ECET 3000

Electrical Principles

Series-Parallel Circuits

1

Introduction

The fundamental circuit theory building blocks that we discussed during our analysis of simple circuits containing **either** series- or parallel-connected elements include:

- **Ohm's Law**
- **Series-connected & Parallel-connected Resistors**
- **Kirchhoff's Voltage & Current Laws (KVL & KCL)**
- **Series-equivalent & Parallel-equivalent Resistances**
- **Voltage Divider & Current Divider Equations**

These same concepts will now be utilized during the analysis of circuits that contain combinations of **both** series- and parallel-connected elements.

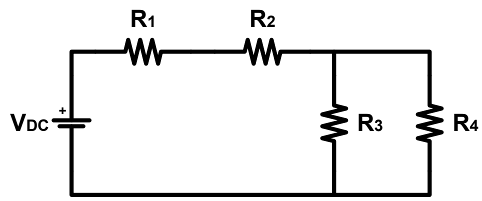
2

Series-Parallel Circuits

A **series-parallel circuit** is a circuit that contains a combination of series-connected and parallel-connected circuit elements.

During this discussion, we will only cover circuits that contain a single DC voltage source and purely resistive loads.

Circuits that contain multiple voltage sources, AC voltage sources, or other types of loads will be presented later in the semester.



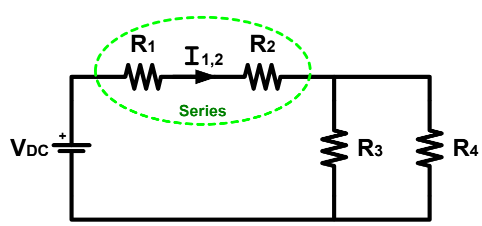
Series-Parallel Circuit Example #1

3

Series Elements

Two or more elements are connected in **series** if the current that flows through one of the elements must entirely flow through the other element(s).

Note that the voltage source is also connected in series with resistors R_1 and R_2 since the current $I_{1,2}$ also flows through the voltage source.



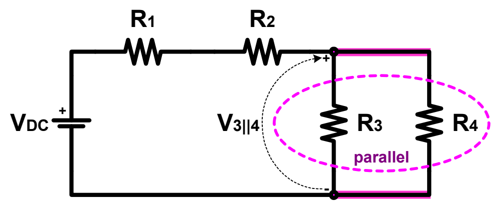
Series-Parallel Circuit Example #1

4

Parallel Elements

Two or more elements are connected in **parallel** if they are each connected across the same two nodes [such that the same voltage appears across each of the elements ($V_3 = V_4 = V_{3||4}$)].

Although a node is a common "point of connection" for two or more circuit elements by ideal wires, a section of ideal wire is equivalent to a node since there is no potential difference across an ideal wire.

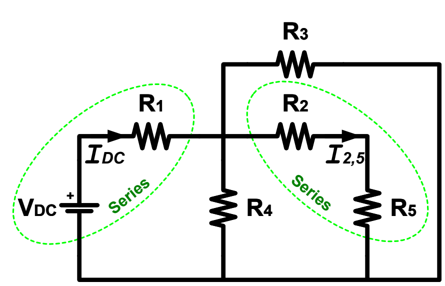


Series-Parallel Circuit Example #1

5

Series-Parallel Circuits

Given the circuit shown below that contains a combination of circuit elements, resistors $\boldsymbol{R_2}$ and $\boldsymbol{R_5}$ are connected in **series** since the current $\boldsymbol{I_{2,5}}$ flows through both of the resistors.

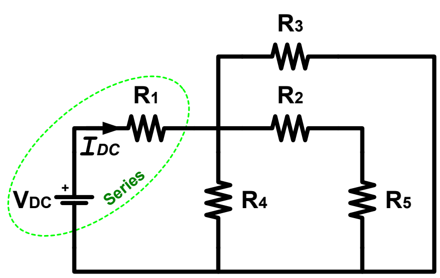


Series-Parallel Circuit Example #2

6

Series-Parallel Circuits

Although R_1 is connected in series with the battery, it is **not** connected in series or in parallel with any of the other resistors in the circuit.

