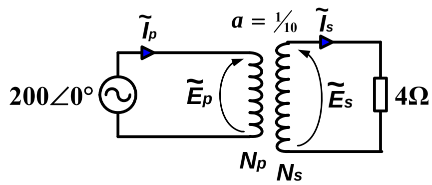


Ideal Transformer Example Problem

Given a transformer that contains windings having 50 and 500 turns:

If a **200∠0° volt source** is connected across the **50-turn winding** and a **4Ω load** is connected across the **500-turn winding**…

- **Load Voltage**: $\tilde{V}_{load} = 2{,}000\angle 0°$ volts
- **Load Current**: $\tilde{I}_{load} = 500\angle 0°$ amps
- **Load Real Power**: $P_{load} = 1{,}000{,}000$ watts
- **Source Current**: $\tilde{I}_{source} = 5{,}000\angle 0°$ amps
- **Source Real Power**: $P_{source} = 1{,}000{,}000$ watts



Transformer Ratings

Rated Voltage ≡ The expected operational winding voltage.

- although the manufacturer designs a transformer (winding turns, core size & material, etc.) to operate with a specific voltage applied to its primary winding, the transformer will still function properly is operated somewhat above or below its rated voltage.
- each winding will have its own rated voltage.

Rated Current ≡ The maximum allowable continuous current that can flow in a winding without damage.

- each winding will have its own rated current.

Rated Apparent Power ≡ **Rated Voltage x Rated Current**

- the apparent power rating should be the same for all windings.
- sometimes the rated apparent power is provided along with the rated voltages, allowing a user to calculate the rated currents.

Transformer Ratings

Turns Ratio ≡ The turns-ratio may be determined as follows:

$$a = \frac{V_{P(rated)}}{V_{S(rated)}}$$

Remember that either winding can be utilized as the primary winding.

– although the number of turns of each winding is rarely provided by the manufacturer, the turns-ratio can be determined based on the voltage ratings of the two windings.

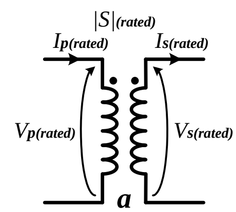
Example: Given a **500VA**, **120V-24V** transformer, determine the transformer's ratings if it is used in a step-down configuration.

Step-Down: Primary ≡ HighV Secondary ≡ LowV

$V_{P(rated)} = 120\text{V}$ $V_{S(rated)} = 24\text{V}$

$$I_{P(rated)} = \frac{S_{rated}}{V_{P(rated)}} = \frac{500\text{VA}}{120\text{V}} = 4.17\text{A} \qquad I_{S(rated)} = \frac{S_{rated}}{V_{S(rated)}} = \frac{500\text{VA}}{24\text{V}} = 20.8\text{A}$$

$$a = \frac{V_{P(rated)}}{V_{S(rated)}} = \frac{120\text{V}}{24\text{V}} = 5$$



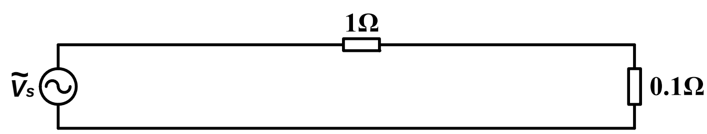
"Power System" Example Problem

A load impedance $Z_{load} = 0.1\Omega$ requires a supply voltage of $100\angle 0°$ volts.

Since the load is far from the actual voltage source, a long pair of wires are used to connect the load to the source.

If the wires have an overall resistance of $R_{wire} = 1\Omega$,

Determine the required <u>source voltage</u> and the overall <u>system efficiency</u>.



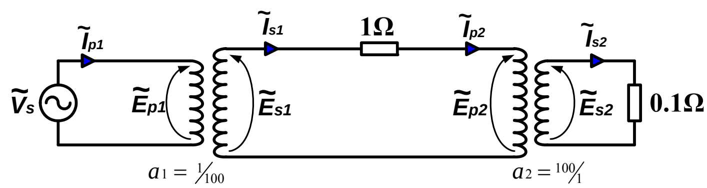
"Power System" Example Problem

A load impedance $Z_{load} = 0.1\Omega$ requires a supply voltage of $100\angle 0°$ volts.

