ECET 3000

Electrical Principles

DC Electric Circuits

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This Presentation

The intent of this presentation is to introduce the audience to the concept of **electric circuit theory** in order to provide a foundation of knowledge, upon which can be built a greater understanding of a large variety of topics, both practical and theoretical, all of which are associated with electrical systems and/or electrical engineering.

– It is assumed that the audience has either already "attended", or has prior knowledge of the concepts and terminology contained within, the introductory presentation for ECET 3000 - Electrical Principles.

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Electric Circuit Theory

But what is **electric circuit theory**?

Electric circuit theory is the theory that has been developed in order to facilitate the **analysis and design of electric circuits**.

Simply stated, an **electric circuit** is an interconnection of electric elements (devices) that provides one or more paths for current flow.

And given an electric circuit, there are two primary quantities that are associated with each element; **voltage** and **current**.

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Circuit Analysis & Circuit Design

Circuit analysis involves the determination of voltage and current values associated with each of the individual circuit elements.

On the other hand, **circuit design** involves the design of an electric circuit, in which the operation of one or more of the individual elements adheres to a predefined criteria that can often be specified in terms of either their associated voltage or current.

Since the target audience is students whose primary field of study is something other than electrical engineering, this presentation will primarily **focus** on the **circuit analysis** aspect of electric circuit theory as it applies specifically to DC electric circuits.

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DC Electric Circuits

The Basic Concepts and Components

5

Current

Current is a measure of the rate at which positive charge flows through a circuit element or along a conductive path.

The standard unit for current is an **ampere** (**amp**), which equates to one coulomb of charge crossing a surface per second.

$$\text{Amps} \equiv \frac{\text{coulombs of charge}}{\text{second}} \qquad 1\,\text{Coulomb} = 6.242x10^{18}$$

Although it is **negative charge** (electrons) **that actually flows** in a circuit, the **traditional circuit theory** concept of **current** was developed with the assumption of **positive charge flow**.

Thus, it is important to remember that the **electrons actually flow** in the **opposite direction** compared to that defined for **current**.

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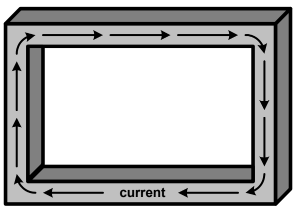
Electric Circuits

An **electric circuit** is an interconnection of electric elements that, when connected together, provide one or more **closed-loop conductive paths** around which current (charge) may flow.



closed-loop current path

A **conductive path** is composed of conductors, which are materials through which charge can easily flow.

Some materials, such as copper, are considered to be "**good conductors**" because they provide little opposition to the flow of charge.

Other materials, such as plastic, are considered to be "**insulators**" because they prevent the flow of charge.

Steady-state current can **only** flow in "closed-loops".

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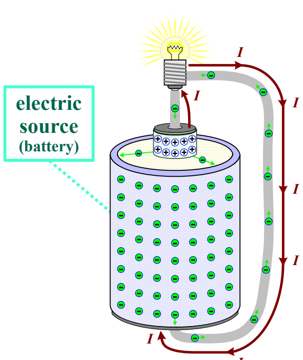
Electric Circuits – Electric Sources

The interconnected "elements" that form an electric circuit, which do not include the **wires** (conductors) that are used as the connecting media, consist of devices that are typically characterized as being either an **electric source** or an **electric load**.



An **electric source** is a device that develops a **voltage** (potential difference) between its terminals, in-turn providing a force that tries to induce the external flow of current from its positive to its negative terminal.

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Voltage

A **voltage** is an electromotive force or a potential difference, and is expressed in the standard unit of **volts**, such that:

$$\text{Volts} \equiv \frac{\text{joules}}{\text{coulomb}}$$

Thus, a:

1 volt potential difference exists between points **a** and **b** if

1 joule of energy is required to move

1 coulomb of charge from point **b** → point **a**.

Note that the amount of energy required to move a unit of charge from point **b** → point **a** is **path independent**.

The voltage produced by a source is often referred to as an **electro-motive force** (**emf**) because it provides a **force** that attempts to **move electrons** (i.e. – create current).

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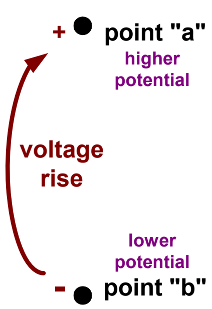
Voltage

With respect to electric circuits, **voltage** is simply the measure of the potential force developed by any circuit element that either "tries to create" or "opposes" the flow of current.



The **voltage** (potential difference) between two points is often labeled using a "+" and a "-" to denote the points of higher and lower potential respectively.

An **arrow** is also often used to show the direction of potential increase ("voltage rise") across a circuit element, which corresponds to the direction of the force developed by the element upon the current that is flowing through of current.

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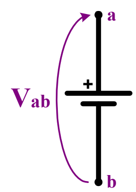
Ideal Voltage Sources

An **ideal voltage source** is a device that maintains a constant voltage potential across its terminals independent of the amount of current that is flowing out of (or through) the source.

One of the standard symbols for an **ideal voltage source** is:



where: **V_{ab}** is the **voltage** developed by the source.

Note that **V_{ab}** is defined as **difference** in potential between points **a** and **b**, or simply the increase in potential (voltage-rise) from **b → a**.

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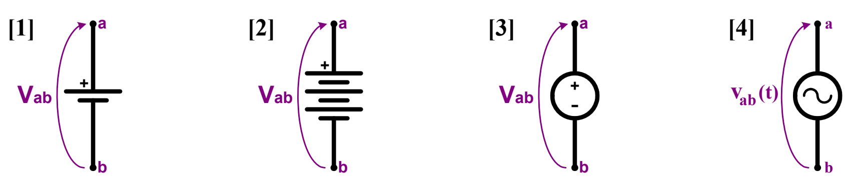
Ideal Voltage Sources

Note that a variety of **symbols** can be used for **voltage sources**:



[1] – The standard symbol used to denote a **single-cell battery**.

[2] – The standard symbol used to denote a **multi-cell battery**, such that the number of = displayed in the symbol often relates to the number of cells combined together to form the overall battery.

[3] – The standard symbol used to denote an **electronic DC source**.

[4] – The standard symbol used to denote an **AC source**.

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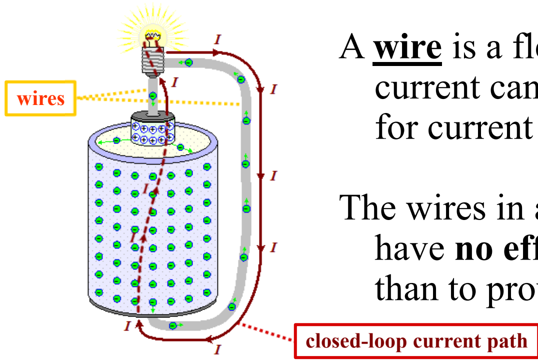
Electric Circuits – Wires

The interconnected "elements" that form an electric circuit, which do <u>not</u> include the **wires** (conductors) that are used as the connecting media, consist of devices that are typically characterized as being either an **electric source** or an **electric load**.



A **<u>wire</u>** is a flexible strand of metal through which current can "easily" flow, thus providing a path for current to flow from one element to another.