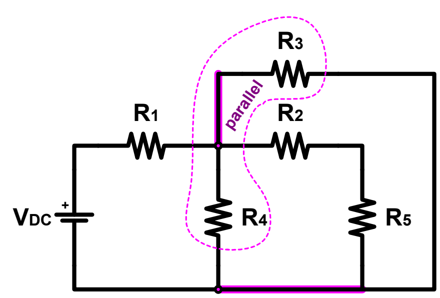


Series-Parallel Circuit Example #2

7

Series-Parallel Circuits

Careful inspection will also reveal that resistors R_3 and R_4 are connected in **parallel** since both resistors are connected across the same two "nodes".

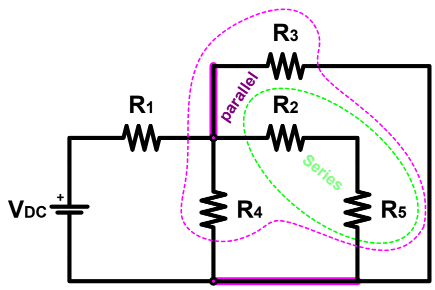


Series-Parallel Circuit Example #2

8

Series-Parallel Circuits

Note – that the **series-combination** of resistors $\boldsymbol{R_2}$ and $\boldsymbol{R_5}$ may be considered as also being connected in parallel with resistors $\boldsymbol{R_3}$ and $\boldsymbol{R_4}$.



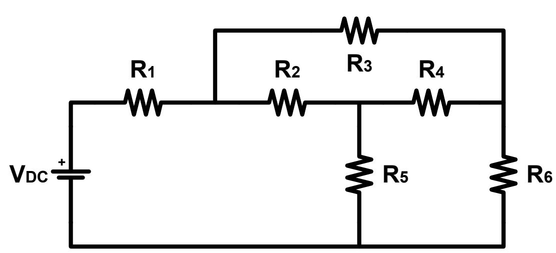
Series-Parallel Circuit Example #2

9

Series-Parallel Circuits

Take a moment to examine the following circuit in order to determine which resistors are connected in series and which are connected in parallel.

There are **no** sets of either series- or parallel-connected resistors, and thus this is not considered a series-parallel circuit.



Although there are techniques available to analyze this type of network, they will not be covered in the course.

Series-Parallel Circuit Example #3

10

The Reduce & Return Approach

Although **series-parallel circuits** may be analyzed by a variety of methods, one of the simplest methods is the:

Reduce and Return Approach.

With this method, the original circuit is **incrementally reduced** in complexity by replacing sets of either series- or parallel-connected elements with their series or parallel equivalents.

$$R_{EQ(series)} = R_1 + R_2 + \cdots + R_N$$

$$R_{EQ(parallel)} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \right)^{-1}$$

11

The Reduce & Return Approach

Each incremental reduction provides a simpler circuit containing fewer elements that, if necessary, can be further reduced until only a **trivial circuit** remains.

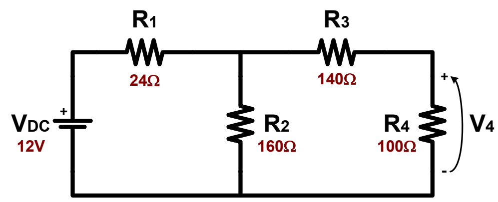
And once the original circuit is reduced down into a trivial circuit, that circuit can be analyzed in order to determine any unknown voltages or currents.

The voltages or currents from the trivial circuit can then be utilized, step-by-step in reverse order back through the simplified circuits, facilitating the analysis of each incrementally more complex circuit until the desired unknown parameters specified in the original circuit are known.

12

Reduce & Return Example

Use the **Reduce and Return Approach** to solve for the voltage V_4 as specified in the following figure:



Circuit #1

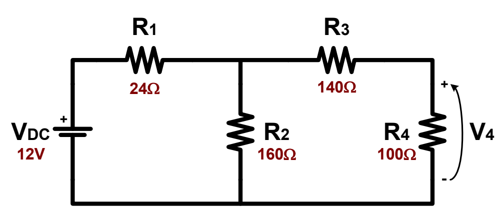
13

Reduce & Return Example

Step 1 – Reduce the Original Circuit

Identify a set of series-connected or parallel-connected resistors and replace them with a single equivalent resistor.

If multiple series or parallel sets of resistors exist in the circuit, the ones furthest from the source are typically reduced first.



