ECET 3000

Electrical Principles

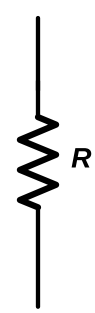
Inductors and Capacitors

Review – Resistors

Resistors are devices that “resist” or oppose the flow of current in an electric circuit.

Thus, an external voltage (potential force) must be utilized in order to push current through the resistor.

A **battery** (voltage source) can be utilized to provide the potential force required to push current through a resistor.

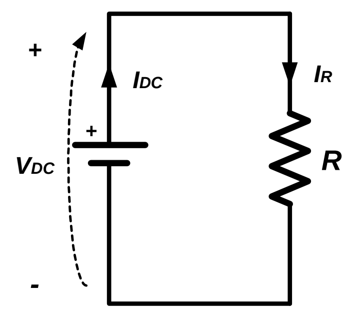


Review – Resistors

When connected together to form a closed-loop path, the battery will begin to push current out of its positive terminal, through the resistor, and back into the battery's negative terminal.

Since the resistor opposes the flow of current, an oppositional force (voltage) will be developed by the resistor that "pushes back" against the flow of current.

Thus, the current will (instantaneously) increase in magnitude until the opposing voltage created by the resistor is equal in magnitude but opposite in direction compared to the voltage supplied by the battery.



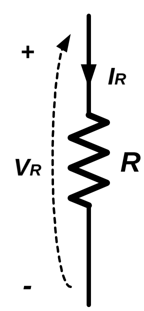
Review – Resistors & Ohm's Law

The voltage-current relationship for a resistor is defined by **Ohm's Law**:

$$V_R = I_R \cdot R$$

such that:

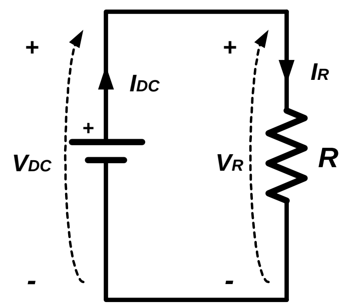
- the magnitude of the resistor's voltage is proportional to the magnitude of the current flowing through the resistor, and
- the direction of the voltage-rise across the resistor is opposite to the direction of current flow through the resistor.



Review – Resistors & Ohm's Law

Since the resistor opposes the flow of current, it takes electrical energy to push current through the resistor. This energy must be supplied by the source of the current.

As the current passes through the resistor, the electrical energy associated with the current flow is absorbed by the resistor and converted into thermal energy, in-turn causing the resistor to heat-up.



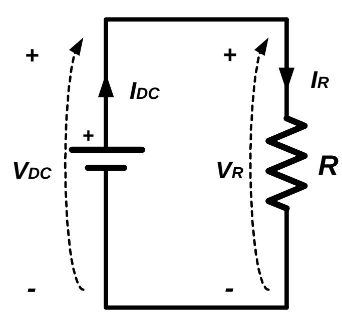
Review – Resistor Energy/Power

The rate at which the electrical energy is "consumed" by the resistor and converted to heat is defined by:

$$P_R = V_R \cdot I_R \quad (watts)$$

Note – by solving Ohm's Law for either V_R or I_R and substituting back into the equation, the resistor power may also be defined as:

$$P_R = V_R \cdot I_R \quad = I_R^2 \cdot R \quad = \frac{V_R^2}{R}$$



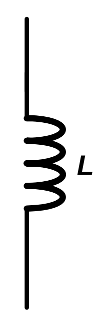
Inductors

Inductors

An **inductor** is an electric circuit element that opposes a <u>change</u> in the magnitude of the current that is flowing through the element.

Note that this is fundamentally different from the operation of a resistor, whose oppositional force is directly proportional to the instantaneous magnitude of the current flowing through the resistor.

Also note that an inductor provides no oppositional force to a steady-state (i.e. – constant) DC current.

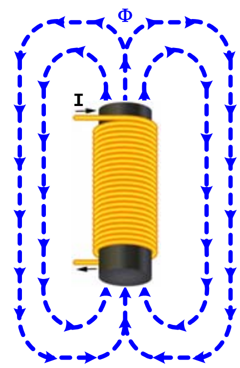


Inductors

An inductor can be constructed as a coil of wire through which a current flows.

When a current flows in the coil, a magnetic field is created in the region around the coil such that the field lines all form closed-loops that pass through the center of the coil.

The magnitude of the field developed by the coil will be directly proportional to the magnitude of the coil current.



