ECET 2111

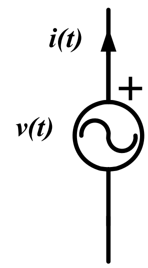
Circuits II

Exam II Review

AC Power

Power from an AC Source

In the case of an AC source where:



$$v(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

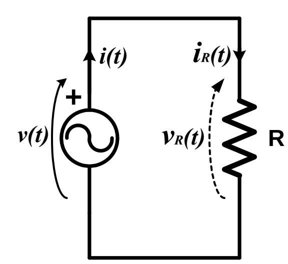
$$i(t) = I_{peak} \cdot \sin(\omega \cdot t + \delta)$$

the general expression for **power** produced by the source is:

$$p(t) = v(t) \cdot i(t)$$
$$= V_{peak} \cdot I_{peak} \cdot \sin(\omega \cdot t + \phi) \cdot \sin(\omega \cdot t + \delta)$$

AC Power and Resistors

If an AC source is connected to a **resistive load**, such that:



$$v_R(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$i_R(t) = I_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$I_{peak} = \frac{V_{peak}}{R}$$

then the **power** consumed by the resistor will be:

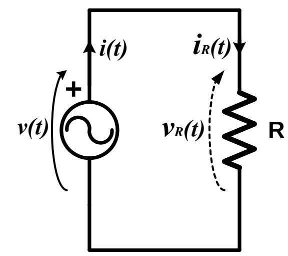
$$p_R(t) = v_R(t) \cdot i_R(t)$$
$$= V_{peak} \cdot I_{peak} \cdot \sin^2(\omega \cdot t + \phi)$$

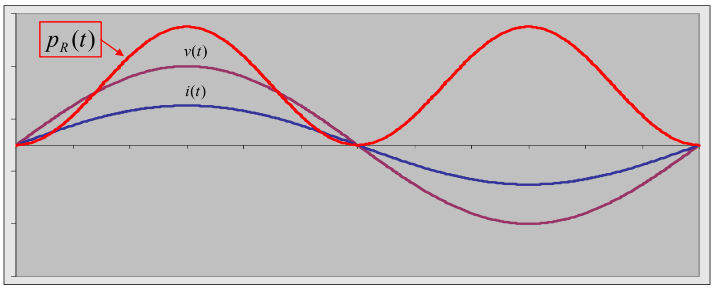
AC Power and Resistors

The figure below shows the **power waveform**:

$$p_R(t) = V_{peak} \cdot I_{peak} \cdot \sin^2(\omega \cdot t + \phi)$$

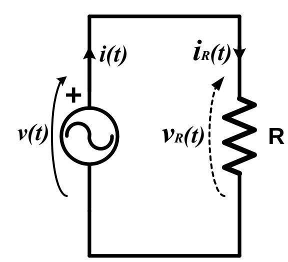
plotted along with the resistor's voltage and current waveforms:

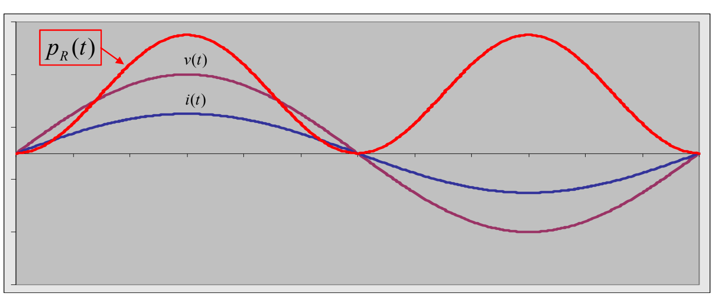




AC Power and Resistors

Additionally, it can be seen that the power waveform varies **periodically**, but with a **frequency** that is **2x larger** than that of the applied voltage or the resultant current.