Circuit #1

14

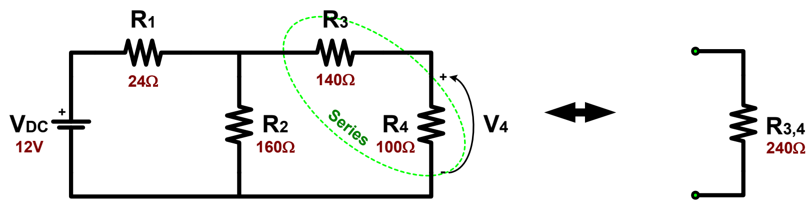
Reduce & Return Example

Step 1 – Reduce the Original Circuit

Resistors R_3 and R_4 are connected in **series**.

Thus, the **series-equivalent resistance** $R_{3,4}$ that may be used to replace the series combination of R_3 and R_4 is:

$$R_{3,4} = R_3 + R_4 = 140\,\Omega + 100\,\Omega = 240\,\Omega$$



Circuit #1

15

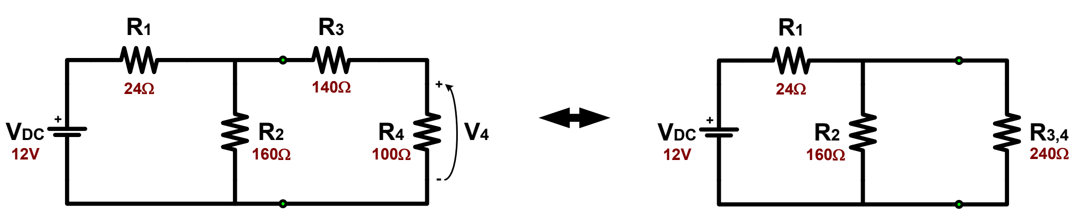
Reduce & Return Example

Step 1 – Reduce the Original Circuit

Redraw the circuit with the new "equivalent resistance" in place.

Note – When replacing a set of resistors with an equivalent resistance, the rest of the circuit remains unchanged.

Note that the variable, V_4, does not exist in the new version of the circuit since R_4 no longer appears as a discrete element in that circuit.



Circuit #1 Circuit #2

16

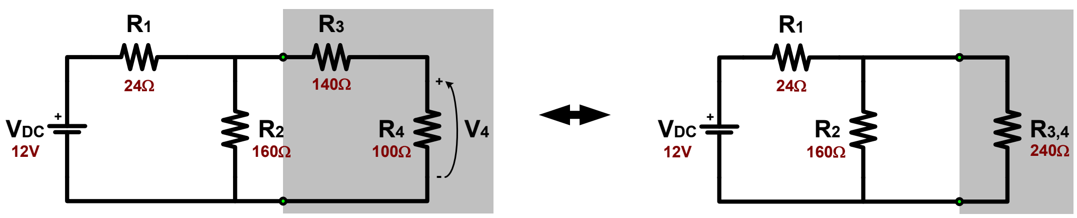
KSU ECET

Reduce & Return Example

Step 1a – Identify New Variables in the Reduced Circuit

Analyze the reduced circuit in order to **identify** and define any **new variables** that may be required during the completion of the overall problem.

It is often useful to highlight the elements that are being reduced in order to help keep track of the circuit changes and to identify the new variables.



Circuit #1 — Circuit #2

17

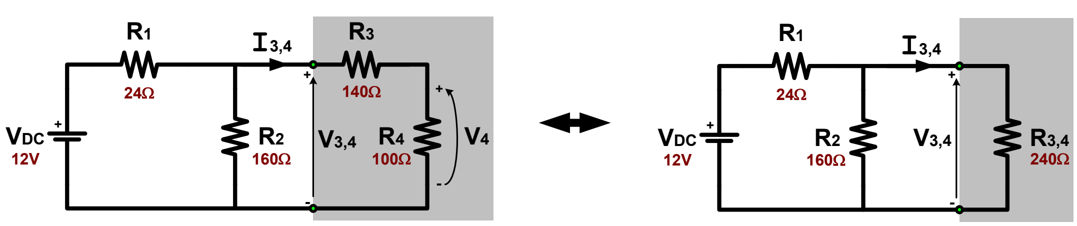
KSU ECET

Reduce & Return Example

Step 1a – Identify New Variables in the Reduced Circuit

New parameters $V_{3,4}$ and $I_{3,4}$ can be defined in both versions of the circuit such that their values will be the same in both circuits based on the concept of series-equivalent resistance.

