ECET 3000

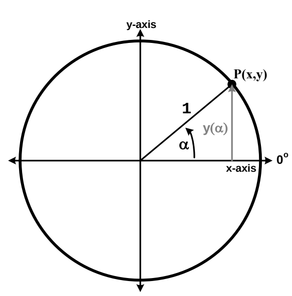
Electrical Principles

Sinusoidally-Varying Waveforms

The Sine Function

Given a point **P(x,y)** plotted on a unit circle centered at the origin of the x-y plane, the instantaneous y-value of the point as a function of its angular position **α** on the circle can be used to define the **sine function**, where:

$$y(\alpha) = \sin(\alpha)$$

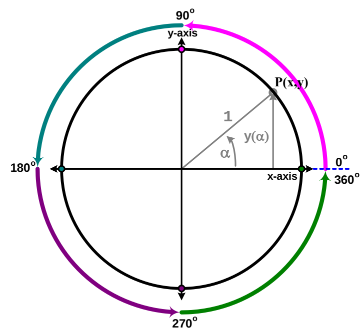


If P(x,y) is plotted on a unit circle, then the "height" of the point with respect to the X-axis is equal to: sin(α)

The Sine Function

The function, **sin(α)**, is a ***periodic function*** that repeats with every **360°** or **2π radian** increase in the angle **α**:

- As **α** varies from **0°→ 360°**, **sin(α)** varies from **0→1→0→ -1→0**, repeating again with every additional **360°** increase in **α**.

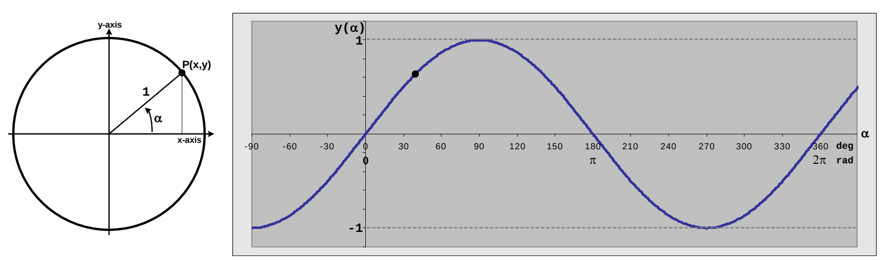


The Sine Function

The function:

$$y(\alpha) = \sin(\alpha)$$

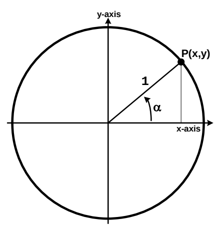
can also be shown by plotting **y(α)** as a function of angle **α**:

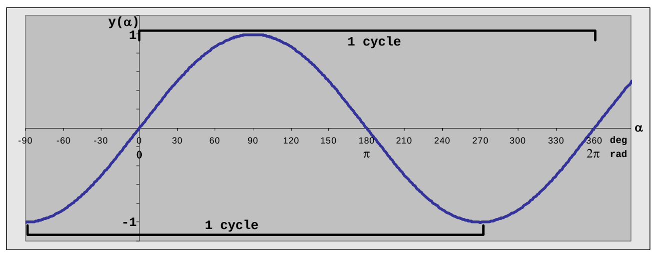


The Sine Function

One **<u>cycle</u>** of a periodic waveform is the smallest portion of the waveform that, if repeated continuously, will reproduce the entire waveform.

- The function **y(α)** completes one cycle of variation every time **α** increases by **360°**.



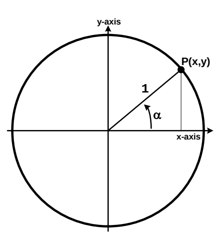


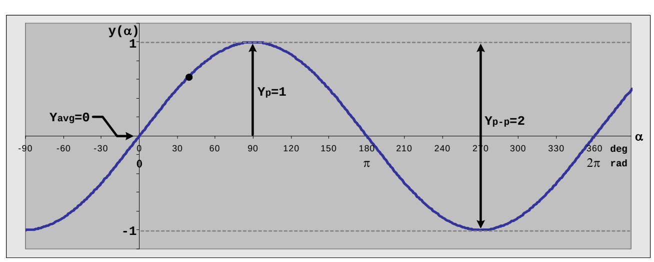
The Sine Function

As shown below, the function:

$$y(\alpha) = \sin(\alpha)$$

- has a **peak magnitude Y_p= 1**,
- has a **peak-to-peak magnitude $Y_{p\text{-}p}$= 2**, and
- has an **average** value **Y_{avg}= 0**.