The wires in a circuit are typically assumed to have **no effect** on the circuit's operation other than to provide the closed-loop current paths.

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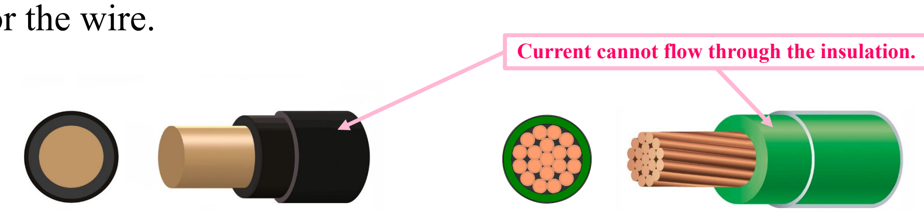
Wires and Conductors

A **wire** is a single, solid, cylindrical, flexible strand of metal through which charge (current) can "easily" flow.

Note – the term **wire** may also refer to a bundle of individual strands that are wrapped together to form a single entity ("**stranded wire**").

A **conductor** is a wire that is encased in one or more layers of **insulation** that provide electrical isolation and physical protection for the wire.



Conductor Constructed with Solid Wire　　Conductor Constructed with Stranded Wire

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Ideal Wires

An **ideal wire** is a theoretical wire that provides no opposition to the flow of current.

Ideal wires have **no effect** on the operation of an electric circuit beyond the fact that they provide the paths for current to flow between the various components that compose an electric circuit.

Although some energy is required to push current through a **practical wire**, the amount of energy is typically negligible compared to that required to push that current through the loads contained within an electric circuit.

Because of this, circuit **wires are typically considered to be ideal** unless they are either very long or insufficiently-sized for the amount of current that they are expected to carry.

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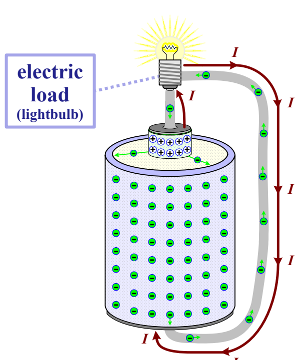
Electric Circuits – Electric Loads

The interconnected "elements" that form an electric circuit, which do not include the **wires** (conductors) that are used as the connecting media, consist of devices that are typically characterized as being either an **electric source** or an **electric load**.



An **electric load** is a device that opposes the flow of current, such that it develops a voltage across its terminals, the polarity of which is opposite to the direction of the current.

The electric **energy** required to push the current through a load is converted into another form of energy, such as heat, light, or motion.

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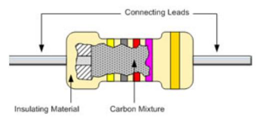
Electric Loads – Resistors

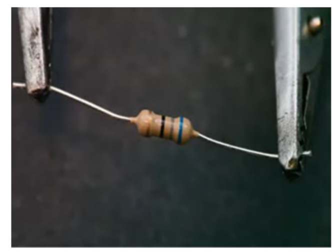
The most common load used in electric circuits is a **resistor**.



http://bmet.wikia.com/wiki/Resistor_Design

A **resistor** develops a voltage that **opposes the flow of current**, the magnitude of which is **linearly proportional** to the amount of **current** that is flowing through the resistor.

The electric energy required to force current through a resistor, in direct opposition to the voltage (force) developed by the resistor, is converted to **heat**.


https://www.youtube.com/watch?v=MUfMPjMAwx8

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Resistors

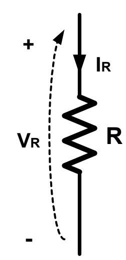
The ability of the resistor to oppose the flow of current is characterized by its **resistance**, ***R***.

Resistance is defined in terms of the voltage developed by the resistor per unit of current flowing through the resistor:

$$R = \frac{V_R}{I_R}\ (\Omega)$$

where: V_R is the **voltage** provided by the resistor, and
I_R is the **current** flowing through the resistor,

the standard unit of which is Ohms (Ω). $\text{Ohms} = \frac{\text{Volts}}{\text{Amps}}$



standard symbol for a resistor

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Ohm's Law

The linear relationship between the voltage, $\boldsymbol{V_R}$, developed by a resistor and the current, $\boldsymbol{I_R}$, that flows through the resistor is referred to as **Ohm's Law**.

This relationship is typically expressed as:

$$V_R = I_R \cdot R$$

Many of the initial circuit theorems that we will utilize are based upon this simple relationship.

> Note that, based on this relationship, a resistor only develops a voltage **when** there is current flowing through the resistor.
>
> But, this should make sense, because the voltage developed by the resistor is in direct response to the flow of current.

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Resistors

Resistors are often labeled with **colored-bands** that are used to determine their **resistance value**.

FOR EXAMPLE: Yellow – Violet – Red – Gold

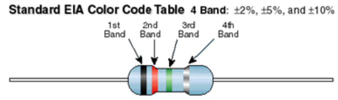
4   **7**   x10 **2**   ±   **5**   %

Yellow – Violet – Red – Gold

= **47**x10**2** ± **5**%

= **4700 Ω** ± **235 Ω**

Standard EIA Color Code Table 4 Band: ±2%, ±5%, and ±10%



Color	1st Band (1st figure)	2nd Band (2nd figure)	3rd Band (multiplier)	4th Band (tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	±2%
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Gray	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	±5%
Silver			10^{-2}	±10%

Chart Provided By XICON

Note that, along with their resistance value, resistors are also assigned a **power rating** that defines the maximum rate at which they can convert electrical energy into heat without damage.

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DC Electric Circuits

Operational Characteristics
of a
Simple Circuit

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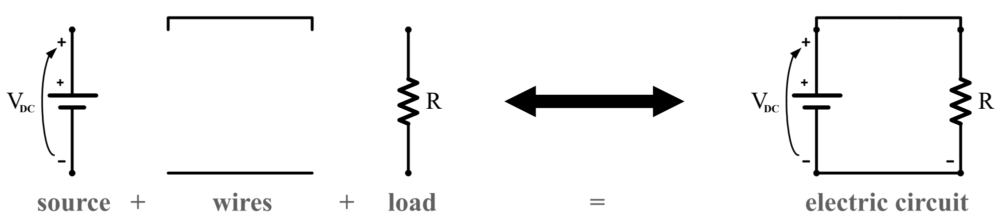
A Simple Electric Circuit

A **simple electric circuit** can be constructed by using two wires to connect an electric source to an electric load:



In this case, the **source is a battery** and the **load is a resistor**.

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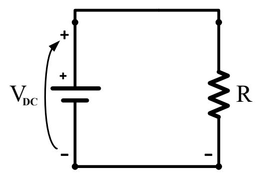
Expected Operation



What is the **expected operation** of the circuit?

1) The voltage (force) produced by the battery (source) will try to push **current** out of the positive terminal.
2) Since current can only flow in a closed-loop path, the **current** that exits the battery will flow clockwise around the only path provided by the circuit, which means the battery current will flow down through the resistor.
3) When the current flows through the resistor, the resistor will develop a **voltage** that opposes the flow of current.