Although V_4, does not exist in circuit #2, if either $V_{3,4}$ or $I_{3,4}$ can be determined using circuit #2, then the results can be used to solve for V_4 in circuit #1.



Circuit #1 — Circuit #2

18

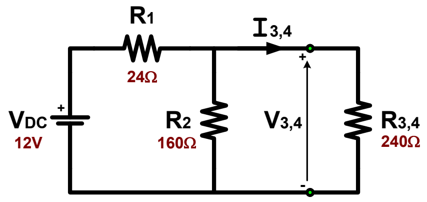
Reduce & Return Example

Steps 2 → ? – Repeat the Process until a trivial circuit remains.

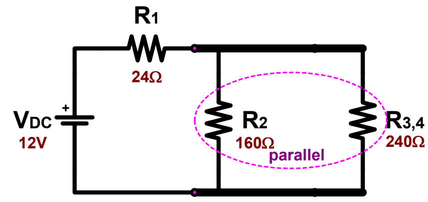
Examine the new circuit to determine if there are any series or parallel connected elements that can also be replaced by their respective equivalents…

Looking at circuit #**2**, it can be seen that resistors R_2 and $R_{3,4}$ are in **parallel**.

Although the circuit can be reduced until only a single resistor remains, it is typically only reduced to the point where a single set of either series- or parallel-connected resistors remain.



Circuit #2



Circuit #2

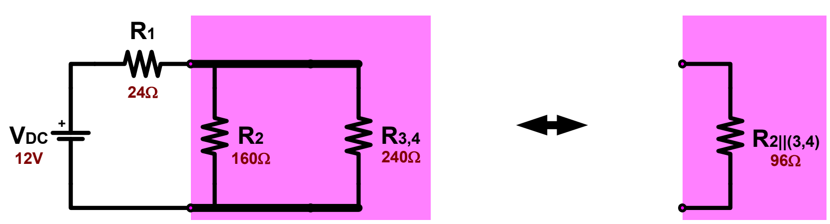
19

Reduce & Return Example

Step 2 – Repeat the Reduction Process …

Thus, a **parallel-equivalent resistance** $R_{2\|(3,4)}$ can be used to replace the R_2 and $R_{3,4}$ combination, where:

$$R_{2\|(3,4)} = \left(\frac{1}{R_2} + \frac{1}{R_{3,4}}\right)^{-1} = \left(\frac{1}{160} + \frac{1}{240}\right)^{-1} = 96\Omega$$



Circuit #2

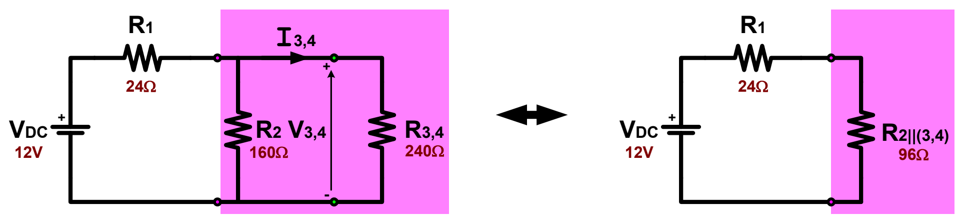
20

Reduce & Return Example

Step 2 – Repeat the Reduction Process …

The newly reduced circuit appears as shown below:

Once again, the circuit should be **analyzed** in order to **identify** any **new variables** that may be required during the completion of the overall problem.



Circuit #2 Circuit #3

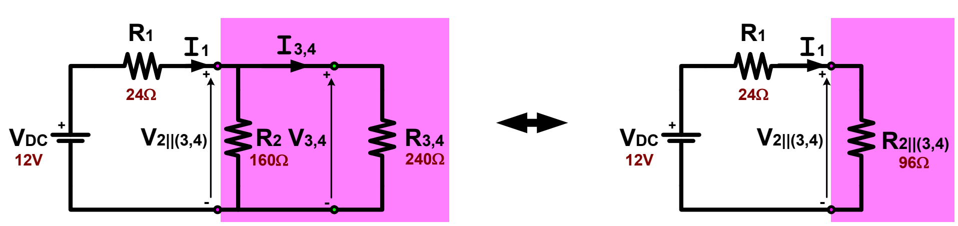
21

Reduce & Return Example

Step 2 – Repeat the Reduction Process …

A **voltage** $V_{2||(3,4)}$ across the parallel-equivalent resistance and a **current** I_1 flowing through the parallel-equivalent resistance can be defined in both versions of the circuit.

