ECET 3000

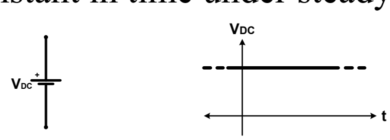
Electrical Principles

Introduction to AC Circuits

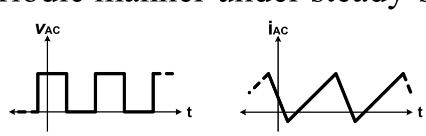
1

Time-varying Voltages and Currents

DC voltages and **DC currents**, such as those supplied by an ideal battery, remain constant in time under steady-state conditions.



On the other hand, **AC voltages** and **AC currents**, have magnitudes that vary in a periodic manner under steady-state conditions.



Note – Although the term "**AC**" actually stands for "**Alternating Current**", it is used to describe both time-varying voltages and currents

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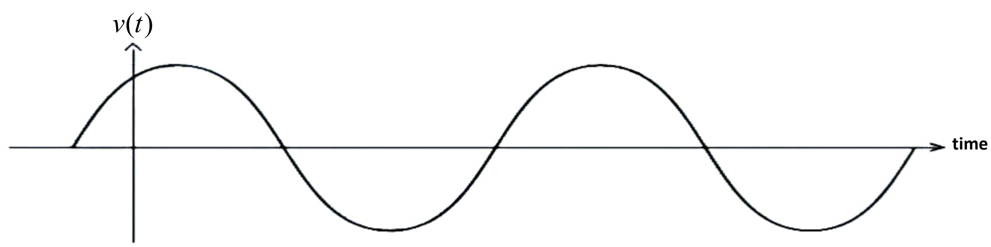
AC Voltage Sources

The most common type of AC voltages (and currents) are those that vary in a **sinusoidal manner**, as shown in the figure below.

Sinusoidally-varying AC voltages and currents are typically created either by rotating machines (generators) or by electronic devices (AC power supplies).



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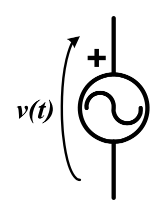
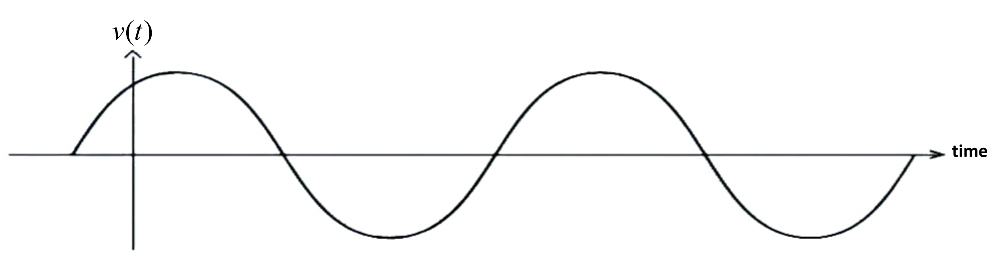
AC Voltage Sources

The **circuit element** typically used to denote a sinusoidally-varying AC voltage source is shown to the right.



Note that, for other types of AC sources (such as square-waves), one cycle of the AC waveform is typically displayed within the circular region.



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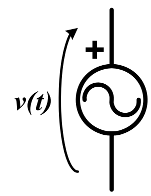
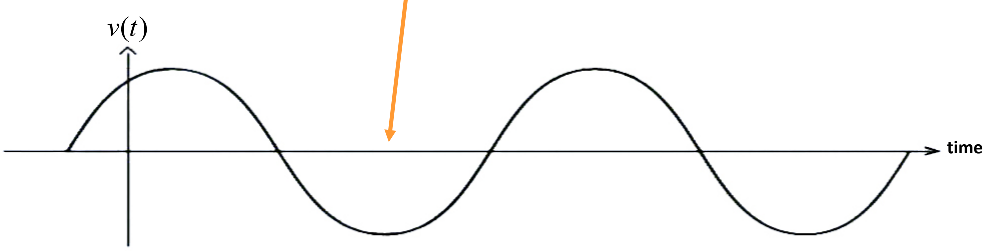
AC Voltage Sources

The "+" sign is used to define the direction of the instantaneous voltage-rise (potential force) provided by the source, as defined by the function $\boldsymbol{v(t)}$.



Note that, when the function goes "negative", the voltage-rise (potential force) provided by the source is actually in the opposite (negative) direction.



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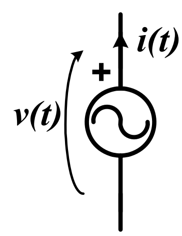
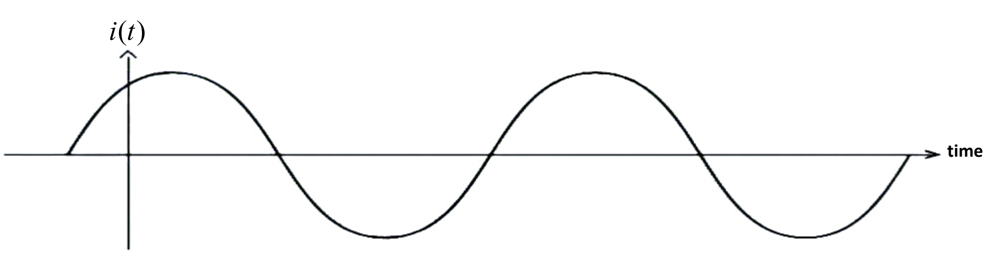
AC Voltage Sources

Since both the magnitude and sign of the voltage potential provided by the "AC" source vary with time, taking on both positive and negative values, the magnitude and direction of the resultant current will also vary with time…