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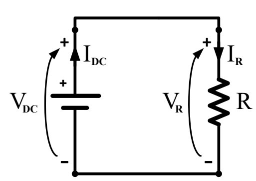
Initially-Defined Voltages and Currents



Based on those expectations:

1) A **current** was added to the circuit that flows up through the battery and out of it's positive terminal.
2) And since that current can only flow clockwise around the circuit, a **current** was also added that flows down through the resistor. A – b
3) Furthermore, since a resistor develops a voltage that opposes the flow of current, a **voltage** was defined (– → +) upward across the resistor.

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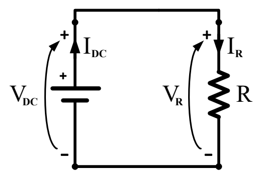
A “Non-Technical” Approach?

We’ll get technical later, but for now, let’s approach this circuit in an intuitive manner by borrowing two simple physics concepts.



It turns out that, analysis of the circuit’s operation only requires the application of two simple concepts and “the law”:

1) When there are **two opposing forces**, the biggest one wins!

2) **Steady-state operation** occurs when equilibrium is reached. (I.e. – when the “forces” are balanced)

3) **Ohm’s Law always holds true**.

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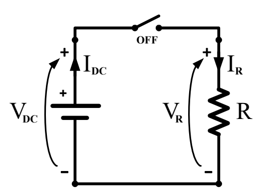
Analysis of the Circuit’s Operation

A switch in the “OFF” position provides a barrier that prevents current from flowing through the switch despite the source voltage.



A switch in the “ON” position acts like an ideal wire, allowing current to flow freely through the switch.

To help facilitate the analysis of the circuit’s operation, a simple **ON/OFF switch** has been added to the circuit.

If the switch is in the “**OFF**” position, it “breaks” the closed-loop current path, in-turn preventing current flow in the circuit.

But, when flipped to the “**ON**” position, the switch “closes” the break in the circuit, thus allowing the flow of current.

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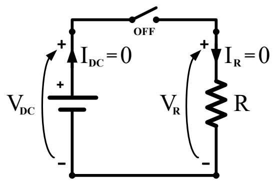
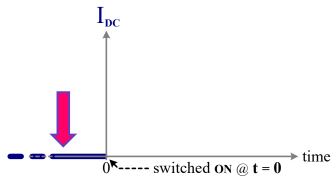
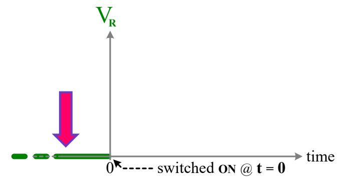
Analysis of the Circuit's Operation







Before the circuit is "switched **ON**" (at time **t = 0**):

There is **no current** flowing in the circuit since the switch is **OFF**.

$$I_{DC} = I_R = 0 \text{ amps}$$

"Initial Conditions"

And since the voltage across a resistor is proportional to the current flowing through the resistor, as defined by Ohm's Law:

$$V_R = I_R \cdot R = 0 \text{ volts}$$

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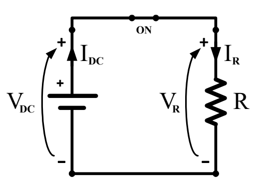
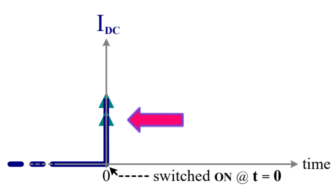
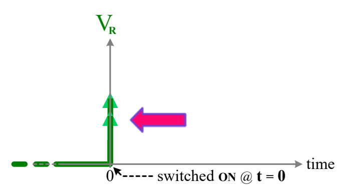
Analysis of the Circuit's Operation







The **instant** that the circuit is "switched **ON**" (at time **t = 0**):

Since the switch no longer provides a barrier that prevents the flow of current, the forces (voltages) provided by the other elements must be considered.

1) The battery provides a **constant voltage**, $\mathbf{V_{DC}}$, the tries to push current out of its positive terminal and clockwise around the closed-loop path.

2) Since the resistor voltage is initially zero, the resistor provides **no initial opposition** to the flow of current, so **current quickly increases** in magnitude, in-turn causing the resistor to develop a voltage... (continued)

The biggest force wins!

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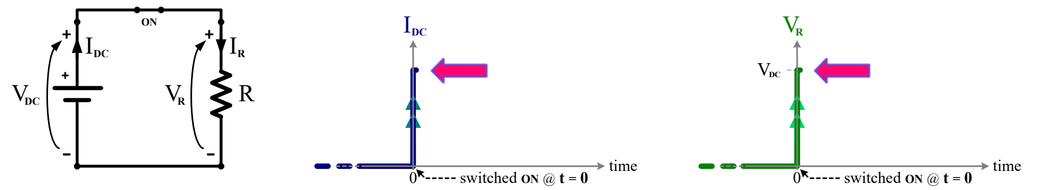
Analysis of the Circuit's Operation



The **instant** that the circuit is "switched **ON**" (at time $\mathbf{t = 0}$):

Since the switch no longer provides a barrier that prevents the flow of current:

3) Now that the resistor develops a voltage, it begins to oppose the flow of current. But as long as the battery provides a larger voltage that tries to induce the flow of current, the current magnitude will keep increasing.

4) **Current increases until the resistor's voltage is equal in magnitude**, but **opposite in direction**, compared to the battery voltage.

Equilibrium is reached (i.e. – steady-state conditions).

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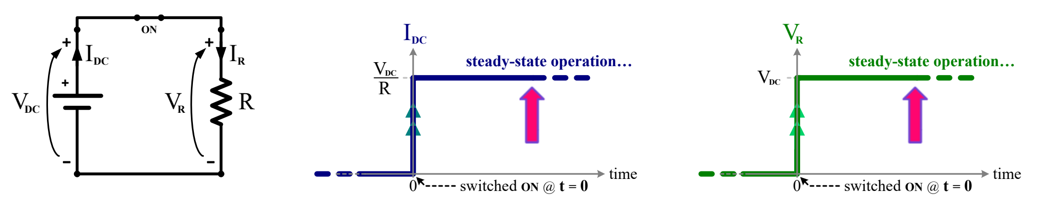
Analysis of the Circuit's Operation



After the circuit is "switched **ON**" (time $\mathbf{t > 0}$):

Now that equilibrium has been reached, such that the resistor voltage equals the battery voltage ($V_R = V_{DC}$), the current cannot increase any further.

Thus, the current levels-off and remains constant at the magnitude required for the resistor to develop a voltage equal to that of the battery:

$$I_{DC} = I_R = \frac{V_R}{R} = \frac{V_{DC}}{R} \text{ amps}$$

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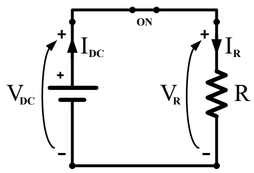
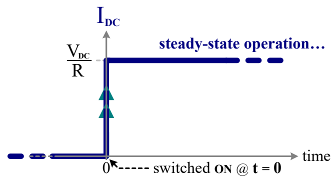
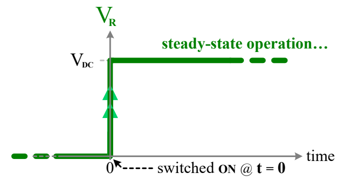
Analysis of the Circuit's Operation







Note that, although this process we described in a step-by-step manner after the circuit was switched "**ON**", the whole series of events actually occurs instantaneously for a circuit that contains an ideal voltage source and a resistor.