Although $V_{3,4}$ and $I_{3,4}$ do not exist in circuit #3, if either $V_{2||(3,4)}$ or I_1 can be determined in circuit #3, then the result can be used to solve for $V_{3,4}$ or $I_{3,4}$ in circuit #2.



Circuit #2 Circuit #3

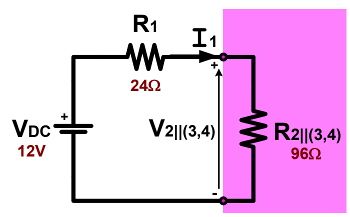
22

Reduce & Return Example

Step 2 – Repeat the Reduction Process …

Since the circuit has now been reduced down to a **trivial circuit** containing only a single pair of series connected resistors, there is no need to further reduce the circuit.

Thus, the **reduction part** of this problem **is complete**.



Circuit #3

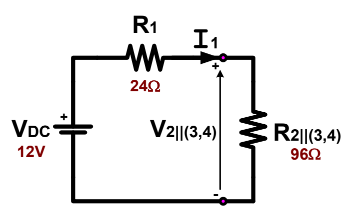
23

Reduce & Return Example

Step 3 – Analyze the Trivial Circuit (#3)

All that remains is a simple circuit (#3) consisting of two series-connected resistors.

We can now determine the unknown parameters I_1 and $V_{2||(3,4)}$, and then begin working backwards through the reduced circuits in order to find the original desired parameter V_4.



Circuit #3

24

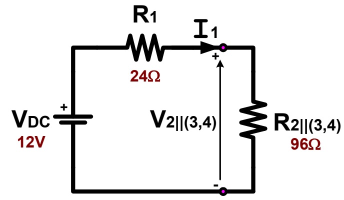
Reduce & Return Example

Step 3 – Analyze the Trivial Circuit (#3)

Solving for I_1 and $V_{2||(3,4)}$:

$$I_1 = \frac{V_{DC}}{R_1 + R_{2||(3,4)}} = \frac{12}{24+96} = 0.1 \text{ amps}$$

$$V_{2||(3,4)} = V_{DC} \cdot \frac{R_{2||(3,4)}}{R_1 + R_{2||(3,4)}} = 12 \cdot \frac{96}{24+96} = 9.6 \text{ volts}$$

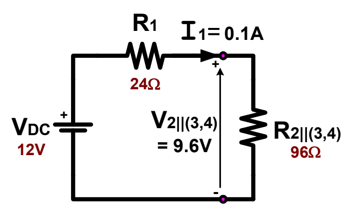


Circuit #3

Reduce & Return Example

Step 3 – Analyze the Trivial Circuit (#3)

Note – Although we found both I_1 and $V_{2||(3,4)}$, only one of those parameters is actually needed to complete this problem.



Circuit #3

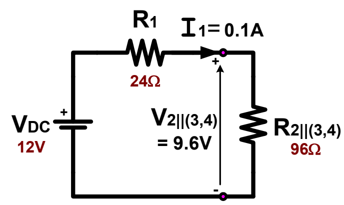
Reduce & Return Example

Step 3 – Analyze the Trivial Circuit (#3)

We will use the value of $V_{2||(3,4)}$ to complete the problem.

$$V_{2||(3,4)} = V_{DC} \cdot \frac{R_{2||(3,4)}}{R_1 + R_{2||(3,4)}} = 12 \cdot \frac{96}{24+96} = 9.6 \text{ volts}$$

Note - for practice, you may want to rework the remainder of this problem by applying the value for I_1 to a current-divider based solution for this problem.



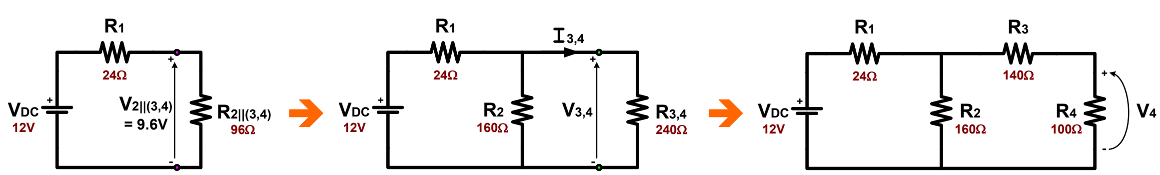
Circuit #3

27

Reduce & Return Example

Step 4 – Return to the Previously Reduced Circuits

Utilize the results obtained from the final circuit (#3) by applying them incrementally to the more-complex circuits (**#3→#2→#1**), solving for any voltages and/or currents necessary to allow for the solution of the desired quantity, V_4.