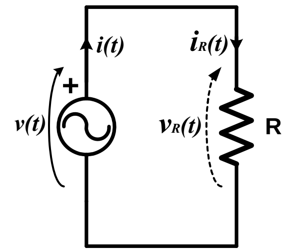


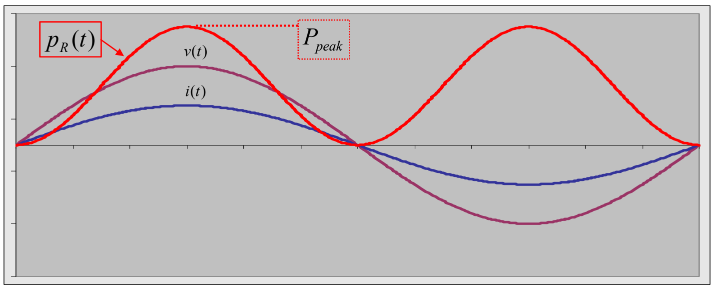
AC Power and Resistors

The peak magnitude of the **AC** power waveform is:

$$P_{peak} = V_{peak} \cdot I_{peak}$$

This should <u>not</u> be confused with the constant power provided to a resistor by a DC source.





AC Power and Resistors

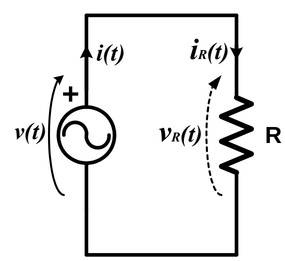
To better understand the resistor's AC power waveform, it is useful to rewrite the power expression:

(by utilizing the trig-identity $sin^2 x = ½·[1 – cos2x]$):

$$p_R(t) = V_{peak} \cdot I_{peak} \cdot \sin^2(\omega \cdot t + \phi)$$

$$= \frac{V_{peak} \cdot I_{peak}}{2} \cdot [1 - \cos(2 \cdot \omega \cdot t)]$$

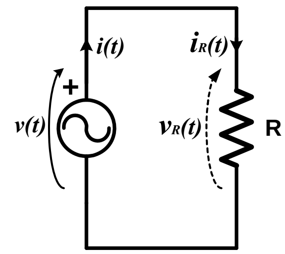
$$= \frac{V_{peak} \cdot I_{peak}}{2} - \frac{V_{peak} \cdot I_{peak}}{2} \cdot \cos(2 \cdot \omega \cdot t)$$



AC Power and Resistors

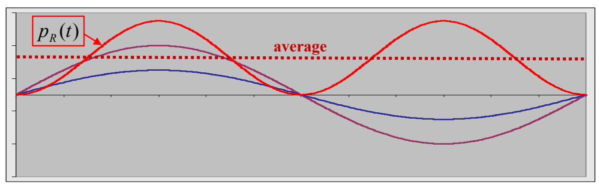
Looking at the resultant AC power waveform:

$$p_R(t) = \boxed{\frac{V_{peak} \cdot I_{peak}}{2}} - \frac{V_{peak} \cdot I_{peak}}{2} \cdot \cos(2 \cdot \omega \cdot t)$$



It can be seen that the waveform has two terms:

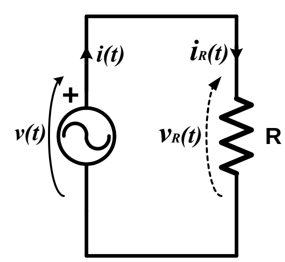
- The **first term** is a constant that relates to the **average** value of the power that is consumed by the resistor.



AC Power and Resistors

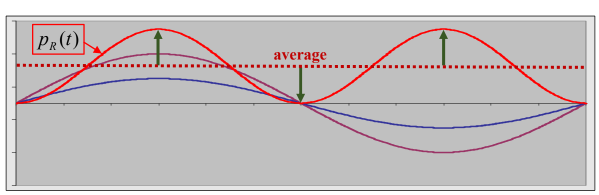
Looking at the resultant AC power waveform:

$$p_R(t) = \frac{V_{peak} \cdot I_{peak}}{2} - \boxed{\frac{V_{peak} \cdot I_{peak}}{2} \cdot \cos(2 \cdot \omega \cdot t)}$$



It can be seen that the waveform has two terms:

- The **second term** is a sinusoidal term that varies at 2x the source frequency and provides the fluctuation in the power waveform.

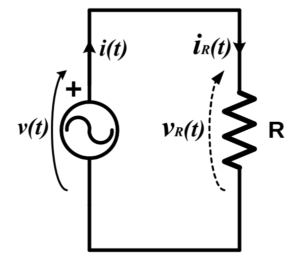


Real Power

In AC systems, it is typically the **average value** of the power that is desired.