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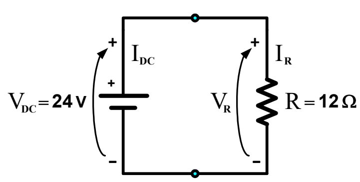
Basic Electric Circuit Example

Given the following **steady-state DC circuit** that contains a 24 V_{DC} source the is connected to a 12 Ω resistor:



determine all of the unknown voltages and currents that are shown in the figure.

Note that I_R must equal I_{DC} since there is only one closed-loop path available for current to flow in the circuit.

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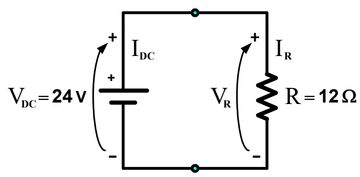
Basic Electric Circuit Example

Given the following **steady-state DC circuit** that contains a 24 V_{DC} source the is connected to a 12 Ω resistor:



Steady-state operation occurs when: $V_R = V_{DC} = 24\text{ V}$

and the **current** required for this, as defined by **Ohm's Law**, is:

$$I_{DC} = I_R = \frac{V_R}{R} = \frac{24\ V}{12\ \Omega} = 2\text{ A}$$

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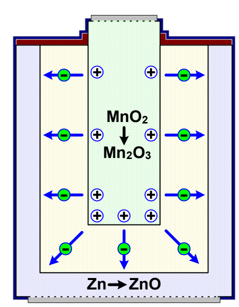
Energy

Energy, which can be defined as the ability of a system to perform work, is a property of an object that can be transferred to another object or converted into a different form, but can neither be created nor destroyed.

For example: The energy that is released during the chemical reaction that occurs in a battery is transferred to the electrons that are deposited on the anode, in-turn providing them with the potential to flow externally from the anode back to the cathode.

The potential energy transferred to the electrons is referred to as "**Electric Energy**"



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Power

Power is defined as the rate at which work is performed or the rate at which energy is converted from one form to another form.

FOR EXAMPLE – Power is the rate at which:

Energy Stored in Chemical Bonds ➞ Electric Energy
(battery)

Electric Energy ➞ Heat
(resistor)

Thus, during the steady-state operation of a system, the amount of work performed (energy converted) equals:

$$\boldsymbol{Energy = Power \cdot Time}$$

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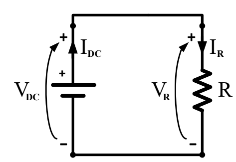
Electric Power – Sources

When a battery (electric source) is connected to a resistor, the battery provides a force that pushes current through the resistor.



But, since a resistor opposes the flow of current, it requires energy to move the current against this oppositional force.

The rate at which the battery produces this electric energy is referred to as Electric Power.

For a DC voltage source, the rate, P_{source}, at which the source produces electric energy (**electric power**) is:

$$P_{source} = V_{source} \cdot I_{source} \ (Watts)$$

The standard units for power are Watts (W):
$Watts = \text{Volts} \cdot \text{Amps}$

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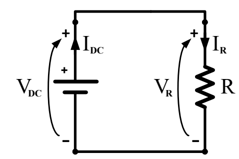
Electric Power – Loads

As current is pushed through the resistor, the electric energy required to overcome the resistor's oppositional force is converted to heat.



The rate, $\boldsymbol{P_R}$, at which the resistor converts the electric energy to heat, is:

$$P_R = V_R \cdot I_R \ (Watts)$$

Note that, when a resistor converts electric energy to heat, this process is often casually stated as:

"the resistor consumes electric energy"
or
"the resistor consumes electric power"

Power is often casually referred to as either being produced or consumed.

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Electric Power

In the previous circuit, the power "**consumed**" by the resistor:

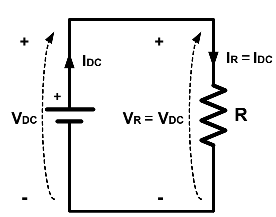
$$P_R = V_R \cdot I_R \ (Watts)$$

must equal to the power "**produced**" by the source:

$$P_{source} = V_{DC} \cdot I_{DC} \ (Watts)$$

in order to maintain an energy balance in the system.

The total energy contained in a closed system must be constant. Thus, energy is neither created nor destroyed in a closed system. (I.e. – energy can only be converted from one form to another)



The electric source only produces the amount of electric energy that is required to force the current through the resistor.

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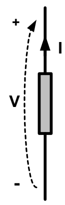
Energy Production/Consumption

In terms of electric circuits:

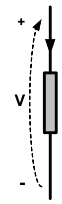
A circuit element "**produces**" electric energy (power) if the voltage rise across the element is in the **same direction** as the current flowing through the element.

A circuit element "**consumes**" electric energy (power) if the voltage rise across the element is in the **opposite direction** compared to the current flowing through the element.

Circuit element produces electric energy/power

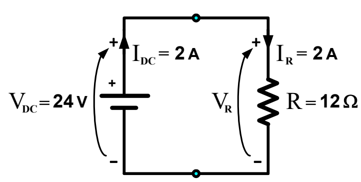


Circuit element consumes electric energy/power



Basic Electric Circuit Example (Revisited)

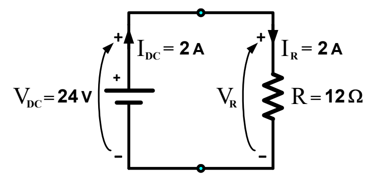
Given the following **steady-state DC circuit** that contains a 24 V_{DC} source the is connected to a 12 Ω resistor:



determine the values of the power "produced" by the source and the power "consumed" by the load.

Basic Electric Circuit Example (Revisited)

Given the following **steady-state DC circuit** that contains a 24 V_{DC} source the is connected to a 12 Ω resistor:



Since:

$$P = V \cdot I$$

the source and load powers are:

$$P_{source} = V_{DC} \cdot I_{DC} = 24\text{V} \cdot 2\text{A} = 48\ \text{W}$$

$$P_{Load} = V_R \cdot I_R = 24\text{V} \cdot 2\text{A} = 48\ \text{W}$$

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Electric Power & Energy

The standard units for electric power is **watts**, such that:

$$1\ \text{Watt} = 1\ \text{Volt} \cdot 1\ \text{Amp} = 1 \frac{\text{Joule}}{\text{Second}}$$

Because a joule is a tiny amount of energy, electric energy is often specified in units of **kilowatt·hours** (kWh), such that:

$$1\ \text{kWh} \equiv \text{power consumed at a rate of 1000 W for 1 hour}$$

Note that: $1\ \text{kWh} = 1{,}000\ \text{W} \cdot 1\ \text{hour} = 1{,}000\ \text{W} \cdot 3{,}600\ \text{sec} = 3{,}600{,}000\ \text{J}$

Electric utility companies (such as Georgia Power) typically bill their customers based on electric energy consumption in units of kWh.

Although prices may vary, a value of $0.10/kWh is often used for cost estimation.