Thus the term → **Alternating Current**





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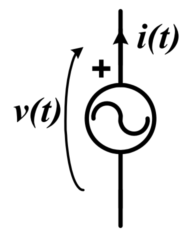
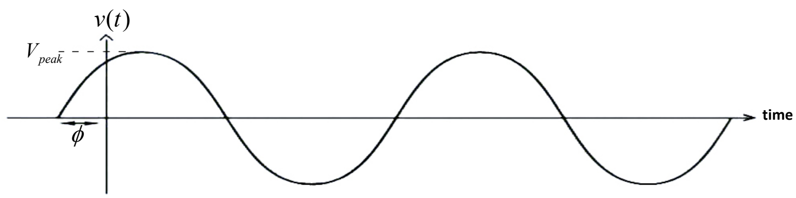
Steady-State AC Voltage Sources

The **voltage** potential of an AC source may be defined as:

$$v(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

where: V_{peak} is the **peak magnitude** of the voltage,
ω is the **angular frequency** ($2\pi f$) of the waveform, and
ϕ is the **phase angle** of the voltage waveform.





Note that the **frequency** of the voltage is:

$$f = \frac{\omega}{2\pi} \text{ Hz}$$

and the **period** of the voltage is:

$$T = \frac{1}{f} \text{ sec}$$

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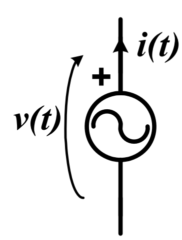
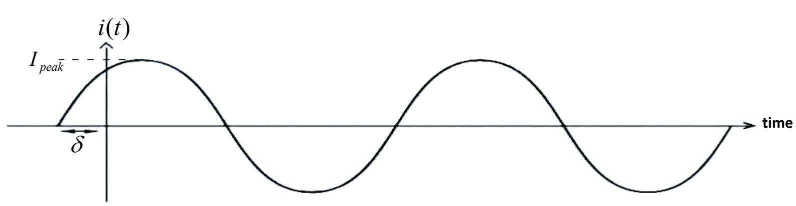
Steady-State AC Voltage Sources

Similarly, the **current** produced by the AC source may be defined as:

$$i(t) = I_{peak} \cdot \sin(\omega \cdot t + \delta)$$

where: I_{peak} is the **peak magnitude** of the current,
ω is the **angular frequency** ($2\pi f$) of the waveform, and
δ is the **phase angle** of the current waveform.





Note that the **frequency** of the current is:

$$f = \frac{\omega}{2\pi} \text{ Hz}$$

and the **period** of the current is:

$$T = \frac{1}{f} \text{ sec}$$

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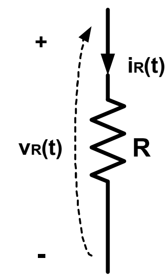
AC Sources and Resistive Loads

Ohm's Law defines the voltage/current relationship for a resistive load. This relationship holds true for all types of voltages and currents, including both AC and DC waveforms.



→ If a time-varying current $\boldsymbol{i_R(t)}$ is flowing through a resistor, then the resistor will develop a time-varying voltage $\boldsymbol{v_R(t)}$ that opposes the flow of current, as defined by the relationship:

$$v_R(t) = i_R(t) \cdot R$$

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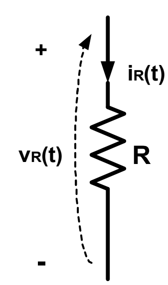
AC Sources and Resistive Loads

Given a resistor whose voltage is:

$$v_R(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$



the current flowing through the resistor must be:

$$i_R(t) = \frac{v_R(t)}{R} = \frac{V_{peak} \cdot \sin(\omega \cdot t + \phi)}{R}$$

$$= \frac{V_{peak}}{R} \cdot \sin(\omega \cdot t + \phi)$$

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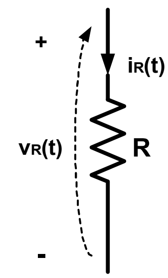
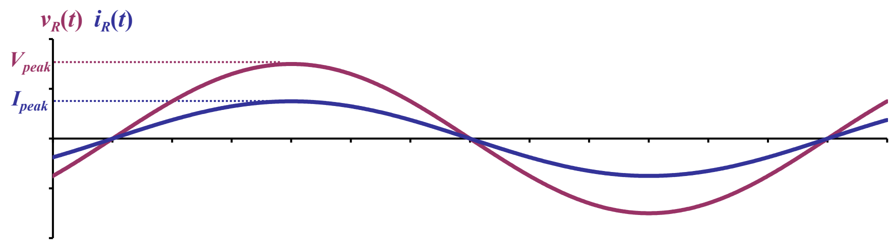
AC Sources and Resistive Loads

Thus, for a resistive load:

$$v_R(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$i_R(t) = \frac{V_{peak}}{R} \cdot \sin(\omega \cdot t + \phi) = I_{peak} \cdot \sin(\omega \cdot t + \phi)$$





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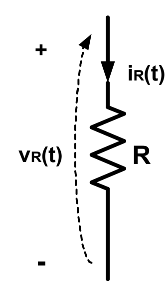
AC Sources and Resistive Loads

Thus, for a resistive load:

$$v_R(t) = \boxed{V_{peak}} \cdot \sin(\omega \cdot t + \phi)$$

$$i_R(t) = \boxed{\frac{V_{peak}}{R}} \cdot \sin(\omega \cdot t + \phi)$$



Note that, the voltage and current **magnitudes** follow the Ohm's Law relationship:

$$I_{peak} = \frac{V_{peak}}{R},$$

while the sinusoidal expressions remain unchanged.