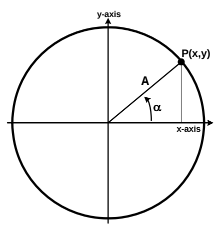
The Sine Function

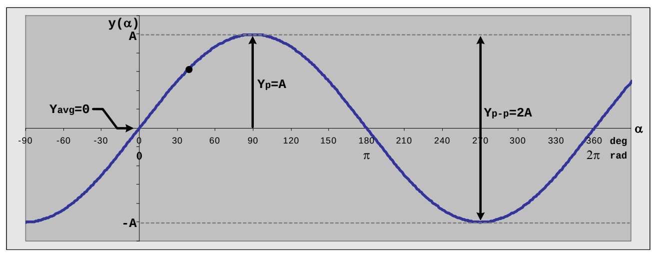
When the sine function is multiplied by a (real) constant ***A***:

$$y(\alpha) = A\sin(\alpha)$$

the **peak magnitude** and the **peak-to-peak magnitude** are both **multiplied** by ***A***.

(The ***average*** value and the ***repetition interval*** both remain unchanged)





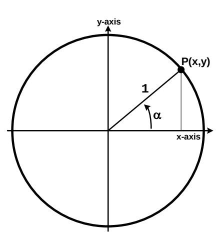
The Sine Function

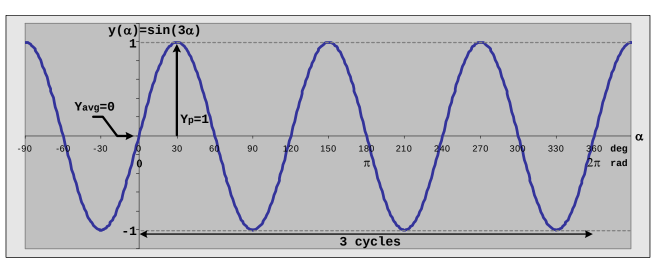
If the angle-term within the sine function is multiplied by a constant ***B***:

$$y(\alpha) = \sin(B\alpha)$$

then the waveform will **repeat *B*** times within the **360°** interval.

(Note: ***B=3*** as shown in the figure below)





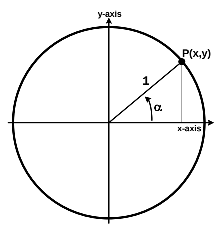
The Sine Function

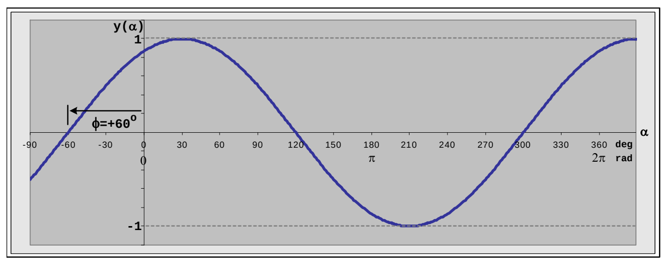
If a constant $\boldsymbol{\phi}$ is added to the angle-term within the sine function:

$$y(\alpha) = \sin(\alpha + \phi)$$

then the entire waveform will **shift** to the left or to the right by the angle $\boldsymbol{\phi}$.

(Note: $\boldsymbol{\phi} = \boldsymbol{60°}$ as shown in the figure below)





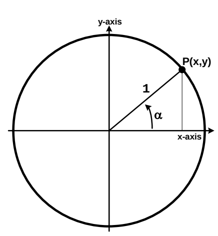
Phase Shift

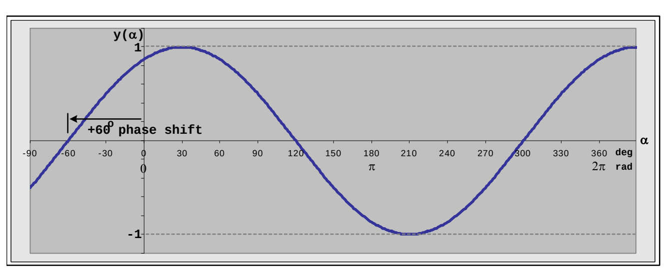
The addition of a **positive** constant $\boldsymbol{\phi}$ to the angle-term within the sine function:

$$y(\alpha) = \sin(\alpha + \phi)$$

results in a **<u>phase shift</u>** of the waveform to the **left** by the angle $\boldsymbol{\phi}$.