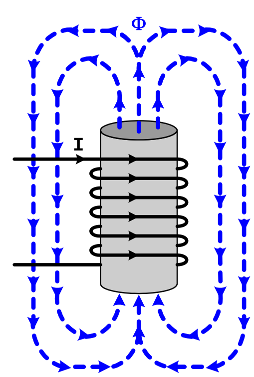
Inductors

Thus, if the current flowing in the coil is constant, then the magnitude of the field developed by the coil will also be constant.

$$\Phi \equiv I$$

But, if the coil current is time-varying, then the magnitude of the field will also be time-varying such that the field and the current maintain their proportionality relationship:

$$\Phi(t) \equiv i(t)$$



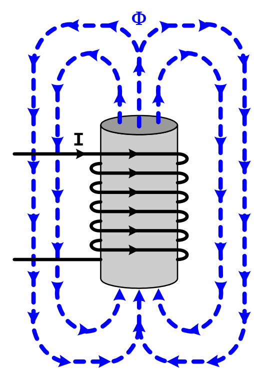
Inductors

But, whenever a conductor is exposed to a time-varying magnetic field, a voltage will be induced across the conductor.

If the conductor is wound into a coil, the voltage can be determined using **Faraday's Law**:

$$v(t) = N \cdot \frac{d\Phi(t)}{dt}$$

where: ***N*** is the number of turns of the coil, and
Φ is the flux (magnetic field) passing through the coil (in Webers)



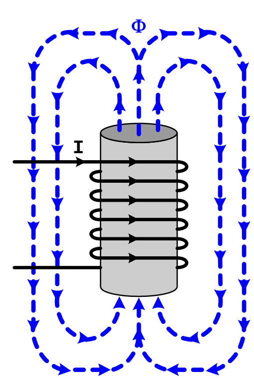
Inductors

Since the time-varying field is proportional to the coil current the induced voltage will also be proportional to the rate of change in the current.

The following <u>non-linear</u> equation is utilized to define the V-I relationship for an inductor:

$$v_L(t) = L \cdot \frac{di_L(t)}{dt}$$

where: ***L*** is the **Inductance** of the coil, and is defined having the standard units of Henries (H).



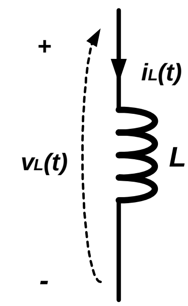
Inductors

The direction of the voltage-rise across the inductor can be determined by applying Lenz's Law:

"***Any induced effect will always oppose it's source***"

Even though the voltage across the inductor will be proportional to the rate of change in the current, the induced voltage must still oppose the current flow…

Therefore, the direction of the voltage-rise across the inductor will be opposite compared to the direction of the current flowing through the inductor.



Inductor V-I Relationship

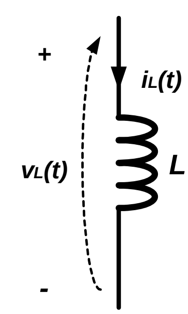
By integrating both sides of the V-I relationship:

$$v_L(t) = L \cdot \frac{di_L(t)}{dt}$$

the inductor current may be defined in terms of the inductor voltage, such that:

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)dt = \frac{1}{L}\int_{0}^{t} v_L(t)dt + I_o$$

where: $\boldsymbol{I_0}$ is the initial current flowing through the coil at time $t=0$.



DC Steady-State Behavior of Inductors

Under DC steady-state conditions, the current flowing through the inductor will be constant.

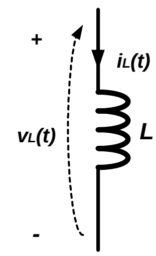
$$i_L(t) = I_L$$

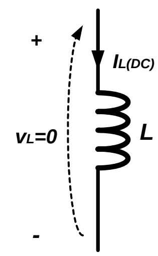
Since the voltage induced across an inductor is proportional to the rate of change in the current:

$$v_L(t) \equiv \frac{di_L(t)}{dt}$$

there will be no voltage-rise across the inductor under DC steady-state conditions:

$$v_{L(\text{DC steady-state})} = 0$$





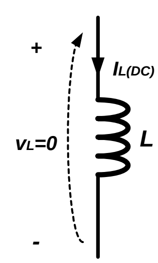
DC Steady-State Behavior of Inductors

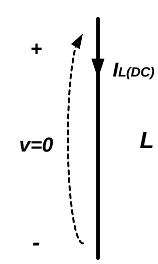
If there is no voltage-rise across the inductor under DC steady-state conditions:

$$v_{L(\text{DC steady-state})} = 0$$

then an inductor acts like an “ideal wire” at steady-state DC.