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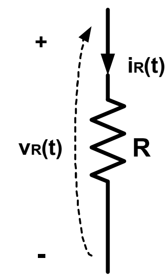
AC Sources and Resistive Loads

Thus, for a resistive load:

$$v_R(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$i_R(t) = \frac{V_{peak}}{R} \cdot \sin(\omega \cdot t + \phi)$$



Based on this result, it can be seen that both the **frequency** and the **phase angle** of the resistor current are equal to those of the applied voltage…

For this reason, AC circuits containing resistive loads are often analyzed in terms of the magnitudes of the voltages and currents.

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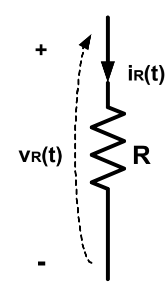
AC Sources and Resistive Loads

And, since **Ohm's Law** holds true for resistive loads supplied with AC voltages,

$$i_R(t) = \frac{v_R(t)}{R}$$



all of the basic **circuit theory** derived for DC circuits can also be applied to AC, resistor-based circuits:

- **Series** and **Parallel Equivalent Resistances**
- **Kirchhoff's Voltage** and **Current Laws**
- **Voltage** and **Current Dividers**
- The **Reduce & Return Approach** for solving **Series-Parallel Circuits**

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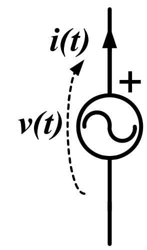
Power in AC Circuits

In electric circuits, **power** can be defined as the **rate** at which **electric energy** is either produced or consumed by an element within the circuit.



Power may be calculated in terms of the voltage and current waveforms associated with a specific circuit element by:

$$p(t) = v(t) \cdot i(t) \quad \text{(Watts)}$$

where: $p(t)$ provides the **instantaneous rate** that an element either produces or consumes electric energy at any time t.

Although it is actually the electric **energy** that is either being produced or consumed by the circuit elements, **power** is often casually referred to as being either produced or consumed within an electric circuit.

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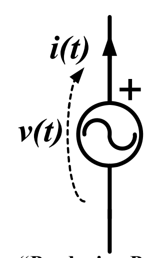
Power in AC Circuits

Note that the expression:

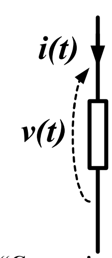
$$p(t) = v(t) \cdot i(t) \quad \text{(Watts)}$$

defines the power "**produced**" by an element when the **current** is defined in the **same direction** as the **voltage-rise** across the element.



"Producing Power"

But, if the **current** is defined in the **opposite direction** as the **voltage-rise** across an element, then ***p*(*t*)** defines the power "**consumed**" by that element.



"Consuming Power"

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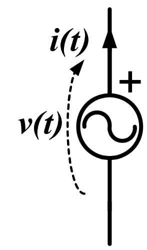
Power from an AC Source

In the case of an AC source where:



$$v(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

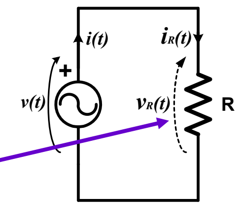
$$i(t) = I_{peak} \cdot \sin(\omega \cdot t + \delta)$$

the general expression for **power** produced by the source is:

$$p(t) = v(t) \cdot i(t)$$

$$= V_{peak} \cdot I_{peak} \cdot \sin(\omega \cdot t + \phi) \cdot \sin(\omega \cdot t + \delta)$$

Since the **power** expression ***p(t)*** is actually quite complex, it may by useful to first consider the case where the voltage source is applied to a **purely resistive load** in order to better understand the true nature of the power expression.



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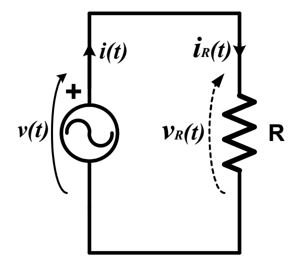
AC Power and Resistors

If an AC source is connected to a **resistive load**, such that:



$$v_R(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$i_R(t) = I_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$I_{peak} = \frac{V_{peak}}{R}$$

Note that the resistor's **voltage** and **current** have the **same phase angle**…

I.e. – there is no phase shift between the voltage and current for a purely resistive load.

then the **power** consumed by the resistor will be:

$$p_R(t) = v_R(t) \cdot i_R(t)$$

$$= V_{peak} \cdot I_{peak} \cdot \sin^2(\omega \cdot t + \phi)$$

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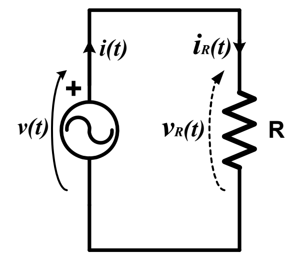
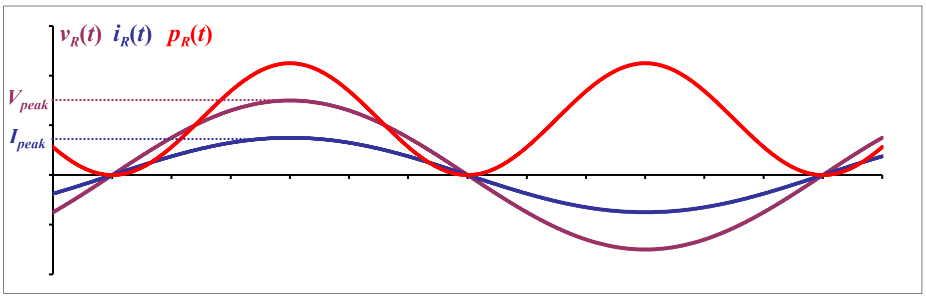
AC Power and Resistors