A long pair of wires are still used to connect the load to the source, but this time a transformer is placed at both the source and load ends of the wires, the turns-ratios of which are $\frac{1}{100}$ and $\frac{100}{1}$ respectively.

If the wires have an overall resistance of $R_{wire} = 1\Omega$,

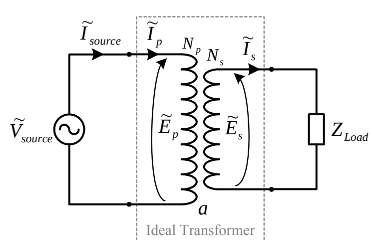
Determine the required <u>source voltage</u> and the overall <u>system efficiency</u>.



Input Impedance

Given an ideal transformer with a source connected across the primary winding and a load connected across the secondary winding…

Determine the overall impedance "seen" by the source.
(I.e. – the **input impedance** of the ideal transformer)



$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

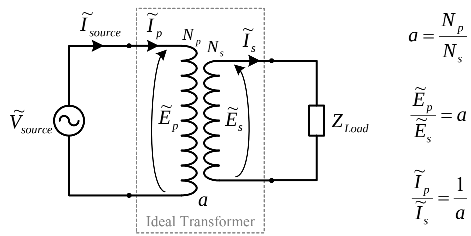
Input Impedance

The **input impedance** of an ideal transformer may be defined as:

$$Z_{in} = \frac{\widetilde{E}_p}{\widetilde{I}_p}$$

If we substitute the following relations into the equation:

$$\widetilde{E}_p = a \cdot \widetilde{E}_s \qquad \widetilde{I}_p = \tfrac{1}{a} \cdot \widetilde{I}_s$$



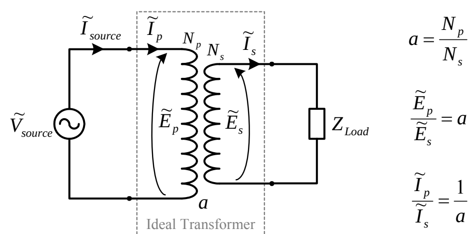
Input Impedance

Then the **input impedance** may be re-written as:

$$Z_{in} = \frac{a \cdot \widetilde{E}_s}{\frac{1}{a} \cdot \widetilde{I}_s} = a^2 \cdot \frac{\widetilde{E}_s}{\widetilde{I}_s}$$

since $\widetilde{E}_s$ and $\widetilde{I}_s$ are equivalent to the load voltage and current respectively:

$$\frac{\widetilde{E}_s}{\widetilde{I}_s} = \frac{\widetilde{V}_{Load}}{\widetilde{I}_{Load}} = Z_{Load}$$

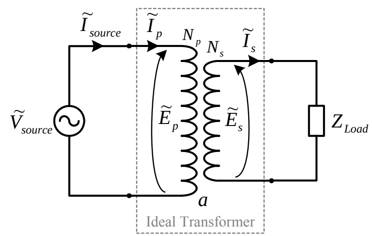


Input Impedance

If expressed in terms of the load impedance, the **input impedance** of the ideal transformer is:

$$Z_{in} = a^2 \cdot \frac{\tilde{E}_s}{\tilde{I}_s} = a^2 \cdot Z_{Load}$$

which equals the turns-ratio squared times the impedance of its connected load: $Z_{in} = a^2 \cdot Z_{Load} = Z'_{Load}$



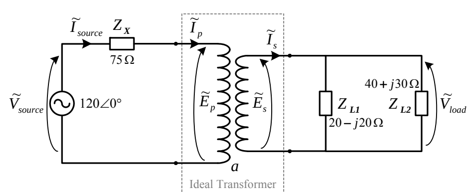
$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:

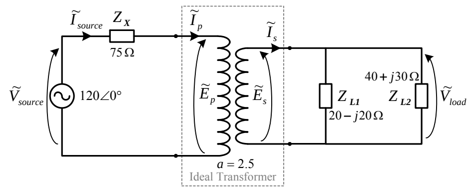


Assuming that the transformer is configured for **step-down** operation,

Determine:
- The **source current**,
- The **load voltage**,
- The **complex power** produced by the **source**, and
- The **total complex power** consumed by Z_{L1} and Z_{L2}.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



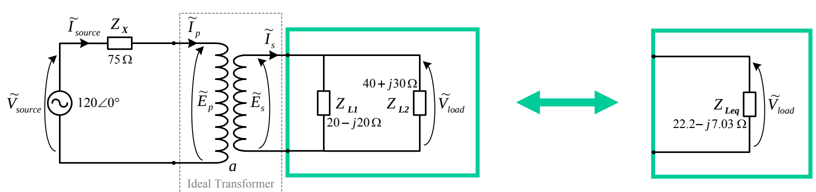
Since the transformer is configured for **step-down operation**, **the primary winding is the high-voltage winding** and **the secondary winding is the low-voltage winding**.