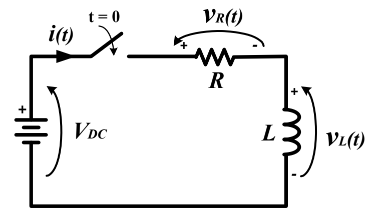
Thus, when analyzing a circuit’s DC steady-state operation, any inductors in the circuit can be replaced by “ideal wires” for that analysis.





DC Transient R-L Circuit Behavior

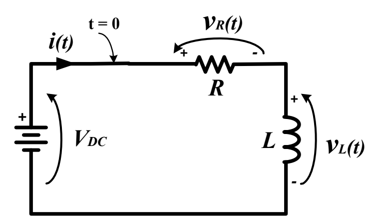
What if a DC voltage source is connected to a series resistor and inductor pair of elements by means of a switch that closes at time *t=0*?



DC Transient R-L Circuit Behavior

When the switch closes and completes the circuit, a KVL loop equation may be written for the circuit as:

$$V_{DC} - v_R(t) - v_L(t) = 0$$



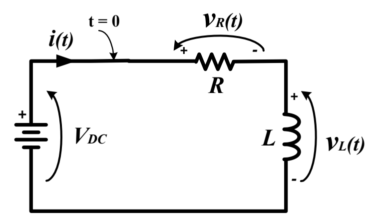
DC Transient R-L Circuit Behavior

The resistor's voltage, as per Ohm's Law, can be defined in terms of the resistor's current as:

$$v_R(t) = i_R(t) \cdot R$$

while inductor's voltage can be defined in terms of the inductor's current as:

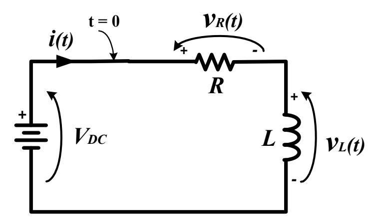
$$v_L(t) = L \cdot \frac{di_L(t)}{dt}$$



DC Transient R-L Circuit Behavior

If the resistor and inductor voltages relationships are substituted into the KVL equation, taking into account that they have the same current flowing through them since they are connected in series, the resultant KVL equation is:

$$V_{DC} - i(t) \cdot R - L \cdot \frac{di(t)}{dt} = 0$$

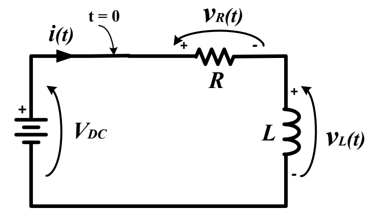


DC Transient R-L Circuit Behavior

By solving for $\frac{di(t)}{dt}$, the KVL equation may be rewritten as:

$$\frac{di(t)}{dt} = \frac{V_{DC}}{L} - \frac{R}{L} \cdot i(t)$$

resulting in a standard 1st order differential equation.



DC Transient R-L Circuit Behavior

The general solution for the 1st order differential equation:

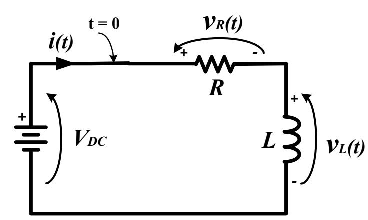
$$\frac{di(t)}{dt} = \frac{V_{DC}}{L} - \frac{R}{L} \cdot i(t)$$

is:

$$i(t) = \frac{V_{DC}}{R} \cdot \left[1 - e^{\frac{-t}{\tau}} \right]$$

where: (time constant)

$$\tau = \frac{L}{R}$$

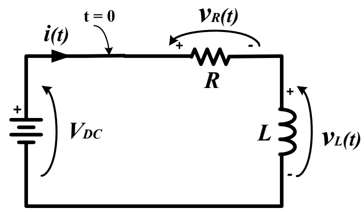


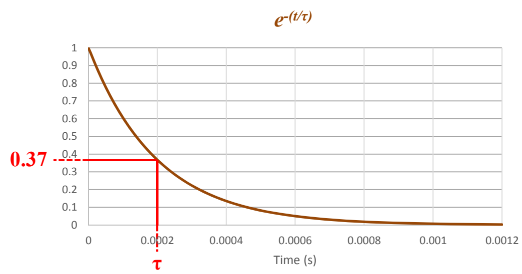
DC Transient R-L Circuit Behavior

The **time constant**, τ, defines the amount of time that it takes for the expression:

$$e^{\frac{-t}{\tau}}$$

to decay to 37% of its initial value at $t=0$.