For example – if a 60W light bulb is used 8 hours/day, 365 days/year, the cost will be:

0.060kW · 8 hours/day · 365 days/year · $0.10/kWh = $17.52/year

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DC Electric Circuits

Series & Parallel Connected Resistors

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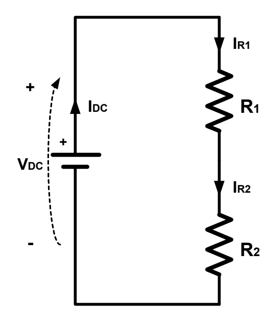
Series-Connected Resistors

Two (or more) circuit elements are connected in **Series** if the current that flows through one of the elements must entirely flow through the other element(s).

Thus, the two resistors shown in the circuit to the right are "**in-series**" with each other since the current flowing through resistor R_1 must also flow through resistor R_2.

$$I_{R1} = I_{R2}$$



Additionally, a similar analysis will show that the resistors are also connected "in-series" with the voltage source since:

$$I_{DC} = I_{R1} = I_{R2}$$

44

Series-Connected Resistors

In terms of the individual resistors, the voltages across the resistors will adhere to Ohm's Law:

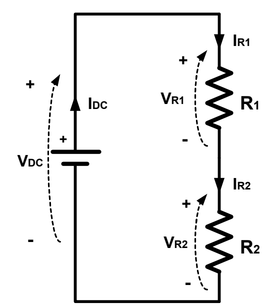
$$V_{R1} = I_{R1} \cdot R_1 \qquad V_{R2} = I_{R2} \cdot R_2$$

Since the current flowing through the resistors is equal to the source current:

$$I_{DC} = I_{R1} = I_{R2}$$

the resistor voltages can be rewritten as:

$$V_{R1} = I_{DC} \cdot R_1 \qquad V_{R2} = I_{DC} \cdot R_2$$



45

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (**KVL**) is a force-balance equation that states:

"The **sum** of the '**voltage rises**' must **equal** to the **sum** of the '**voltage drops**' defined in a continuous direction **around any closed-loop path** in an electric circuit."

$$\sum V_{rises} = \sum V_{drops}$$

Note that the concept of either a **voltage rise** or a **voltage drop** relates to the actual change in the voltage potential across a circuit element in an arbitrary "direction of travel".

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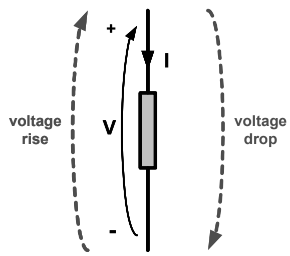
Voltage Rise vs. Voltage Drop

Given a resistor through which current is flowing (as shown below), the increase in voltage potential across that resistor is in the "upward" direction.

If considering the change in potential **upward** across the resistor, then it could be defined as a **voltage rise** since the potential increases in the upward direction.



But, if considering the change in potential **downward** across the resistor, then it could be defined as a **voltage drop** since the potential decreases in the downward direction.

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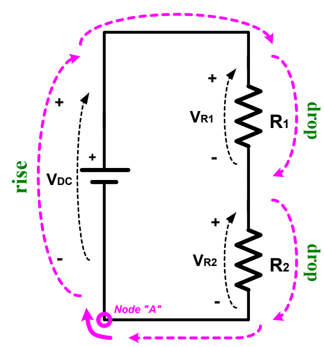
KVL & Series-Connected Resistors

There is only one closed-loop path around which current can flow in the given series circuit.

If **Kirchhoff's Voltage Law** is applied to the series circuit such that the voltages are summed in a **CW direction** around the closed-loop path beginning at node *"A"*, the following voltage relationship can be defined:

$$V_{DC} = V_{R1} + V_{R2}$$



For steady-state operation to occur, the total force (voltage) provided by the source that tries to push current around the closed-loop path must equal to the sum the forces (voltages) developed by the resistors that oppose the flow of current.

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Series-Connected Resistors

Based on the KVL equation:

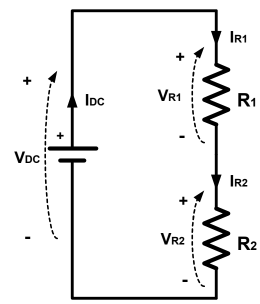
$$V_{DC} = V_{R1} + V_{R2}$$

along with the Ohm's Law equations:

$$V_{R1} = I_{DC} \cdot R_1 \qquad V_{R2} = I_{DC} \cdot R_2$$

for the series circuit, the relationship between the source voltage and current can be defined by:

$$\begin{aligned} V_{DC} &= V_{R1} + V_{R2} \\ &= I_{DC} \cdot R_1 + I_{DC} \cdot R_2 \\ &= I_{DC} \cdot \left(R_1 + R_2\right) \end{aligned}$$



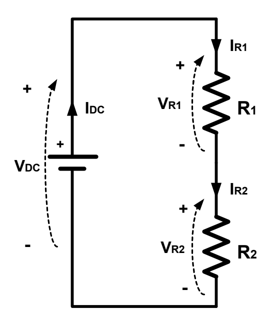
49

Series-Connected Resistors

Thus, given a circuit containing two series-connected resistors, the current that will flow in the circuit is:

$$I_{DC} = \frac{V_{DC}}{\left(R_1 + R_2\right)}$$

It can be seen from this expression that the **total "resistance"** provided by the series-connected resistors is equal to the sum of the individual resistances.



This result can be expanded for a set of *"N"* series-connected resistors, such that:

$$R_{series(total)} = R_1 + R_2 + \cdots + R_N$$

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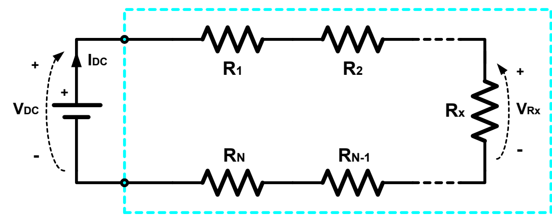
Series-Equivalent Resistance

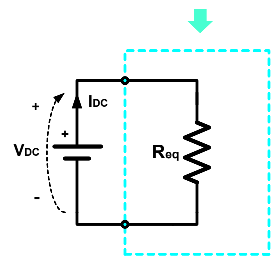
Given a circuit containing "*N*" series-connected resistors:

A single "**equivalent resistance**" can be used in place of the set of series resistors without affecting the overall operation of the source, provided that:



$$R_{eq(series)} = R_1 + R_2 + \cdots + R_N$$

I.e. – the **series-equivalent resistance** of a set of series-connected resistors is the sum of the individual resistances.



$$R_{eq(series)} = \sum_{x=1}^{N} R_x$$

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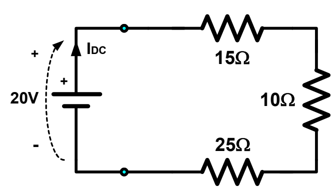
Series-Resistor Example

Given the circuit containing **three series-connected resistors**:



Determine:
- the **source current**,
- the total **electric power** produced by the source,
- the **voltage** across each resistor,
- the **electric power** consumed by each resistor, and
- the **total power** consumed by the resistors.

Also confirm:
- that **KVL** holds true based on the results.