The figure below shows the **power waveform**:

$$p_R(t) = V_{peak} \cdot I_{peak} \cdot \sin^2(\omega \cdot t + \phi)$$

plotted along with the resistor's voltage and current waveforms:





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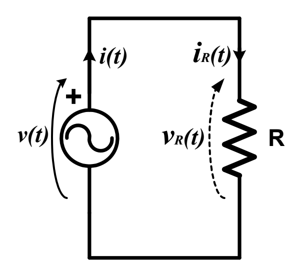
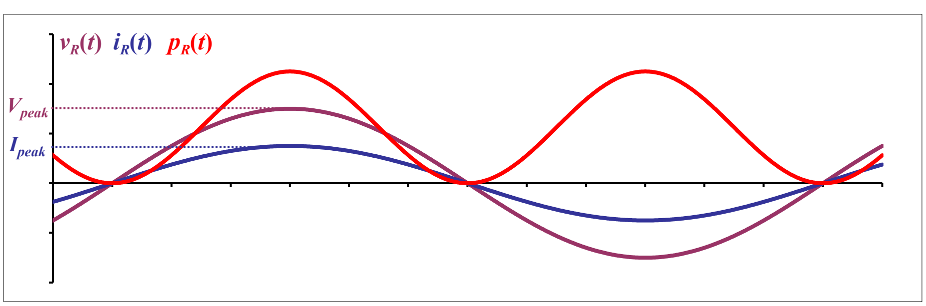
AC Power and Resistors

As shown, **power supplied to the resistor** is always **non-negative**, which is expected since a resistor can only consume electric power.

Since $p(t)$ is the power "consumed" by the resistor, a negative value would imply that power is actually being "produced" by the resistor, a result which can not occur.





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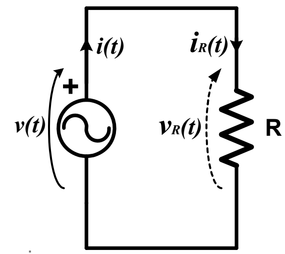
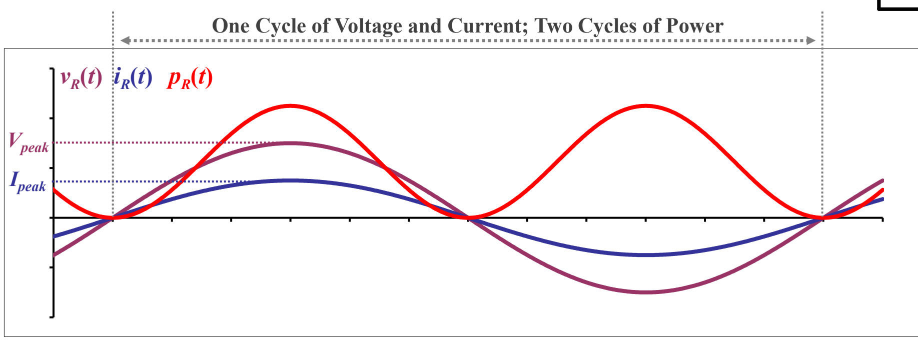
AC Power and Resistors

Additionally, it can be seen that the power waveform varies **periodically**, but with a **frequency** that is **2x larger** than that of the applied voltage or the resultant current.





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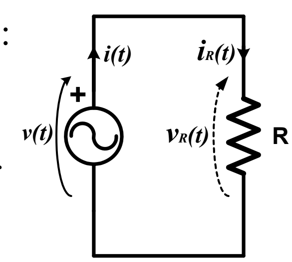
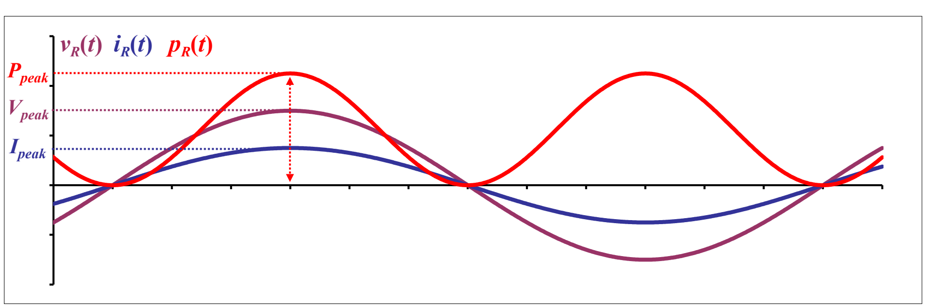
AC Power and Resistors

The **peak magnitude** of the **AC power** waveform is:

$$P_{peak} = V_{peak} \cdot I_{peak}$$

This should <u>not</u> be confused with the constant power provided to a resistor by a DC source.





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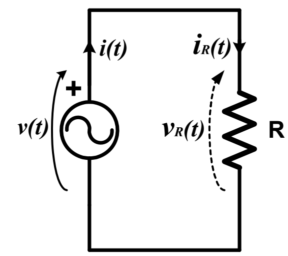
AC Power and Resistors

To help clarify the characteristics of the **resistor's AC power waveform**, it is useful to rewrite the power expression:

(by utilizing the trig-identity $sin^2x = ½·[1 – cos2x]$):



$$p_R(t) = V_{peak} \cdot I_{peak} \cdot \sin^2(\omega \cdot t + \phi)$$

$$= \frac{V_{peak} \cdot I_{peak}}{2} \cdot [1 - \cos(2 \cdot \omega \cdot t)]$$

$$= \boxed{\frac{V_{peak} \cdot I_{peak}}{2} - \frac{V_{peak} \cdot I_{peak}}{2} \cdot \cos(2 \cdot \omega \cdot t)}$$