$V_{DC} = 10\text{ V}$

$R = 100\ \Omega$

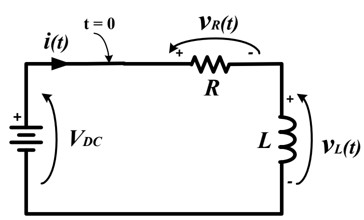
$L = 20\text{ mH}$

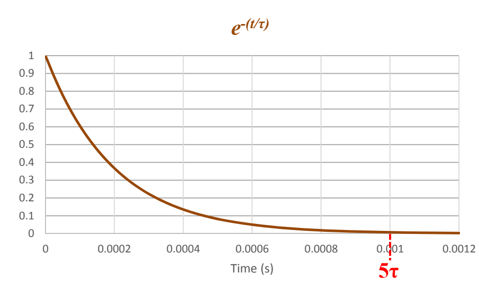
$\tau = \frac{L}{R} = 0.0002\text{ sec}$

DC Transient R-L Circuit Behavior

Note that, after five time constants ($t \geq 5\tau$), the term $e^{-t/\tau}$ has effectively decayed to zero.

Because of this, the R-L transient circuit is assumed to have reached **steady-state** conditions when $t \geq 5\tau$.





$V_{DC} = 10\text{ V}$

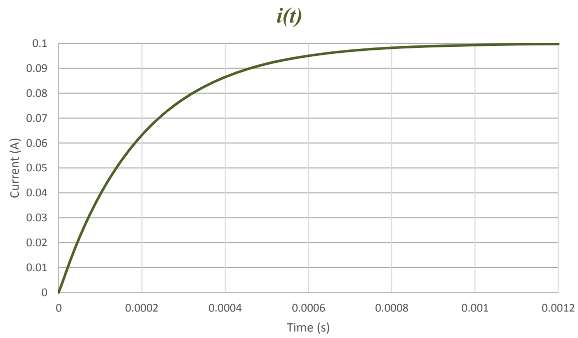
$R = 100\ \Omega$

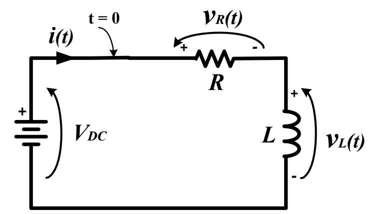
$L = 20\text{ mH}$

$\tau = \frac{L}{R} = 0.0002\text{ sec}$

DC Transient R-L Circuit Behavior

The **current** for the R-L circuit may be sketched as follows:





$$i(t) = \frac{V_{DC}}{R} \cdot \left[1 - e^{\frac{-t}{\tau}}\right]$$

$V_{DC} = 10\ \text{V}$

$R = 100\ \Omega$

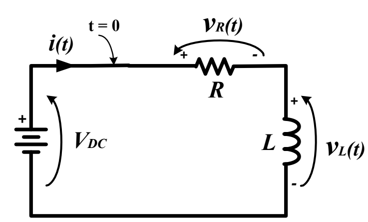
$L = 20\ \text{mH}$

$\tau = \frac{L}{R} = 0.0002\ \text{sec}$

DC Transient R-L Circuit Behavior

By substituting:

$$i(t) = \frac{V_{DC}}{R} \cdot \left[1 - e^{\frac{-t}{\tau}}\right]$$

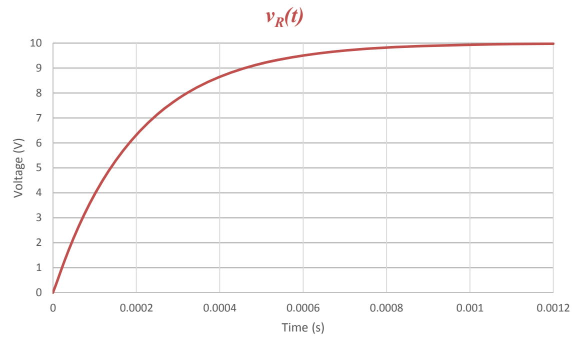


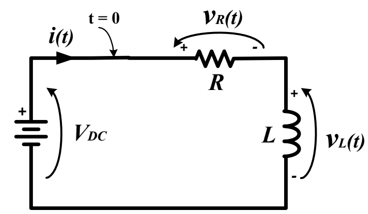
back into the V-I relationships for the resistor and the inductor, the resistor and inductor voltages can be expressed as:

$$v_R(t) = V_{DC} \cdot \left[1 - e^{\frac{-t}{\tau}}\right] \qquad v_L(t) = V_{DC} \cdot e^{\frac{-t}{\tau}}$$