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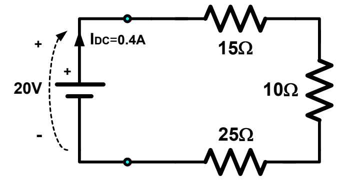
Series-Resistor Example

Given the circuit containing **three series-connected resistors:**

a) Determine the **source current**, $\boldsymbol{I_{DC}}$.



$$I_{DC} = \frac{V_{DC}}{(R_1 + R_2 + R_3)} = \frac{20\text{V}}{15\Omega + 10\Omega + 25\Omega}$$

$$= \frac{20\text{V}}{50\Omega} = 0.4\text{A}$$

b) Determine the total **electric power**, $\boldsymbol{P_{source}}$, "produced" by the source.

$$P_{source} = V_{DC} \cdot I_{DC} = (20\text{V}) \cdot (0.4\text{A}) = 8\text{W}$$

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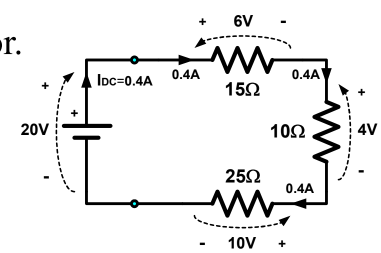
Series-Resistor Example

Given the circuit containing **three series-connected resistors:**

c) Determine the **voltage** across each resistor.



$$I_{DC} = I_{15\Omega} = I_{10\Omega} = I_{25\Omega} = 0.4\text{A}$$

$$V_{15\Omega} = I_{15\Omega} \cdot 15\Omega = (0.4\text{A}) \cdot (15\Omega) = 6\text{V}$$

$$V_{10\Omega} = (0.4\text{A}) \cdot (10\Omega) = 4\text{V}$$

$$V_{25\Omega} = (0.4\text{A}) \cdot (25\Omega) = 10\text{V}$$

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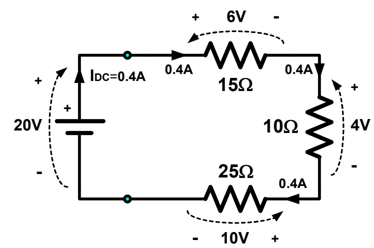
Series-Resistor Example

Given the circuit containing **three series-connected resistors:**

d) Determine the **electric power** consumed by each resistor.



$$P_{15\Omega} = V_{15\Omega} \cdot I_{15\Omega} = (6\text{V}) \cdot (0.4\text{A}) = 2.4\text{W}$$

$$P_{10\Omega} = (4\text{V}) \cdot (0.4\text{A}) = 1.6\text{W}$$

$$P_{25\Omega} = (10\text{V}) \cdot (0.4\text{A}) = 4\text{W}$$

e) Determine the **total power** consumed by the resistors.

$$P_{R(total)} = P_{15\Omega} + P_{10\Omega} + P_{25\Omega} = 2.4\text{W} + 1.6\text{W} + 4\text{W} = 8\text{W}$$

$$P_{R(total)} = 8\text{W} = P_{source} \quad \Rightarrow$$ **The total power "produced" by the source equals the total power "consumed" by the resistors**

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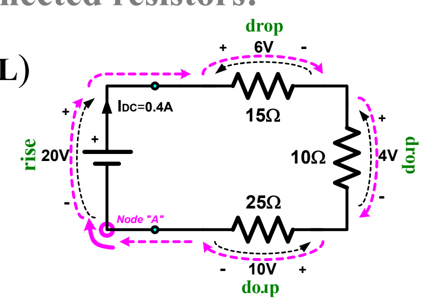
Series-Resistor Example

Given the circuit containing **three series-connected resistors:**

f) Confirm **Kirchhoff's Voltage Law** (**KVL**) using the results from this circuit.



$$\sum V_{rises} = \sum V_{drops}$$

KVL: CW Direction Summation

$$20\text{V}_{\text{rise}} = 6\text{V}_{\text{drop}} + 4\text{V}_{\text{drop}} + 10\text{V}_{\text{drop}}$$

$$= 20\text{V}_{\text{drop(total)}}$$

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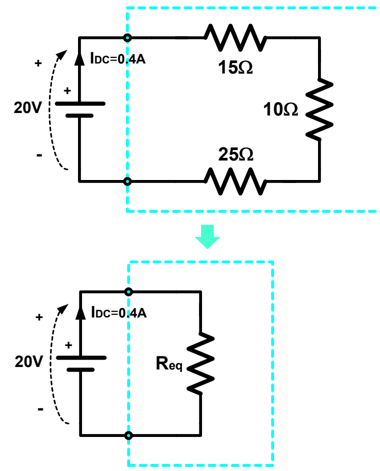
Series-Resistor Example

Given the circuit containing **three series-connected resistors:**

g) Determine the value of an **equivalent resistance** that can be used in place of the three resistors without affecting the overall operation of the source.



$$R_{eq} = \frac{V_{DC}}{I_{DC}} = \frac{20\text{V}}{0.4\text{A}} = 50\Omega$$

$$R_{eq} = 50\Omega$$
$$= 15\Omega + 10\Omega + 25\Omega$$
$$= R_{series(total)}$$

The series equivalent resistance value must be greater than the value of the largest resistor in the set of series resistors.

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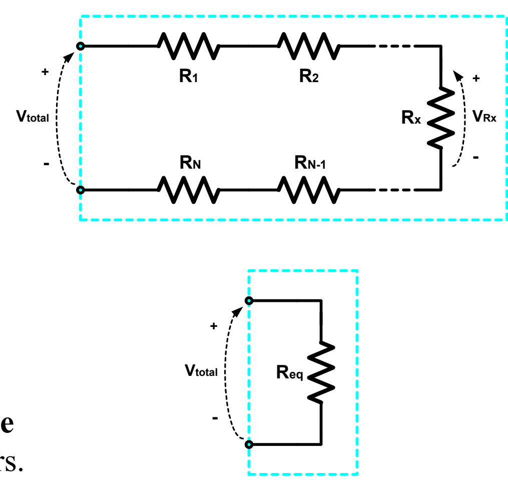
Voltage-Divider Equation

Given a set of series-connected resistors across which the total voltage, V_{total}, is known:

The voltage, V_{Rx}, across resistor R_x can be determined using the **voltage-divider** equation:

$$V_{Rx} = V_{total} \cdot \frac{R_x}{R_{eq(series)}}$$

where $R_{eq(series)}$ is the **equivalent resistance** of the set of series resistors.



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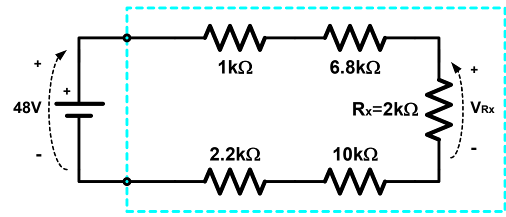
Voltage-Divider Equation Example

If a 48V source is connected across a set of series-connected resistors, as shown in the figure:



Determine the voltage, $\boldsymbol{V_{Rx}}$, across the 2kΩ resistor using the **voltage-divider** equation.

$$R_{eq(series)} = 1\text{k}\Omega + 6.8\text{k}\Omega + 2\text{k}\Omega + 10\text{k}\Omega + 2.2\text{k}\Omega = 22\text{k}\Omega$$

$$V_{Rx} = V_{total} \cdot \frac{R_x}{R_{eq(series)}} = 48 \cdot \frac{2\text{k}\Omega}{22\text{k}\Omega} = 4.36\text{V}$$

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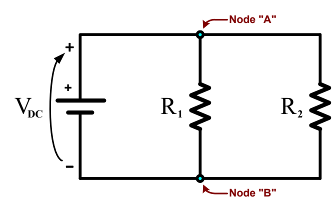
Parallel-Connected Resistors

Two (or more) circuit elements are connected in **Parallel** if they are connected across the same two nodes in the circuit.