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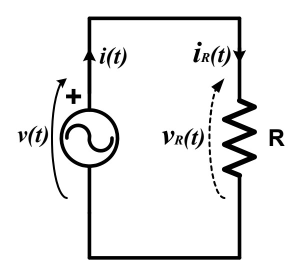
AC Power and Resistors

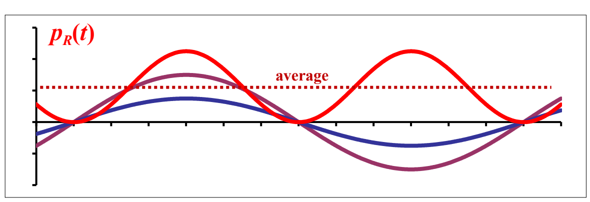
Looking at the resultant AC power waveform:

$$p_R(t) = \boxed{\frac{V_{peak} \cdot I_{peak}}{2}} - \frac{V_{peak} \cdot I_{peak}}{2} \cdot \cos(2 \cdot \omega \cdot t)$$



It can be seen that the waveform has two terms:

- The **first term** is a **constant** that relates to the **average** value of the power that is consumed by the resistor.



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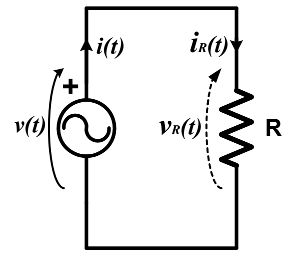
AC Power and Resistors

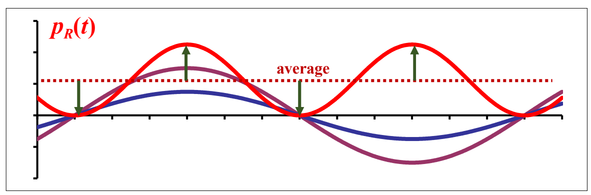
Looking at the resultant AC power waveform:

$$p_R(t) = \frac{V_{peak} \cdot I_{peak}}{2} - \boxed{\frac{V_{peak} \cdot I_{peak}}{2} \cdot \cos(2 \cdot \omega \cdot t)}$$



It can be seen that the waveform has two terms:

- The **second term** is a sinusoidal term that varies at **2x the source frequency** and provides the fluctuation in the power waveform.

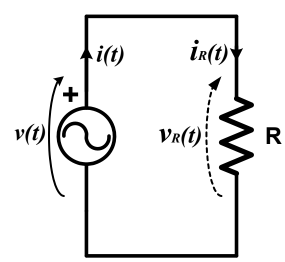


Real Power

In AC systems, it is typically the **average value** of the power that is desired.

This **average power** value is called **Real Power**.

The **real power** consumed by a resistive load is:



$$P_{R(AC)} = Avg[p_R(t)] = \frac{V_{peak} \cdot I_{peak}}{2} \quad \text{(Watts)}$$

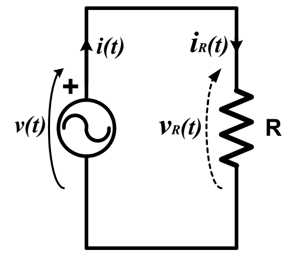
The average value of the AC power is typically of concern because the operational characteristics of many AC loads are based upon the average power that the loads consume.

For example – an incandescent lightbulb is primarily a resistive load. The amount of light that the bulb emits is based upon the temperature of the bulb's filament. But, due to the finite rate at which the bulb can dissipate the heat it produces, the bulb maintains an average temperature which is based upon the average power that be bulb consumes.

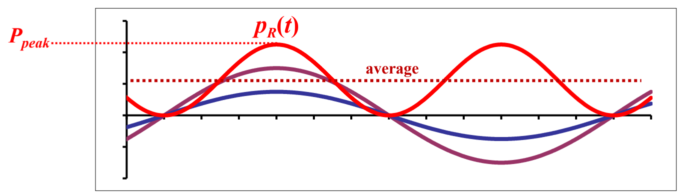
AC vs. DC Power to Resistors

Note that the **real power** consumed by the resistor is **½ that of the peak power** value:

$$P_{R(AC)} = \frac{P_{peak}}{2} = \frac{V_{peak} \cdot I_{peak}}{2} \quad \text{(Watts)}$$



This result is expected since the power waveform fluctuates evenly between zero and its peak value.



There are some AC loads for which this fluctuating power may be of concern.

For example, the torque produced by a single-phase **AC motor** fluctuates proportionally with the amount of electric energy it converts to mechanical. But, this may be detrimental if the mechanical system that the motor is driving is sensitive to vibrations.

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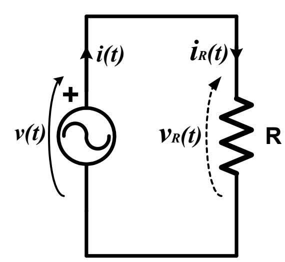
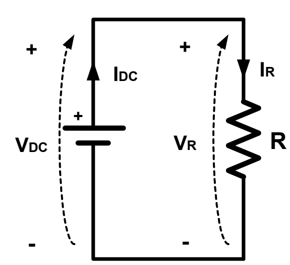
AC vs. DC Power to Resistors

But, this result can also be confusing if comparing the power consumed in AC and DC circuits:

$$P_{R(AC)} = \frac{V_{peak} \cdot I_{peak}}{2} \qquad P_{R(DC)} = V_{DC} \cdot I_{DC}$$



Also, based upon the results, given an AC source whose peak value is equal to the magnitude of a DC source, the **AC source** is only **<u>½ as effective</u>** as the **DC source** in terms of the **average power** supplied to a resistor:



If: $V_{peak} = V_{DC}$ ➡

$$P_{R(AC)} = \frac{V_{peak} \cdot I_{peak}}{2} \quad \text{(Watts)}$$

$$P_{R(DC)} = V_{DC} \cdot I_{DC} \quad \text{(Watts)}$$

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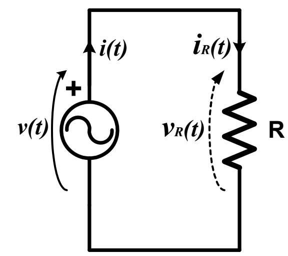
Effective Voltage

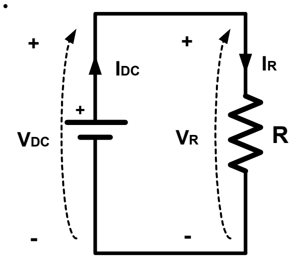
Since the **average AC power** is proportional to the square of the source's peak voltage:

$$P_{R(AC)} = \frac{V_{peak} \cdot I_{peak}}{2} = \frac{V_{peak} \cdot V_{peak}}{2 \cdot R} = \frac{V_{peak}^2}{2 \cdot R}$$



if the peak value of the AC voltage is increased such that it is $\sqrt{2}$ **times larger** than the DC voltage:

$$V_{peak} = \sqrt{2} \cdot V_{DC}$$



then the AC and DC sources will supply the **same average power** to the resistor.

$$P_{R(AC)} = \frac{V_{peak}^2}{2 \cdot R} = \frac{\left(\sqrt{2} \cdot V_{DC}\right)^2}{2 \cdot R} = \frac{2 \cdot V_{DC}^2}{2 \cdot R} = \frac{V_{DC}^2}{R} = P_{R(DC)}$$

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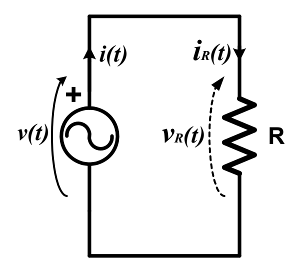
Effective / RMS Voltage Magnitudes

Thus, an **effective voltage** can be defined for a sinusoidally-varying AC source, such that:

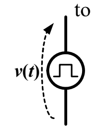
$$V_{effective} = \frac{V_{peak}}{\sqrt{2}}$$



Note that the effective value of the AC source is equal to the **RMS** (***root-mean-squared***) value of the source voltage:

$$V_{effective} = V_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T v^2(t) \cdot dt} = \frac{V_{peak}}{\sqrt{2}}$$

An AC source's **effective** or **RMS voltage** is equal to the magnitude of the DC source that will deliver the **same average power** to a resistor as the AC source.



This statement holds true for **all** periodic AC sources, not just sinusoidally-varying sources.

Note that this is the reason why most **handheld voltmeters** display the **RMS magnitude** of AC voltages and currents.

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Effective Voltage

FOR EXAMPLE:

A **100V$_{peak}$** AC source has an **effective voltage** of:

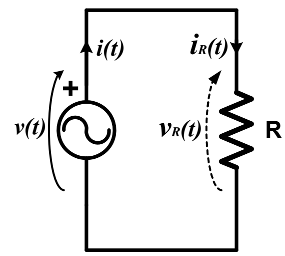
$$V_{effective} = V_{RMS} = \frac{V_{peak}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7\ volts$$

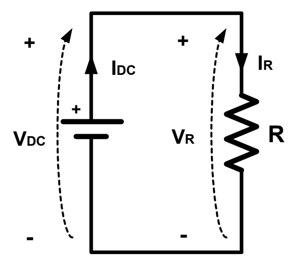
since it delivers an average power of 50W to a 100Ω resistor:

$$P_{R(AC)} = \frac{V_{peak}^2}{2 \cdot R} = \frac{100^2}{2 \cdot 100} = 50\ Watts$$

which is equal to that from a **70.7V** DC source:

$$P_{R(DC)} = \frac{V_{DC}^2}{R} = \frac{70.7^2}{100} = 50\ Watts$$





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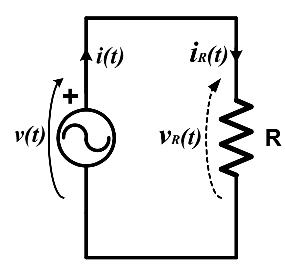
RMS Magnitudes

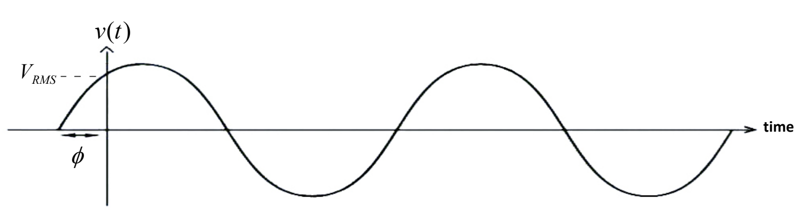
The **voltage waveform** may be expressed in terms of its ***RMS voltage magnitude***:

$$v(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$

where: $V = \frac{V_{peak}}{\sqrt{2}}$ is the RMS magnitude of the AC voltage.



By default, the magnitude of AC voltages (and currents) will be defined in terms of their **RMS magnitudes**.
For example, given: $V = 120$ volts → $V = V_{RMS}$ → $V_{peak} = 170$ volts.