DC Transient R-L Circuit Behavior

The **resistor voltage** for the R-L circuit may be sketched as follows:





$$v_R(t) = V_{DC} \cdot \left[1 - e^{\frac{-t}{\tau}}\right]$$

$V_{DC} = 10 \text{ V}$

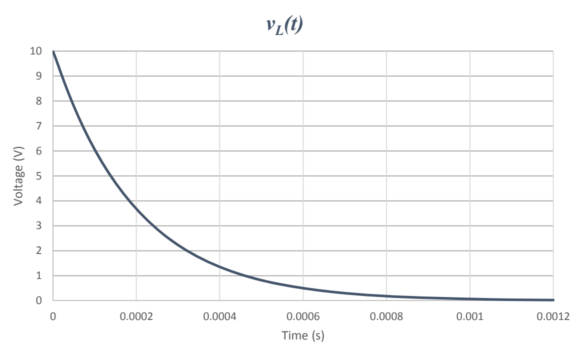
$R = 100\ \Omega$

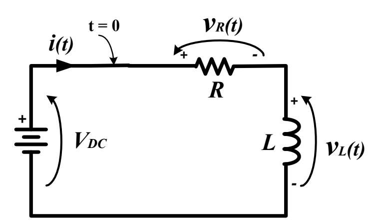
$L = 20 \text{ mH}$

$\tau = \frac{L}{R} = 0.0002 \text{ sec}$

DC Transient R-L Circuit Behavior

The **inductor voltage** for the R-L circuit may be sketched as follows:





$$v_L(t) = V_{DC} \cdot e^{\frac{-t}{\tau}}$$

$V_{DC} = 10 \text{ V}$

$R = 100\ \Omega$

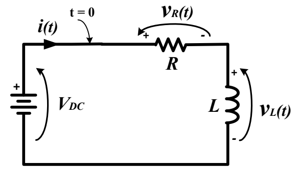
$L = 20 \text{ mH}$

$\tau = \frac{L}{R} = 0.0002 \text{ sec}$

R-L Circuit Steady-State Behavior

When t ≥ 5τ, the inductor voltage will have decayed to zero and the resistor voltage will equal V_{DC}.

Thus, under **steady-state conditions**, the inductor is electrically equivalent to an ideal wire.

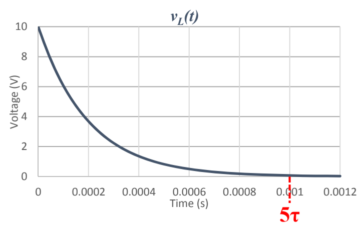


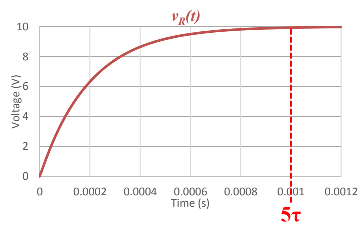
$V_{DC} = 10\ \text{V}$

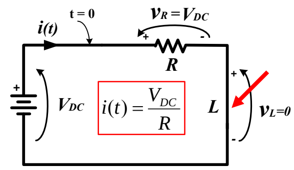
$R = 100\ \Omega$

$L = 20\ \text{mH}$

$\tau = \frac{L}{R} = 0.0002\ \text{sec}$







steady-state operation

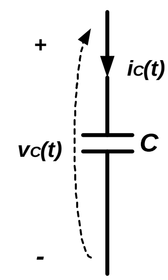
Capacitors

Capacitors

A capacitor is a device that can be created by configuring two conductive plates such that they are held in position, parallel to each other, but separated by a thin layer of insulation.

For a voltage (potential difference) to exist across the capacitor's plates, an electric field must exist between the plates, as defined by:

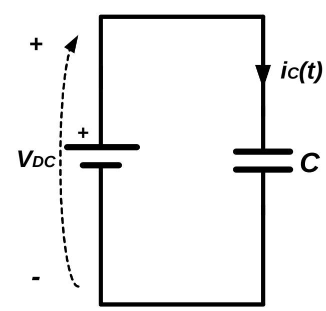
$$V_C = \int_b^a \overline{E} \cdot dl$$



Capacitors

Assuming that the plates are neutrally charged when the it is first subjected to an external voltage source, then no voltage will appear across the capacitor's terminals to oppose the flow of current "through" the capacitor.

Note – current does not actually flow "through" the capacitor since the two plates are separated by an insulator. Instead, electrons are drawn out of the top plate and into the positive terminal of the source and pushed into the bottom plate from the negative terminal of the source, resulting in the "appearance" of current flowing through the capacitor.