Thus, the two resistors in the above circuit are "**in-parallel**" with both each other, and with the voltage source, since they are all connected across nodes **A** and **B**.

A <u>node</u> is a common point of connection in an electric circuit.

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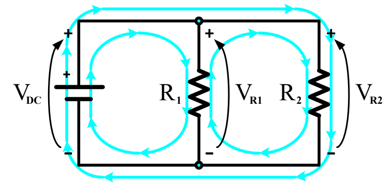
Parallel-Connected Resistors

Two (or more) circuit elements are connected in **Parallel** if they are connected across the same two nodes in the circuit.



By applying Kirchhoff's Voltage Law around each closed-loop path in the circuit, it can be proven that:

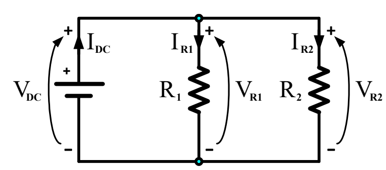
$$V_{DC} = V_{R1} = V_{R2}$$

The same voltage must exist across all parallel-connected circuit elements.

61

Parallel-Connected Resistors

In terms of the individual resistors, the **current** flowing through each resistor must adhere to Ohm's Law:



$$I_{R1} = \frac{V_{R1}}{R_1} \qquad I_{R2} = \frac{V_{R2}}{R_2}$$

Since the voltage across each resistor is equal to the source voltage:

$$V_{DC} = V_{R1} = V_{R2}$$

the resistor currents can be rewritten as:

$$I_{R1} = \frac{V_{DC}}{R_1} \qquad I_{R2} = \frac{V_{DC}}{R_2}$$

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Kirchhoff's Current Law

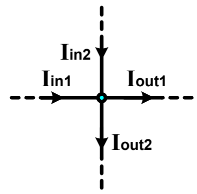
Kirchhoff's Current Law (**KCL**) is a mass-balance equation that states:

"The **sum of the currents** defined **entering a node** must **equal** to the **sum of the currents** defined **exiting that node** for any fully-defined node in an electric circuit."

where: a "**fully-defined node**" is a node for which a current is defined in each branch connected to that node.

$$\sum I_{entering} = \sum I_{exiting}$$

$$I_{in1} + I_{in2} = I_{out1} + I_{out2}$$

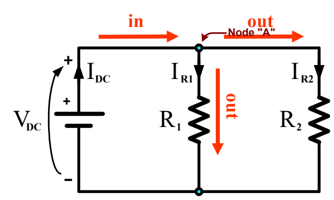


63

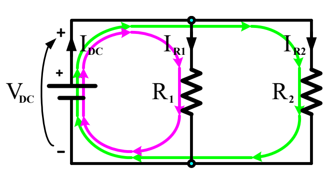
Parallel-Connected Resistors

If **Kirchhoff's Current Law** is applied to node "**A**", it can be proven that:

$$I_{DC} = I_{R1} + I_{R2}$$



I.e. – the total current produced by the source must equal to the sum of the currents flowing through the parallel resistors.



Current Loops

Note that, due to the parallel connection, there are actually two closed-loop paths around which current can flow, and the voltage provided by the source pushes current around both of those paths.

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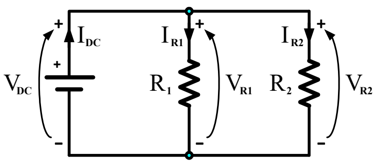
Parallel-Connected Resistors

The **KCL** and **Ohm's Law** equations:

$$I_{DC} = I_{R1} + I_{R2} \qquad I_{R1} = \frac{V_{DC}}{R_1} \qquad I_{R2} = \frac{V_{DC}}{R_2}$$



can be used to define a relationship between the source voltage and source current based on the values of the parallel resistances:

$$I_{DC} = I_{R1} + I_{R2} = \frac{V_{DC}}{R_1} + \frac{V_{DC}}{R_2}$$

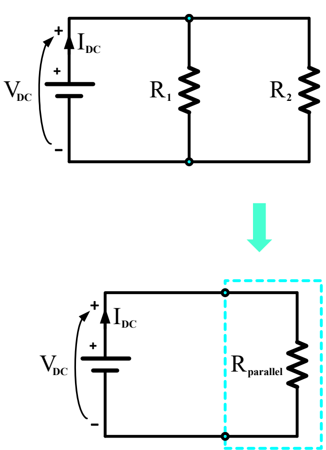
$$= V_{DC} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_{DC}}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}}$$

65

Parallel-Equivalent Resistance

Thus, given the circuit containing two **parallel-connected resistors**, the current that will flow from the source is:

$$I_{DC} = \frac{V_{DC}}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}} = \frac{V_{DC}}{R_{parallel}}$$



And, the **total resistance** provided by the parallel-connected resistors is equal to the inverse of the sum of the inverses of the two resistances:

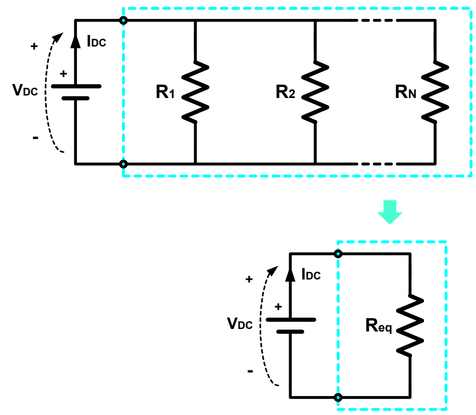
$$R_{parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

66

Parallel-Equivalent Resistance

Given a circuit containing "*N*" **parallel-connected resistors**:

A single "**equivalent resistance**" can be used in place of the set of *N* parallel resistors without affecting the overall operation of the source, provided that:

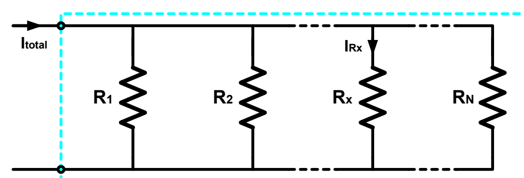


$$R_{eq(parallel)} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \right)^{-1}$$

67

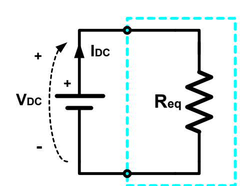
Current-Divider Equation

Given a set of parallel-connected resistors through which the total current, $\boldsymbol{I_{total}}$, is known:



The current, $\boldsymbol{I_{Rx}}$, through resistor $\boldsymbol{R_x}$ can be determined using the **current-divider** equation:

$$I_{Rx} = I_{total} \cdot \frac{R_{eq(parallel)}}{R_x}$$



where $\boldsymbol{R_{eq(parallel)}}$ is the **equivalent resistance** of the set of parallel resistors.