Thus, the operational **turns-ratio** of the transformer is:

$$a = \frac{V_{Rated\,(Pri)}}{V_{Rated\,(Sec)}} = \frac{120\text{V}}{48\text{V}} = 2.5$$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



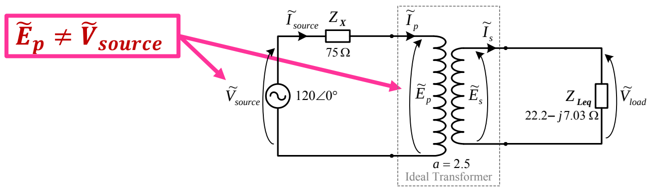
Since Z_{L1} is connected in parallel with Z_{L2}, the two impedances can be replaced by a single **equivalent impedance** Z_{Leq}:

$$Z_{Leq} = \left(\frac{1}{Z_{L1}} + \frac{1}{Z_{L2}}\right)^{-1} = \left(\frac{1}{20 - j20} + \frac{1}{40 + j30}\right)^{-1} = (22.2 - j7.03)\ \Omega$$

Note that the voltage across Z_{Leq} equals the original load voltage V_{load}.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



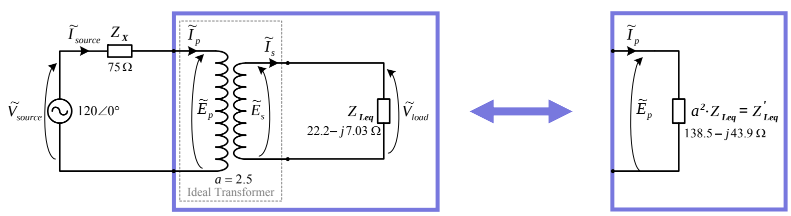
Since there is a 75Ω impedance connected in series between the source and the primary winding, the **primary voltage E_p** is a function of both the source voltage and the source current:

$$\tilde{E}_p = \tilde{V}_{source} - \tilde{I}_{source} \cdot Z_x$$

Because of this, the circuit cannot be easily analyzed in its current configuration.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:

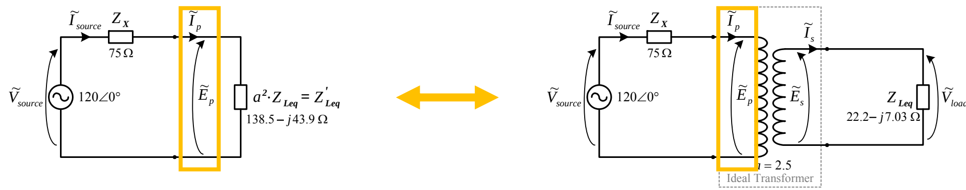


Thus, to facilitate the analysis of the circuit, the ideal transformer and load combination will initially be replaced by an overall equivalent impedance that equals the **input impedance** seen looking into the transformer's primary terminals:

$$Z_{in} = Z'_{Leq} = a^2 \cdot Z_{Leq} = 2.5^2 \cdot (22.2 - j7.03) = (138.5 - j43.9)\ \Omega$$

Ideal Transformer Example Problem

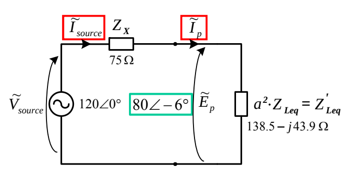
Given the following circuit that contains a **120V–48V** ideal transformer:



Note that, when the transformer–load combination is replaced by their equivalent impedance, the voltage across the equivalent impedance and the current flowing through the equivalent impedance are equal to the primary winding voltage and primary winding current respectively.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