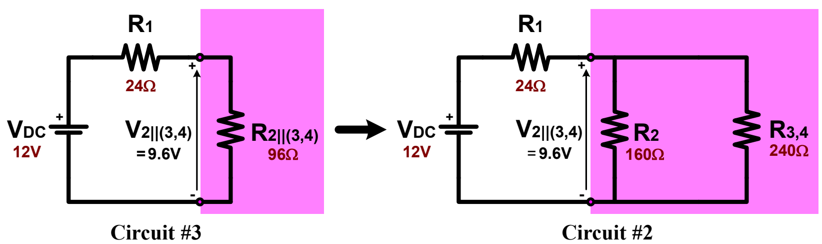
Circuit #3 Circuit #2 Circuit #1

28

Reduce & Return Example

Step 4 – Return to the Previously Reduced Circuits (Circuit #2)

As stated earlier, $V_{2||(3,4)}$ in the final circuit (**#3**) is equivalent to $V_{2||(3,4)}$ as defined in the incrementally more complex circuit **#2**.



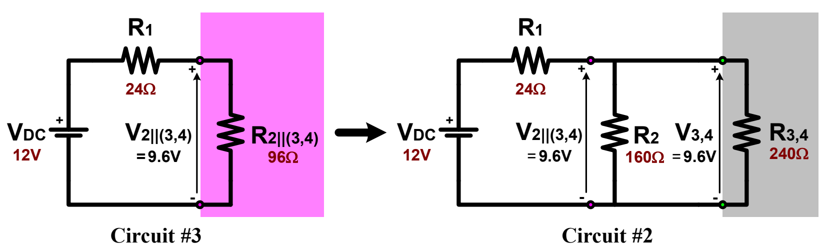
29

Reduce & Return Example

Step 4 – Return to the Previously Reduced Circuits (Circuit #2)

Since resistor R_2 is connected in parallel with $R_{3,4}$ in circuit **#2**, $V_{2||(3,4)}$ is equivalent to $V_{3,4}$, as also defined in that circuit.

And, since we now know $V_{3,4}$ in circuit **#2**, we can return to the original circuit (**#1**) and us $V_{3,4}$ in order to solve V_4.



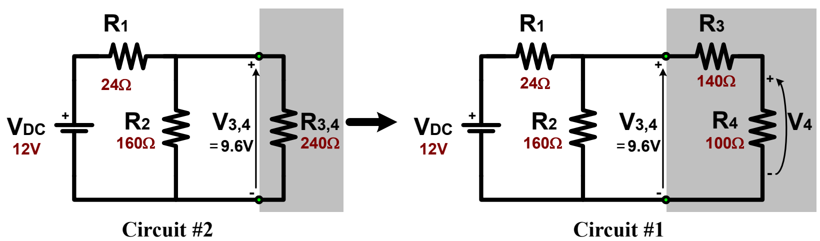
30

Reduce & Return Example

Step 4 – Return to the Previously Reduced Circuits (Circuit #1)

Thus, apply the solution from circuit **#2** to the original circuit (**#1**) and then use the result to solve for the voltage V_4.

We can do this because $V_{3,4}$ in circuit **#2** is equivalent to $V_{3,4}$ in the original circuit (**#1**), which is also the total voltage across the series-connected resistors R_3 and R_4.



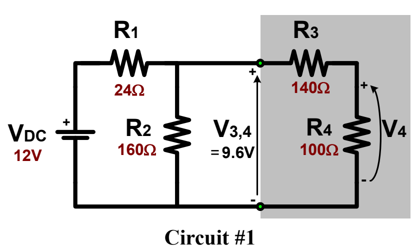
31

Reduce & Return Example

Step 4 – Return to the Previously Reduced Circuits (Circuit #1)

And so now that $V_{3,4}$ is known, V_4 can be determined simply by using a voltage divider equation as follows:

$$V_4 = V_{3,4} \cdot \frac{R_4}{R_3 + R_4} = (9.6) \cdot \frac{100}{140 + 100} = \boxed{4 \text{ volts}}$$



32