This **average power** value is called **Real Power**.

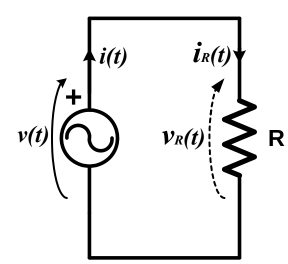
The **real power** consumed by a resistive load is:



$$P_{R(AC)} = Avg[p_R(t)] = \frac{V_{peak} \cdot I_{peak}}{2} \quad (Watts)$$

AC vs. DC Power to Resistors

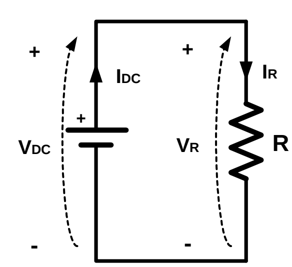
In terms of **average power** supplied to a resistor, an AC source is ½ as **effective** as a DC source whose magnitude is equal to the peak value of the AC source.



If: $V_{peak} = V_{DC}$ →

$$P_{R(AC)} = \frac{V_{peak} \cdot I_{peak}}{2} \quad (Watts)$$

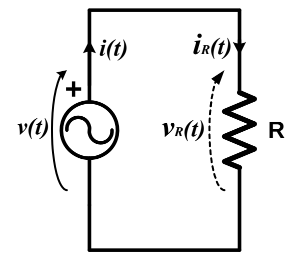
$$P_{R(DC)} = V_{DC} \cdot I_{DC} \quad (Watts)$$



Effective Voltage

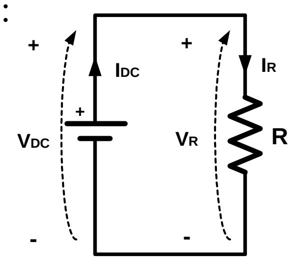
Since the **average AC power** is proportional to the square of the source's peak voltage:

$$P_{R(AC)} = \frac{V_{peak} \cdot I_{peak}}{2} = \frac{V_{peak} \cdot V_{peak}}{2 \cdot R} = \frac{V_{peak}^2}{2 \cdot R}$$



if the peak value of the AC voltage is increased such that it is $\sqrt{2}$ **times larger** than the DC voltage:

$$V_{peak} = \sqrt{2} \cdot V_{DC}$$

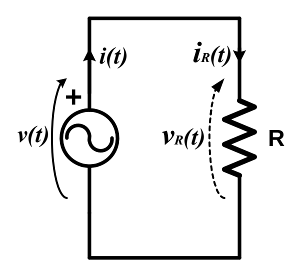


then the AC source will supply the **same average power** to the resistor as the DC source.

Effective / RMS Voltage Magnitudes

Based on this result, an **effective voltage** can be defined for a sinusoidally-varying AC source, such that:

$$V_{effective} = \frac{V_{peak}}{\sqrt{2}}$$

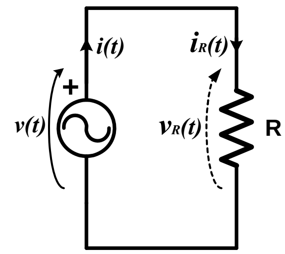


Note that the effective value of the AC source is equal to the **RMS** (***root-mean-squared***) value of the source voltage, as defined by the function:

$$V_{effective} = V_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T v^2(t) \cdot dt} = \frac{V_{peak}}{\sqrt{2}}$$

RMS Magnitudes

The **voltage waveform** may be expressed in terms of its ***RMS voltage magnitude***:

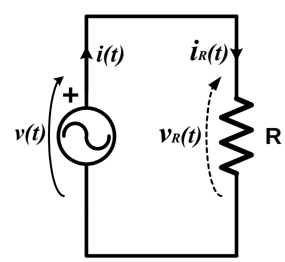


$$v(t) = \sqrt{2} \cdot V \cdot \sin(\omega \cdot t + \phi)$$

where: $V = \frac{V_{peak}}{\sqrt{2}}$ is the RMS magnitude of the AC voltage.