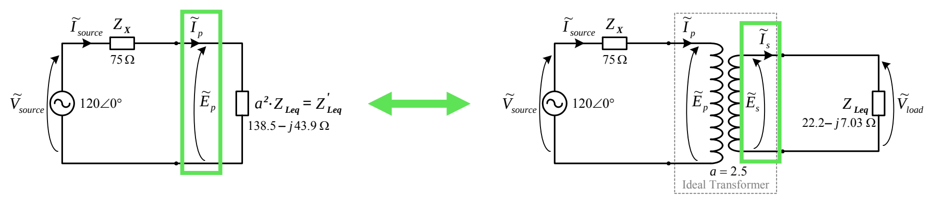
Since the circuit has been reduced-down to two series impedances, the primary voltage and current can be determined as follows:

$$\tilde{I}_{source} = \frac{\tilde{V}_{source}}{(Z_x + Z'_{Leq})} = \frac{120\angle 0°}{75 + (138.5 - j43.9)} = 0.5505\angle 11.6°\ amps = \tilde{I}_p$$

$$\tilde{E}_p = \tilde{I}_p \cdot Z'_{Leq} = (0.5505\angle 11.6°) \cdot (138.5 - j43.9) = 80\angle -6°\ \text{volts}$$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:

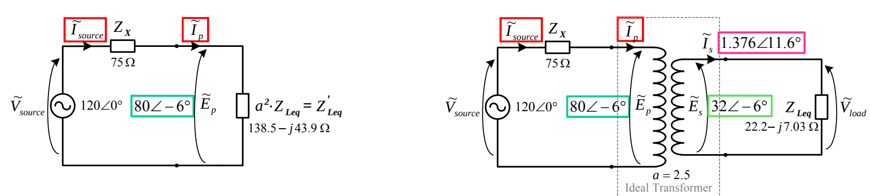


Now that the overall circuit has been simplified down to a relatively trivial circuit, the unknown voltages and currents remaining in the simplified circuit can be determined by applying basic circuit theory.

Additionally, once the remaining voltages and currents are determined, the basic turns-ratio equations can be utilized in order to relate the primary-side voltages and currents to the secondary-side quantities.

Ideal Transformer Example Problem

Given the following circuit that contains a **120V–48V** ideal transformer:



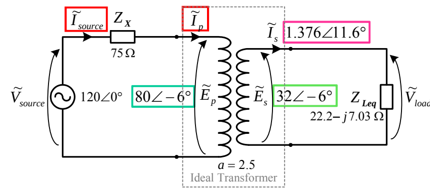
Since $\boldsymbol{E_p}$ and $\boldsymbol{I_p}$ in the reduced circuit equal $\boldsymbol{E_p}$ and $\boldsymbol{I_p}$ in the original circuit, the **secondary voltage** $\boldsymbol{E_s}$ and **secondary current** $\boldsymbol{I_s}$ can be determined:

$$\tilde{E}_s = \frac{\tilde{E}_p}{a} = \frac{80\angle{-6°}}{2.5} = \boxed{32\angle -6° \text{ volts}} \qquad \tilde{I}_s = a \cdot \tilde{I}_p = 2.5 \cdot 0.5505\angle 11.6° = \boxed{1.376\angle 11.6° \text{ amps}}$$

Note that the **load voltage** $\tilde{V}_{load} = \tilde{E}_s$ and the **total load current** $\tilde{I}_{load} = \tilde{I}_s$

Ideal Transformer Example Problem

Given the following circuit that contains a **120V−48V** ideal transformer:



Now that the source and load voltages and currents are known, the **source and load complex powers** can be determined:

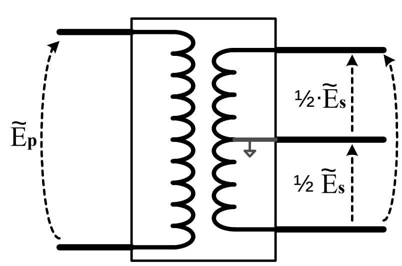
$$\boldsymbol{S_{source}} = \boldsymbol{\tilde{V}_{source}} \cdot \boldsymbol{\tilde{I}^{*}_{source}} = (120\angle 0°) \cdot (\mathbf{0.5505\angle{-11.6°}}) = \mathbf{66.1\angle{-11.6°}} = \mathbf{64.7 - j13.3}$$

$$\boldsymbol{S_{load}} = \boldsymbol{\tilde{V}_{load}} \cdot \boldsymbol{\tilde{I}^{*}_{load}} = (32\angle{-6°}) \cdot (\mathbf{1.376\angle{-11.6°}}) = \mathbf{44.04\angle{-17.6°}} = \mathbf{42 - j13.3}$$

1Φ 3-Wire Systems

The **1Φ**, **3-wire system** utilizes a **single-phase transformer** whose secondary winding is **center-tapped** and grounded.