(Note: $\boldsymbol{\phi} = \boldsymbol{60°}$ as shown in the figure below)





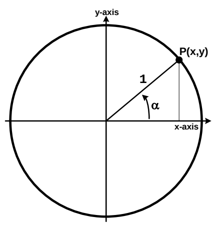
Phase Shift

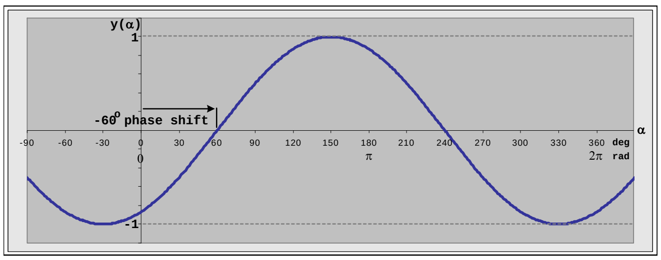
The addition of a **negative** constant $\boldsymbol{\phi}$ to the angle-term within the sine function:

$$y(\alpha) = \sin(\alpha + \phi)$$

results in a **phase shift** of the waveform to the **right** by the angle $\boldsymbol{\phi}$.

(Note: $\boldsymbol{\phi = -60°}$ as shown in the figure below)



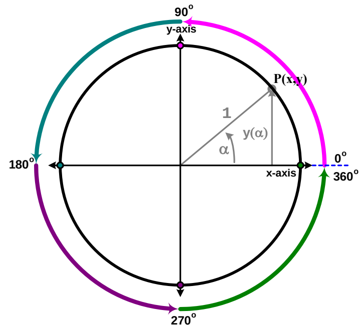


Sine as a Function of Time

Refer back to point **P(x,y)** that is rotated around a unit circle:

Each time **P** rotates one complete revolution around the circle, the function ***y(α)*** progresses through one cycle of its waveform.

What if **P** is continuously rotated around the circle at a constant rate?

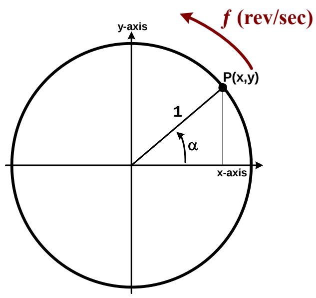


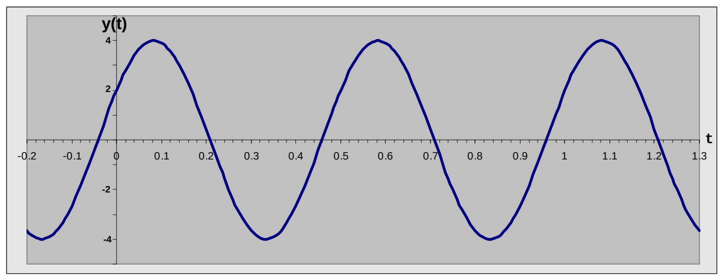
Sine as a Function of Time

If **P** is rotating at a constant rate, ***f***, such that ***f*** is the number of revolutions per second that **P** rotates…

Then the sine function ***y(t)*** will progress through ***f*** cycles of its waveform each second.

(As shown below, ***f*** =2 revolutions per second)



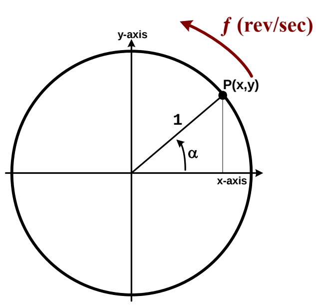


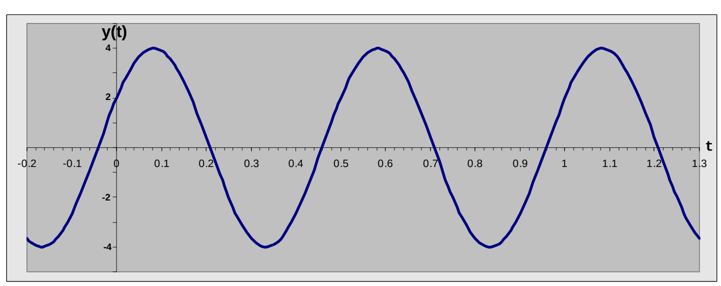
Frequency

The **frequency**, ***f***, of a periodic function is defined as the number of cycles that the function will progress through in one second.