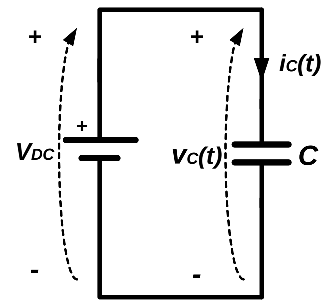


Capacitors

As current flows through the capacitor, opposing charges begin to build-up on the plates, in-turn creating an electric field between the plates and a potential difference (voltage) across the plates.

The direction of the voltage-rise across the capacitor will be such that it opposes the continued flow of current through the capacitor.

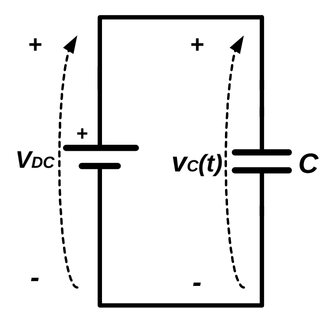
Thus, as the voltage increases, the rate at which current flows through the capacitor decreases.



Capacitors

Eventually, the voltage potential developed by the capacitor will build-up such that it is equal but opposite in direction compared to the voltage provided by the source, at which point current will stop flowing through the capacitor.

DC steady-state operation has been reached once current stops flowing through the capacitor, at which point it is said to be "***fully charged***".



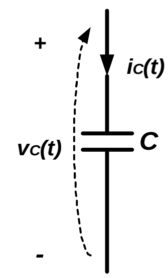
Capacitor V-I Relationship

The rate at which the plates are charged is proportional to the rate of charge flow (current) into the capacitor.

$$\frac{dQ}{dt} \equiv i_C$$

Additionally, the voltage across the capacitor is proportional to the electric field between the plates, which is, in-turn, proportional to the charge stored on the plates.

$$v_C \equiv E \equiv Q$$

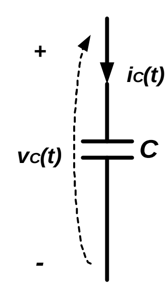


Capacitor V-I Relationship

Note that, in order for a capacitor to develop a force (voltage) that opposes the flow of current, a charge must first build-up on the capacitor's plates.

In order for the charge to build-up, current must first flow into the capacitor, the magnitude of which will determine the rate of charge build-up and, in-turn, the rate at which the voltage builds-up.

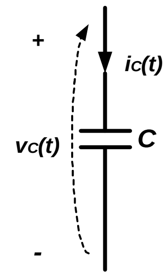
$$\frac{dv_C(t)}{dt} \equiv i_C(t)$$



Capacitor V-I Relationship

Although a capacitor can be charged such that constant voltage can exist across its terminals without any additional current flow through the capacitor, a current is required in order to change the magnitude of the capacitor's voltage.

For this reason, capacitors are often characterized as "opposing a change" in voltage.



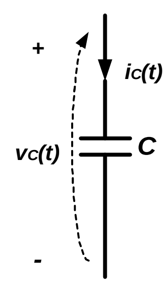
Capacitor V-I Relationship

Since current is defined as a rate of charge flow, the total charge stored on the capacitor's plates may be defined in terms of the capacitor current as:

$$Q = \int_{-\infty}^{\infty} i_c(t) \cdot dt$$

And, since the capacitor's voltage is proportional to its stored charge, the capacitor's voltage will also be proportional to the integral of the current:

$$v_c(t) \equiv \int_{-\infty}^{\infty} i_c(t) \cdot dt$$

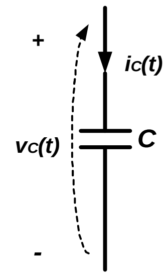


Capacitor V-I Relationship

The exact relationship between the capacitor's voltage and current is described by:

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt$$

where: ***C*** is the capacitor's **Capacitance** defined in the standard units of Farads



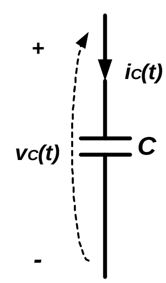
Capacitor V-I Relationship

The relationship can also be manipulated to solve for the current as a function of the voltage:

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt$$

$$\Updownarrow$$

$$i_C(t) = C \cdot \frac{dv_C(t)}{dt}$$

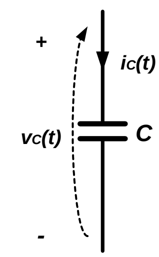


DC Steady-State Behavior of Capacitors

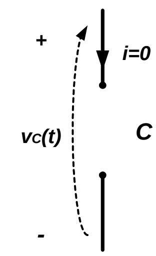
If there is no current flowing through the capacitor under DC steady-state conditions:

$$i_{C(\text{DC steady-state})} = 0$$

then a capacitor acts like an "open circuit" at steady-state DC.