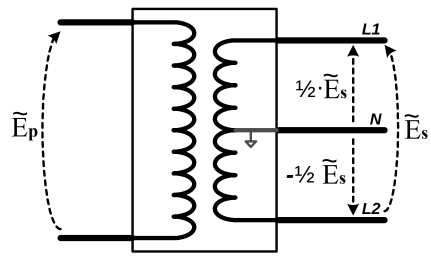
In this configuration, the entire secondary voltage exists across the secondary winding while one-half of that voltage exists across each half of the secondary winding.



1Φ 3-Wire Systems

Note that the voltages across each half of the secondary winding are typically defined as the voltage-rises from the neutral conductor to the energized conductors.

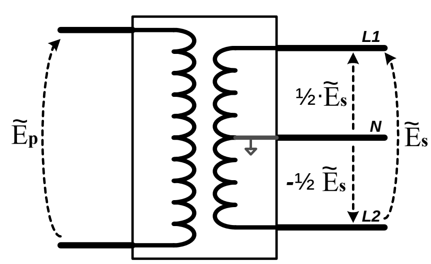
If defined as such, then the two line-to-neutral voltages would be the negative of each other or out-of-phase by 180°.



1Φ 3-Wire Systems

"Energized" (ungrounded) conductors **L1** and **L2** are connected to the end-points of the secondary-winding and a "neutral" conductor is connected to the center-tap.

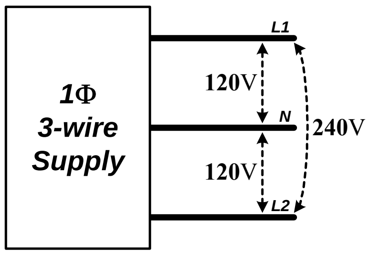
Thus, single-phase load could be supplied by utilizing any two of the three secondary-connected lines.



1Φ 3-Wire Systems

The typical 1Φ, 3-wire, residential system operates with a standard **line-to-line voltage** magnitude of **240V** and a **line-to-neutral voltage** magnitude of **120V**.

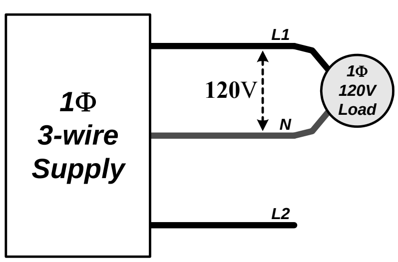
Thus, the system can be used to supply 1Φ loads requiring either a 120V or a 240V potential.



1Φ 3-Wire Systems

Since most residential loads (televisions, computers, microwaves, lights, ceiling fans, etc.) consume relatively low power during operation, they are designed to operate at 120V.

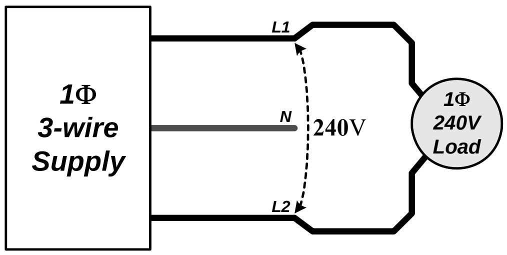
These low-power loads are connected between the neutral conductor and either one of the energized conductors.



1Φ 3-Wire Systems

A few residential loads (clothes dryer, AC compressor, electric oven, etc.) consume larger amounts of power.

These loads are designed to be supplied at 240V in order to minimize the supply current, and thus are connected across the energized conductors.



1Φ 3-Wire Systems

When serving multiple 120V loads, the loads should be distributed evenly to balance the overall current-flow through the secondary-winding of the transformer.