Frequency is assigned the standard unit **Hertz**, where:

Hertz = cycles/sec

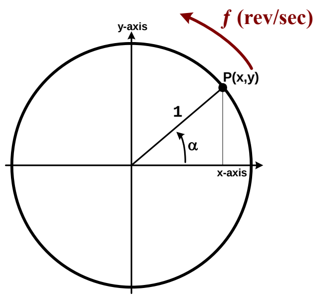


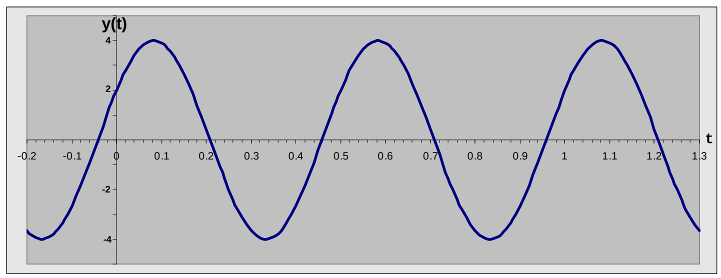


Sine as a Function of Time

If **P** is rotating at a constant rate ***f*** (rev/sec)…

Then the angle $\boldsymbol{\alpha}$ increases at a rate of $2\pi \cdot f$ (rad/sec) since each rotation relates to an angular increase of 2π radians.



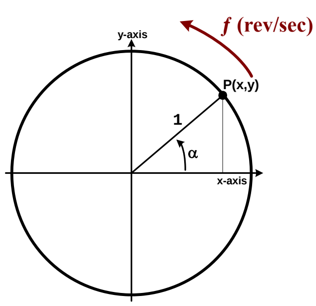


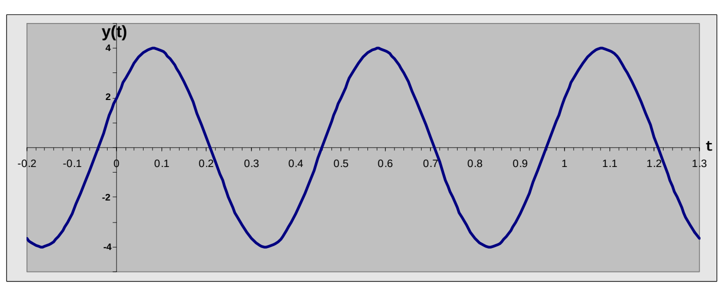
Angular Velocity

The **<u>Angular Velocity</u>** ($\boldsymbol{\omega}$) of a sine function is the angular rate at which the sine-angle increases:

$$\boldsymbol{\omega = 2\pi \cdot f} \text{ (rad/sec)}$$

Note – Although angular velocity, $\boldsymbol{\omega}$, can be expressed in units of deg/sec, the standard units for $\boldsymbol{\omega}$ is rad/sec.





Sine as a Function of Time

If ***Angular Velocity*** (***ω***) defines the rate at which the angle of the sine function increases in radians/sec…

$$\boldsymbol{\omega = 2\pi \cdot f} \text{ (rad/sec)}$$

then $\boldsymbol{\omega \cdot t}$ defines a ***radian*** angle if ***t*** is expressed in seconds.

And, since the term $\boldsymbol{\omega \cdot t}$ defines an angle, it can be used in place of the angle ***α*** in the sine function provided:

$$\boldsymbol{\alpha = \omega \cdot t} \text{ (radians)}$$

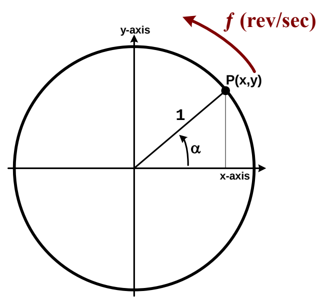
Sine as a Function of Time

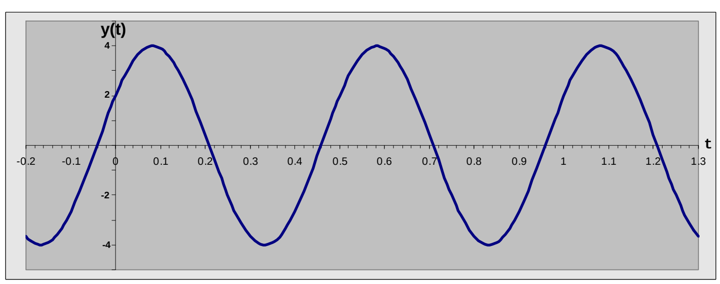
Thus, we can express sine as a function of time, ***y(t)***, such that:

$$y(t) = \sin(\omega \cdot t)$$

where:

$$\omega = 2\pi \cdot f$$

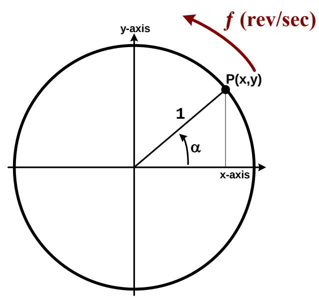


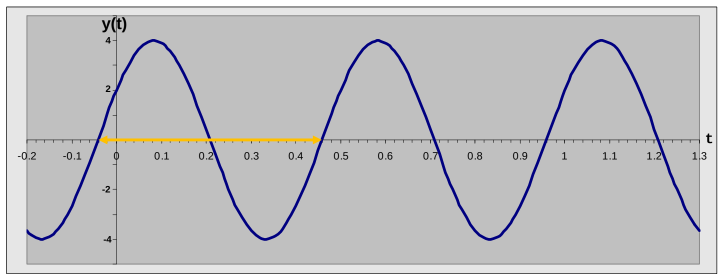


Period

The **period**, ***T***, of a periodic function is the length of time it takes for the function to progress through one cycle of its waveform.

Note – Period is typically defined in standard units of seconds.





Period

Since the function:

$$y(t) = \sin(\omega \cdot t)$$

progresses through ***f*** cycles of its waveform each second, then the **period** of the function (i.e. – the amount of time required to complete each cycle) can be determined from frequency as:

$$T = \frac{1}{f}\left(\frac{1}{Hertz}\right) = \frac{1}{f}\left(\frac{1}{cycles/_{sec}}\right) = \frac{1}{f}\left(\frac{\text{sec}}{cycle}\right)$$

Period

Thus, given a sine function having frequency ***f***:

$$y(t) = \sin(\omega \cdot t) \qquad \omega = 2\pi \cdot f$$

the **period** of the function can be determined by:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \left(\frac{\sec}{cycle} \right)$$

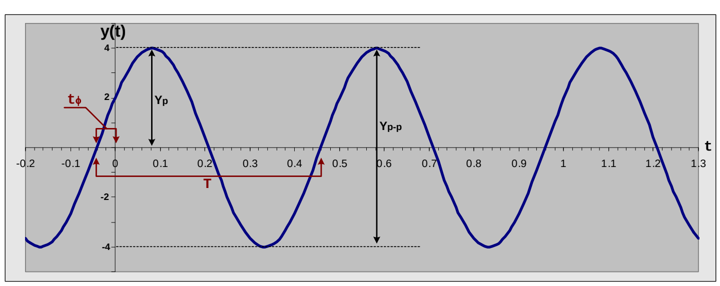
Or, given a periodic waveform having **period *T***, the **frequency *f*** of the waveform can be determined by:

$$f = \frac{1}{T} \left(\frac{\sec}{cycle} \right)^{-1} = \frac{1}{T} \left(\frac{cycles}{\sec} \right) = \frac{1}{T} (Hertz)$$

Sine as a Function of Time

Given a sinusoidally varying waveform with a peak magnitude $\boldsymbol{Y_p}$, an angular frequency **ω**, and a phase angle $\boldsymbol{\phi}$, the waveform may be expressed as a time function ***y(t)***, where:

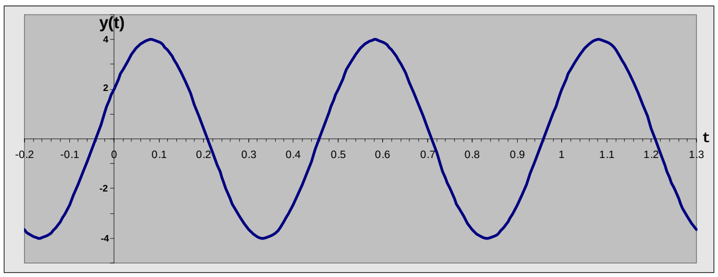
$$y(t) = Y_p \cdot \sin(\omega \cdot t + \phi)$$