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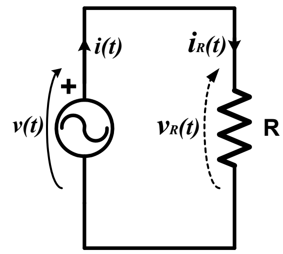
RMS Magnitudes

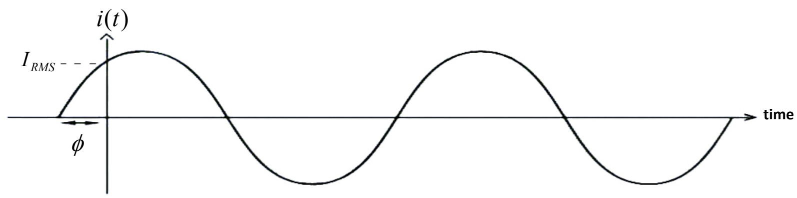
Similarly, the **current waveform** may also be expressed in terms of its ***RMS current magnitude***:



$$i(t) = \sqrt{2} \cdot I \cdot \sin(\omega \cdot t + \phi)$$

where: $I = \frac{I_{peak}}{\sqrt{2}}$ is the RMS magnitude of the AC current.

If the **voltages** are defined in terms of their **RMS magnitudes**, then the resultant **currents** will also be defined in terms of their **RMS magnitudes**.



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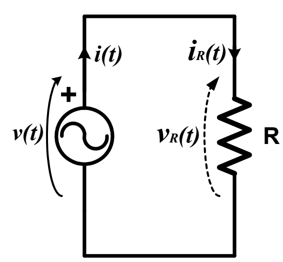
RMS Magnitudes & Resistor Power

If the voltages and currents are expressed in terms of their **RMS magnitudes**:



$$V_{peak} = \sqrt{2} \cdot V \qquad I_{peak} = \sqrt{2} \cdot I$$

then the **Real Power** delivered to a resistor is:

$$P_{R(AC)} = Avg[p_R(t)] = Avg[V \cdot I - V \cdot I \cdot \cos(2 \cdot \omega \cdot t)]$$

$$\rightarrow \boxed{P_{R(AC)} = V \cdot I}$$

This result is similar to the DC power formula: $P_{R(DC)} = V_{DC} \cdot I_{DC}$

Note that this also provides the motivation for expressing AC waveforms in terms of their RMS (effective) magnitudes instead of their peak magnitudes.

$$v(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$
$$i(t) = \sqrt{2} \cdot I \cdot \sin(\omega \cdot t + \phi)$$

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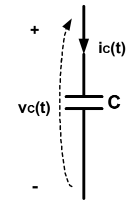
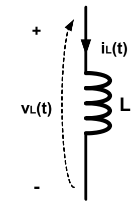
AC Sources and Reactive Loads

What if the AC source is supplying a load that is purely reactive…

I.e. – an ideal **Capacitor** or **Inductor**?





Similar to resistive loads, a sinusoidal (AC) voltage source will cause a sinusoidal (AC) current to flow through both capacitors and inductors.

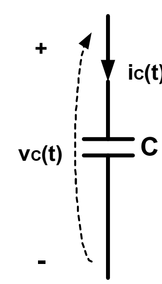
But, their voltage and current waveforms do not follow the linear Ohm's Law relationship. Instead, their voltage and current waveforms are governed by a **differential relationship**.

AC Sources and Capacitors

For an ideal **capacitor**, the voltage-current relationship is defined by the following equations:

$$i_C(t) = C \cdot \frac{dv_C(t)}{dt}$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt = \frac{1}{C}\int_{0}^{t} i_C(t)dt + V_o$$



Thus, given an AC source, we may obtain a solution for **steady-state AC operation** from these relationships.

AC Sources and Capacitors

If a **sinusoidal voltage** exists across a capacitor:

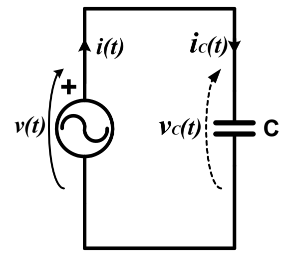
$$v_C(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$

then the associated capacitor **current** will be:

$$i_C(t) = \sqrt{2} \cdot V \cdot \omega \cdot C \cdot \cos(\omega \cdot t + \phi°)$$

which can be rewritten as:

$$i_C(t) = \sqrt{2} \cdot V \cdot \omega \cdot C \cdot \sin(\omega \cdot t + \phi° + 90°)$$



In order to allow for the direct comparison of the voltage and current waveforms, they must be expressed in terms of the same sinusoidal function (sine).
Thus, the current was rewritten in terms of sine by applying the identity:

$$\cos(\theta) = \sin(\theta + 90°)$$

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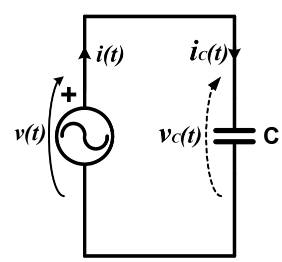
AC Sources and Capacitors

Given the resultant capacitor **voltage** and **current** waveforms:

$$v_C(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$

$$i_C(t) = \sqrt{2} \cdot V \cdot \omega \cdot C \cdot \sin(\omega \cdot t + \phi° + 90°)$$



It can be seen that:

- The voltage and current magnitudes do **not** follow the linear Ohm's Law relationship that holds true for resistors, and
- The capacitor current is phase-shifted by **+90°** compared to the capacitor voltage.

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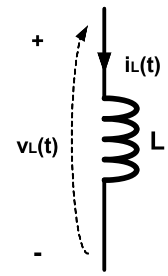
AC Sources and Inductors

For an ideal **inductor**, the voltage-current relationship is defined by the following equations:

$$v_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)dt = \frac{1}{L}\int_{0}^{t} v_L(t)dt + I_o$$



Thus, given an AC source, we may obtain a solution for **steady-state AC operation** from these relationships.