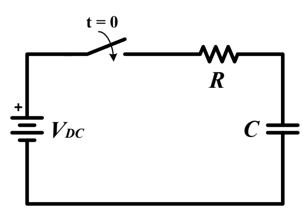


Thus, when analyzing a circuit's DC steady-state operation, any capacitors in the circuit can be removed with the connection-terminals left "open-circuited" for that analysis.



DC Transient R-C Circuit Behavior

What if a DC voltage source is connected to a series resistor-capacitor pair of elements through a switch that closes at time $t=0$?



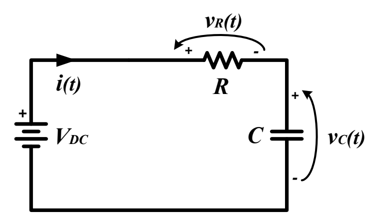
DC Transient R-C Circuit Behavior

When the switch closes and completes the circuit, a KVL loop equation may be written for the circuit as:

$$V_{DC} - v_R(t) - v_C(t) = 0$$

Solving for the resistor voltage:

$$v_R(t) = V_{DC} - v_C(t)$$



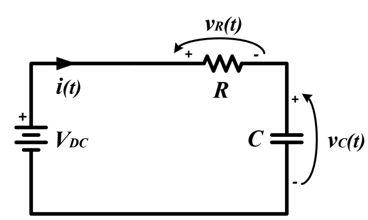
DC Transient R-C Circuit Behavior

The V-I relationship for the resistor, as defined by Ohm's Law, is:

$$i_R(t) = \frac{v_R(t)}{R}$$

which may be re-written as:

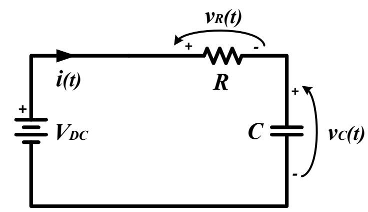
$$i_R(t) = \frac{V_{DC} - v_C(t)}{R}$$



DC Transient R-C Circuit Behavior

The V-I relationship for the capacitor is defined by:

$$i_C(t) = C \cdot \frac{dv_C(t)}{dt}$$



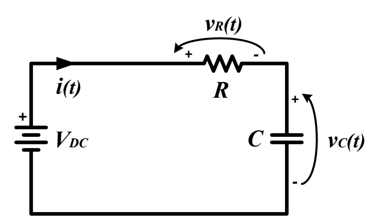
DC Transient R-C Circuit Behavior

Since they are in series, the same current flows through the resistor and the capacitor:

$$i_C(t) = i_R(t) = i(t)$$

If the currents are expressed in terms of the voltages, we get:

$$C \cdot \frac{dv_C(t)}{dt} = \frac{V_{DC} - v_C(t)}{R}$$



DC Transient R-C Circuit Behavior

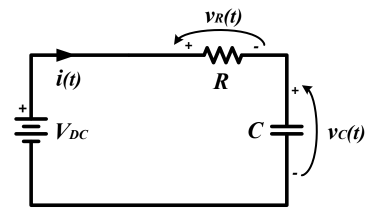
The expression:

$$C \cdot \frac{dv_C(t)}{dt} = \frac{V_{DC} - v_C(t)}{R}$$

may be re-written as:

$$\frac{dv_C(t)}{dt} = \frac{V_{DC}}{RC} - \frac{1}{RC} \cdot v_C(t)$$

resulting in a standard 1st order differential equation.



DC Transient R-C Circuit Behavior

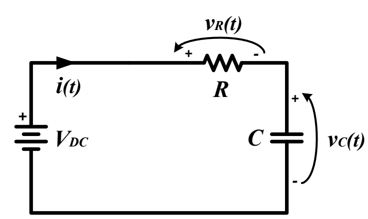
The general solution for the 1st order differential equation:

$$\frac{dv_C(t)}{dt} = \frac{V_{DC}}{RC} - \frac{1}{RC} \cdot v_C(t)$$

is:

$$v_C(t) = V_{DC} \cdot \left[1 - e^{\frac{-t}{\tau}} \right]$$

where: $\tau = RC$ (time constant)