Determining Sine from a Time Plot

Given a sinusoidal waveform, ***y(t)***, plotted as a function of time ***t***, determine the following characteristics of the waveform:

Peak Magnitude
Period
Angular Frequency
Peak-to-Peak Magnitude
Frequency
Phase Shift



Determining Sine from a Time Plot

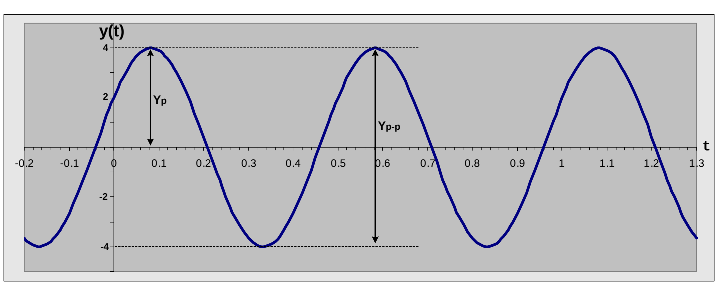
From the plot of ***y(t)*** shown below:

Peak Magnitude

$Y_p = 4$ volts

Peak-to-Peak Magnitude

$Y_{p\text{-}p} = 4 - (-4) = 8$ volts



Determining Sine from a Time Plot

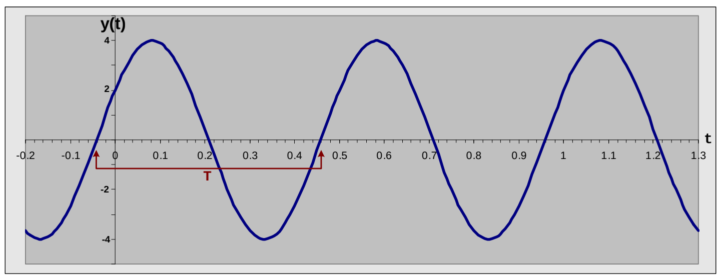
From the plot of ***y(t)*** shown below:

Period

$T = 0.46 - (-0.04) = 0.50$ sec

Frequency

$f = T^{-1} = (0.50)^{-1} = 2$ Hertz (cycles/sec)

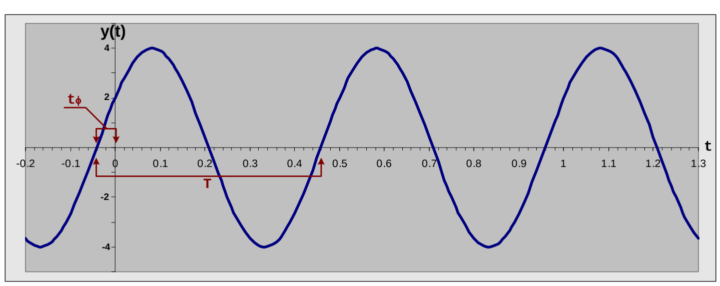


Determining Sine from a Time Plot

Utilizing the value of the frequency ***f***:

Angular Frequency

$\omega = 2\pi \cdot f = 4\pi$ (radians/sec)



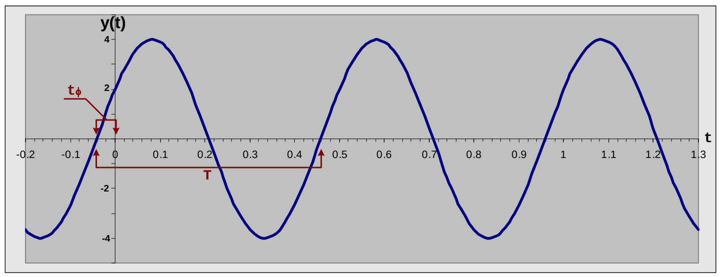
Determining Sine from a Time Plot

From the plot of ***y(t)*** shown below:

Phase Shift

$t_\phi = 0.04$ (seconds), converted to degrees

$$\phi = \frac{t_\phi}{T} \cdot 360° = \frac{0.04}{0.50} \cdot 360° = 28.8°$$



Determining Sine from a Time Plot

Thus, given the plot of the function ***y(t)***, where:

$$y(t) = Y_p \cdot \sin(\omega \cdot t + \phi)\ volts$$

the exact expression for ***y(t)*** is:

$$y(t) = 4 \cdot \sin(4\pi \cdot t + 28.8°)\ volts$$