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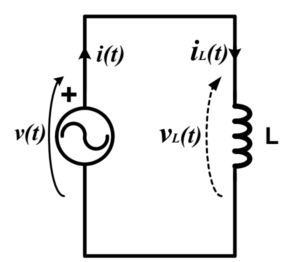
AC Sources and Inductors

If a **sinusoidal voltage** exists across an inductor:

$$v_L(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$



the associated inductor current will be:

$$i_L(t) = -\sqrt{2} \cdot \frac{V}{\omega \cdot L} \cdot \cos(\omega \cdot t + \phi°)$$

which can be re-written as:

$$i_L(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot L} \cdot \sin(\omega \cdot t + \phi° - 90°)$$

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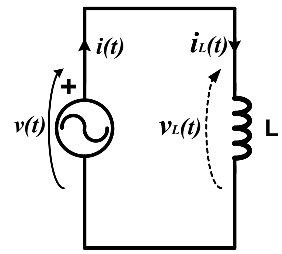
AC Sources and Inductors

Given the resultant inductor voltage and current waveforms:

$$v_L(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$

$$i_L(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot L} \cdot \sin(\omega \cdot t + \phi° - 90°)$$



It can be seen that:

- The voltage and current magnitudes do not follow the linear Ohm's Law relationship that holds true for resistors, and
- The inductor current is phase-shifted by –90° compared to the inductor voltage.

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Analysis of AC Sourced R-L-C Circuits

The following set of equations define the general *V-I* relationships for resistors, capacitors, and inductors:

$$v_R(t) = i_R(t) \cdot R \qquad v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt \qquad v_L(t) = L \cdot \frac{di_L(t)}{dt}$$

Although each these relationships may seem relatively simple on their own, when all three types of the circuit elements exist within the same circuit, analysis of the circuit's operation can become quite complex.

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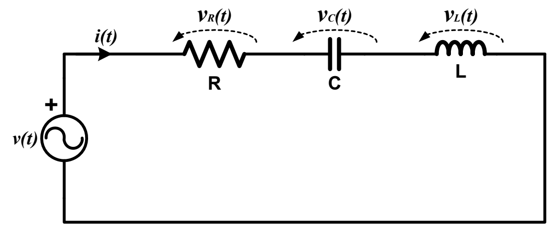
Analysis of AC Sourced R-L-C Circuits

FOR EXAMPLE: Given a simple, series-connected, R-L-C, AC circuit, a **2nd-order differential equation** must be solved in order to determine the source current.

$$v(t) = v_R(t) + v_C(t) + v_L(t)$$

$$v(t) = i(t) \cdot R + \frac{1}{C}\int_{-\infty}^{t} i(t)dt + L \cdot \frac{di(t)}{dt}$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{LC} i(t) = \frac{1}{L} \cdot \frac{dv(t)}{dt}$$



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Analysis of AC Sourced R-L-C Circuits

Note that, whenever a circuit containing **resistors** and **inductors** and/or **capacitors** is supplied by a sinusoidal source:

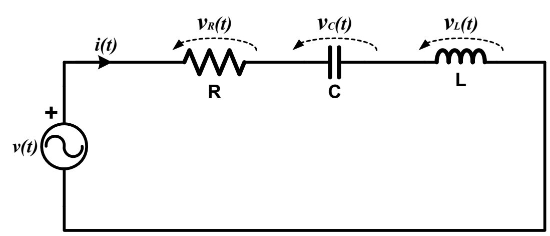
$$v(t) = V \cdot \sin(\omega t + \phi) \text{ volts}$$

the resulting current will also be sinusoidal in nature:

$$i(t) = I \cdot \sin(\omega t + \delta) \text{ amps}$$

where the **angle** $\boldsymbol{\delta}$ will fall within the range $\boldsymbol{\phi - 90° \le \delta \le \phi + 90°}$.

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{LC} i(t) = \frac{1}{L} \cdot \frac{dv(t)}{dt}$$



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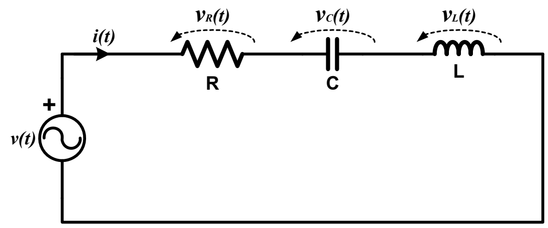
Analysis of AC Sourced R-L-C Circuits

FOR EXAMPLE: Given a simple, series-connected, R-L-C, AC circuit, a **2nd-order differential equation** must be solved in order to determine the source current.

Note that, although solving the 2nd-order differential equation is manageable, this is for a **simple** series circuit.

But, what if this was a series-parallel circuit…?

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{LC} i(t) = \frac{1}{L} \cdot \frac{dv(t)}{dt}$$



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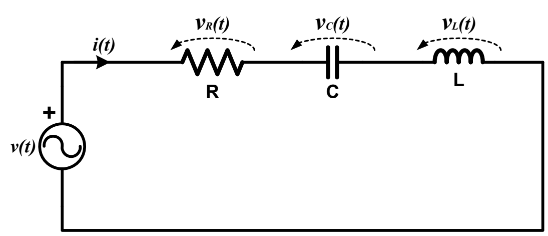
Analysis of AC Sourced R-L-C Circuits

FOR EXAMPLE: Given a simple, series-connected, R-L-C, AC circuit, a **2nd-order differential equation** must be solved in order to determine the source current.

But, provided that the source is sinusoidally time-varying, a **Phasor Analysis** can be performed on the circuit, the method of which allows for the ***V-I* relationships** for all three elements to be **linearized** into simple "Ohm's Law" based relationships.

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{LC} i(t) = \frac{1}{L} \cdot \frac{dv(t)}{dt}$$



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