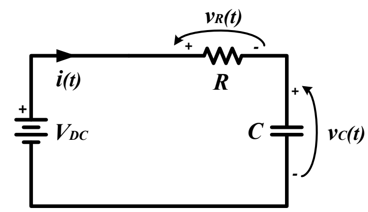
DC Transient R-C Circuit Behavior

The current can then be determined by substitution the solution for $v_C(t)$ into:

$$i(t) = C \cdot \frac{dv_C(t)}{dt}$$

with the following result:

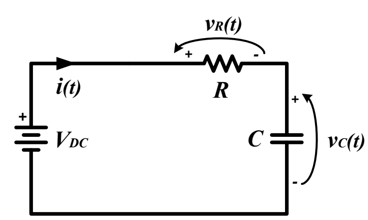
$$i(t) = \frac{V_{DC}}{R} \cdot e^{\frac{-t}{\tau}}$$



DC Transient R-C Circuit Behavior

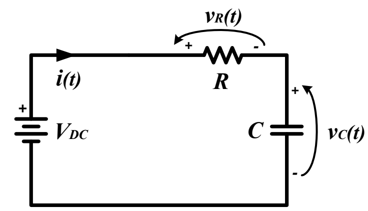
The resistor voltage may then be defined as:

$$v_R(t) = i(t) \cdot R = V_{DC} \cdot e^{\frac{-t}{\tau}}$$



DC Transient R-C Circuit Behavior

Thus, for a DC voltage source connected to a series resistor-capacitor pair by means of a switch that closes at time *t=0*:



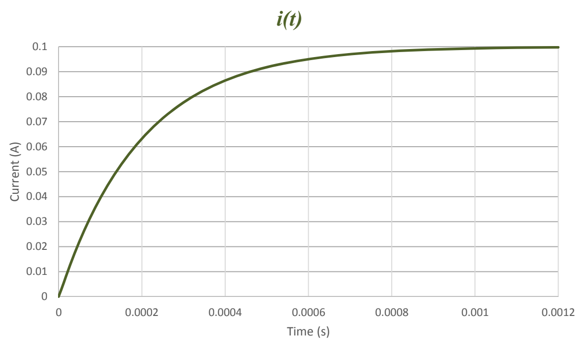
$$i(t) = \frac{V_{DC}}{R} \cdot e^{\frac{-t}{\tau}} \qquad \tau = RC$$

$$v_C(t) = V_{DC} \cdot \left[1 - e^{\frac{-t}{\tau}}\right]$$

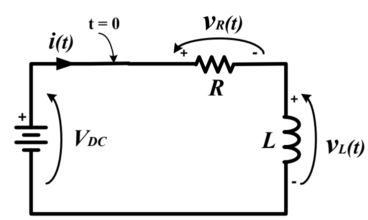
$$v_R(t) = V_{DC} \cdot e^{\frac{-t}{\tau}}$$

DC Transient R-L Circuit Behavior

The **current** for the R-L circuit may be sketched as follows:



$$i(t) = \frac{V_{DC}}{R} \cdot \left[1 - e^{\frac{-t}{\tau}}\right]$$

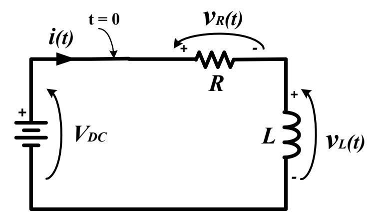


$$V_{DC} = 10 \text{ V}$$
$$R = 100\ \Omega$$
$$L = 20 \text{ mH}$$
$$\tau = \tfrac{L}{R} = 0.0002 \text{ sec}$$

DC Transient R-L Circuit Behavior

By substituting:

$$i(t) = \frac{V_{DC}}{R} \cdot \left[1 - e^{\frac{-t}{\tau}}\right]$$

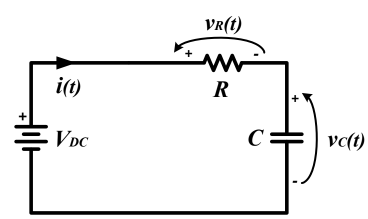


back into the V-I relationships for the resistor and the inductor, the resistor and inductor voltages can be expressed as:

$$v_R(t) = V_{DC} \cdot \left[1 - e^{\frac{-t}{\tau}}\right] \qquad v_L(t) = V_{DC} \cdot e^{\frac{-t}{\tau}}$$

DC Transient R-C Circuit Behavior

The voltage and current waveforms for the R-C circuit may be sketched as follows:



$$V_{DC} = 10 \text{ V}$$
$$R = 100\ \Omega$$
$$C = 60\ \mu\text{F}$$
$$\tau = R \cdot C = 0.006 \text{ sec}$$