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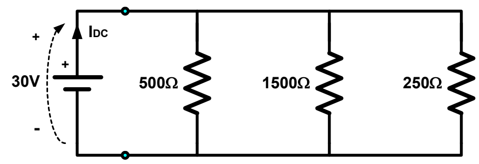
Parallel-Resistor Example

Given the circuit containing **three parallel-connected resistors**:



Determine:

- the **equivalent resistance** of the three resistors,
- the **source current**,
- the total **electric power** produced by the source,
- the **current** through each resistor,
- the **electric power** consumed by each resistor,
- the **total power** consumed by the resistors,

And confirm:

- **Kirchhoff's Current Law** using the previous results, and
- the **current-divider equation** using the 1500Ω resistor.

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Parallel-Resistor Example

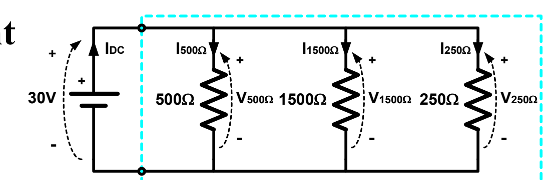
Given the circuit containing **three parallel-connected resistors**:

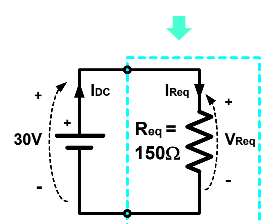
a) Determine the parallel **equivalent resistance**, $\boldsymbol{R_{eq}}$, for the three resistors:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$= \left(\frac{1}{500} + \frac{1}{1500} + \frac{1}{250} \right)^{-1}$$

$$= (0.006\overline{6})^{-1} = 150\ \Omega$$





The parallel equivalent resistance value must be <u>smaller</u> than the value of the smallest resistor in the set of parallel resistors.

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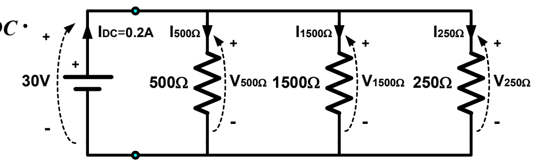
Parallel-Resistor Example

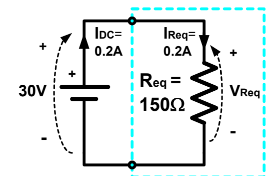
Given the circuit containing **three parallel-connected resistors**:

b) Determine the **source current**, I_{DC}.



$$I_{DC} = \frac{V_{Req}}{R_{eq}} = \frac{V_{DC}}{R_{eq}} = \frac{30\text{V}}{150\Omega} = 0.2\text{A}$$

c) Determine the total **electric power** "produced" by the source.



$$P_{source} = V_{DC} \cdot I_{DC} = (30\text{V}) \cdot (0.2\text{A}) = 6\text{W}$$

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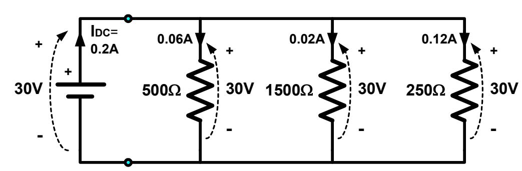
Parallel-Resistor Example

Given the circuit containing **three parallel-connected resistors**:

d) Determine the **current** through each resistor.



$$V_{DC} = V_{500\Omega} = V_{1500\Omega} = V_{250\Omega} = 30\text{V}$$

$$I_{500\Omega} = \frac{V_{500\Omega}}{500\Omega} = \frac{V_{DC}}{500\Omega} = \frac{30\text{V}}{500\Omega} = 0.06\text{A}$$

$$I_{1500\Omega} = \frac{V_{DC}}{500\Omega} = \frac{30\text{V}}{1500\Omega} = 0.02\text{A}$$

$$I_{250\Omega} = \frac{V_{DC}}{250\Omega} = \frac{30\text{V}}{250\Omega} = 0.12\text{A}$$

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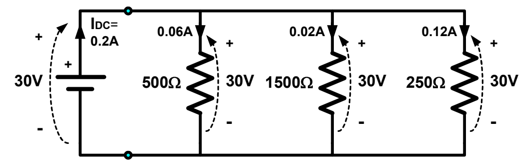
Parallel-Resistor Example

Given the circuit containing **three parallel-connected resistors**:

e) Determine the **electric power** consumed by each resistor.



$$P_{500\Omega} = V_{500\Omega} \cdot I_{500\Omega} = (30\text{V}) \cdot (0.06\text{A}) = 1.8\text{W}$$

$$P_{1500\Omega} = V_{1500\Omega} \cdot I_{1500\Omega} = (30\text{V}) \cdot (0.02\text{A}) = 0.6\text{W}$$

$$P_{250\Omega} = V_{250\Omega} \cdot I_{250\Omega} = (30\text{V}) \cdot (0.12\text{A}) = 3.6\text{W}$$

Note that, if voltage is held constant, power increases as resistance decreases.

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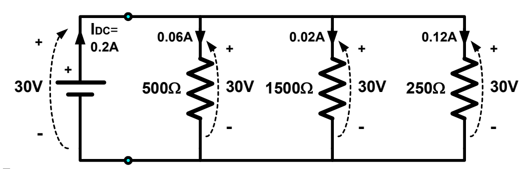
Parallel-Resistor Example

Given the circuit containing **three parallel-connected resistors**:

f) Determine the **total power** consumed by the resistors.



$$P_{R(total)} = P_{500\Omega} + P_{1500\Omega} + P_{250\Omega} = 1.8\text{W} + 0.6\text{W} + 3.6\text{W} = 6\text{W}$$

$$P_{R(total)} = 6\text{W} = P_{source}$$

The **total power "produced"** by the source equals the **total power "consumed"** by the resistors.

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Parallel-Resistor Example

Given the circuit containing **three parallel-connected resistors**:

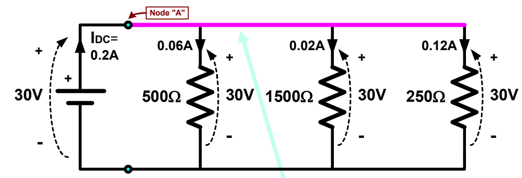
g) Confirm **Kirchhoff's Current Law (KCL)** using the results from this circuit.

$$\sum I_{entering} = \sum I_{exiting}$$

KCL: Node A

$$0.2\text{A} = 0.06\text{A} + 0.02\text{A} + 0.12\text{A}$$

$$0.2\text{A} = 0.2\text{A}$$



An **ideal wire** is equivalent to an extended **node**.

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Parallel-Resistor Example

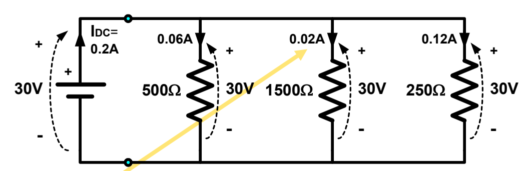
Given the circuit containing **three parallel-connected resistors**:

h) Confirm the validity of the **current-divider equation** by solving for the current in the 1500Ω resistor.

$$I_{Rx} = I_{total} \cdot \frac{R_{eq(parallel)}}{R_x}$$

$$I_{1500\Omega} = 0.2\text{A} \cdot \frac{150\Omega}{1500\Omega}$$

$$= 0.02\text{A}$$



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