RMS Magnitudes

Similarly, the **current waveform** may also be expressed in terms of its ***RMS current magnitude***:

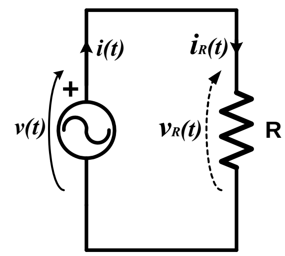


$$i(t) = \sqrt{2} \cdot I \cdot \sin(\omega \cdot t + \phi)$$

where: $I = \frac{I_{peak}}{\sqrt{2}}$ is the RMS magnitude of the AC current.

RMS Magnitudes & Resistor Power

When the voltages and currents are expressed in terms of their **RMS magnitudes**:



$$V_{peak} = \sqrt{2} \cdot V \qquad I_{peak} = \sqrt{2} \cdot I$$

the **power** delivered to a resistor is:

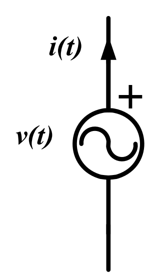
$$p_R(t) = V \cdot I - V \cdot I \cdot \cos(2 \cdot \omega \cdot t)$$

with an average (**Real Power**) value of:

$$P_{R(AC)} = Avg[p_R(t)] = V \cdot I$$

AC Power – General Case

As previously stated, the **general expression** for the **power** produced by an AC source is:



$$p(t) = v(t) \cdot i(t)$$

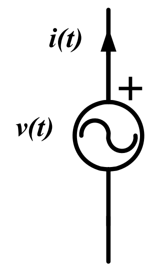
$$= V_{peak} \cdot I_{peak} \cdot \sin(\omega \cdot t + \phi) \cdot \sin(\omega \cdot t + \delta)$$

where: $v(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$

$i(t) = I_{peak} \cdot \sin(\omega \cdot t + \delta)$

AC Power – General Case

If the voltages & currents are expressed in terms of their **RMS magnitudes**, the power expression becomes:



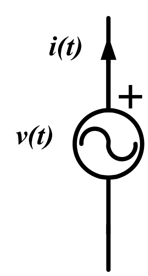
$$p(t) = v(t) \cdot i(t)$$
$$= \sqrt{2} \cdot V \cdot \sqrt{2} \cdot I \cdot \sin(\omega \cdot t + \phi) \cdot \sin(\omega \cdot t + \delta)$$
$$= 2 \cdot V \cdot I \cdot \sin(\omega \cdot t + \phi) \cdot \sin(\omega \cdot t + \delta)$$

which may be modified using several trigonometric identities into the following form:

$$p(t) = V \cdot I \cdot \cos(\phi - \delta)$$
$$- V \cdot I \cdot \cos(\phi - \delta) \cdot \cos(2 \cdot \omega \cdot t)$$
$$+ V \cdot I \cdot \sin(\phi - \delta) \cdot \sin(2 \cdot \omega \cdot t)$$

AC Power – General Case

The modified power expression is often simplified by defining a new variable, $\boldsymbol{\theta}$, where:



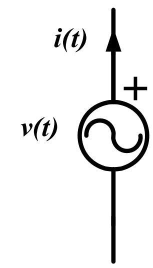
$$\theta = \phi - \delta$$

and substituting it into the equation, resulting in the **general power expression**:

$$p(t) = V \cdot I \cdot \cos(\theta)$$
$$- V \cdot I \cdot \cos(\theta) \cdot \cos(2 \cdot \omega \cdot t)$$
$$+ V \cdot I \cdot \sin(\theta) \cdot \sin(2 \cdot \omega \cdot t)$$

AC Power – General Case

The angle $\boldsymbol{\theta}$, is defined by the difference between the phase angles of the voltage and current,



$$\theta = \angle\tilde{V} - \angle\tilde{I} = \phi - \delta$$

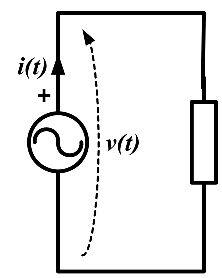
such that: $$v(t) = V_{peak} \cdot \sin(\omega \cdot t + \phi)$$

$$i(t) = I_{peak} \cdot \sin(\omega \cdot t + \delta)$$

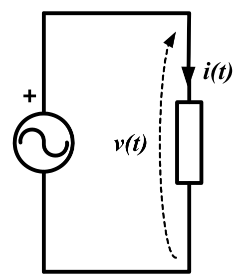
The angle $\boldsymbol{\theta}$ is often referred to as the **power angle**:

AC Power – General Case

This general expression defines the instantaneous **power produced by an AC source**.



$$p(t) = V \cdot I \cdot \cos(\theta) - V \cdot I \cdot \cos(\theta) \cdot \cos(2 \cdot \omega \cdot t) + V \cdot I \cdot \sin(\theta) \cdot \sin(2 \cdot \omega \cdot t)$$

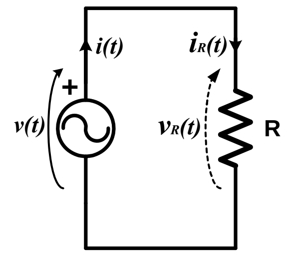


Likewise, if the source is connected across a load that may have resistive, capacitive, and/or inductive components, then the solution also defines the instantaneous **power consumed by the AC supplied load**.

AC Power and Resistors

The resultant power waveform has two terms:

$$p_R(t) = \boxed{V_R \cdot I_R} - \boxed{V_R \cdot I_R \cdot \cos(2 \cdot \omega \cdot t)}$$



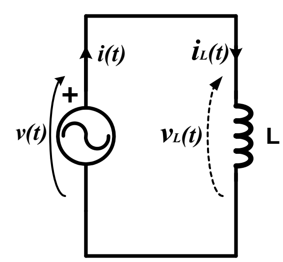
- the first of which is a **constant** that provides the **average** power supplied to the resistor, which is defined to be **Real Power**, $\boldsymbol{P_R}$, and
- the second of which is a **purely sinusoidal** term that has a **zero average** value and varies at 2x the frequency of the source voltage.

$$P_R = V_R \cdot I_R \quad \text{Watts}$$

AC Power and Inductors

The resultant power waveform has only one term:

$$p_L(t) = \boxed{V_L \cdot I_L \cdot \sin(2 \cdot \omega \cdot t)}$$



which is a **purely sinusoidal** term that has a **zero average** value and varies at twice (2x) the frequency of the source voltage.

Since the power waveform has a zero-average value, the inductor consumes zero real power:

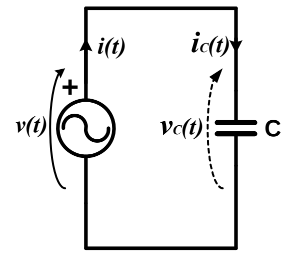
$$P_L = 0 \quad \text{Watts}$$

but power is flowing into and out of the inductor.

AC Power and Capacitors

The resultant power waveform has only one term:

$$p_C(t) = \boxed{-V_C \cdot I_C \cdot \sin(2 \cdot \omega \cdot t)}$$



which is a **purely sinusoidal** term that has a **zero average** value and varies at twice (2x) the frequency of the source voltage.

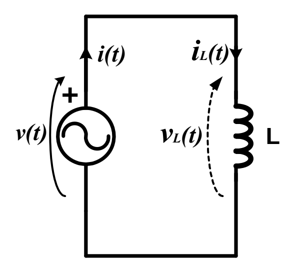
Since the power waveform has a zero-average value, the capacitor consumes zero real power:

$$P_C = 0 \quad \text{Watts}$$

but power is flowing into and out of the capacitor.

Reactive Power

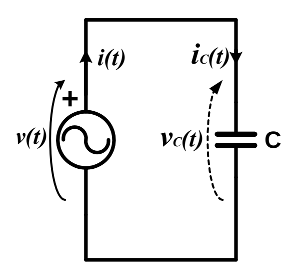
Reactive Power (*Q*) is defined as the magnitude of the power that is flowing into and out of a reactive load when supplied by an AC source.



Thus, given:

$$p_L(t) = V_L \cdot I_L \cdot \sin(2 \cdot \omega \cdot t)$$

$$p_C(t) = -V_C \cdot I_C \cdot \sin(2 \cdot \omega \cdot t)$$



the reactive power for the inductive and capacitive loads can be defined as:

$$Q_L = +V_L \cdot I_L \quad Vars$$

$$Q_C = -V_C \cdot I_C \quad Vars$$

Reactive Power

Reactive Power is given the unit of "***Vars***", which stands for:

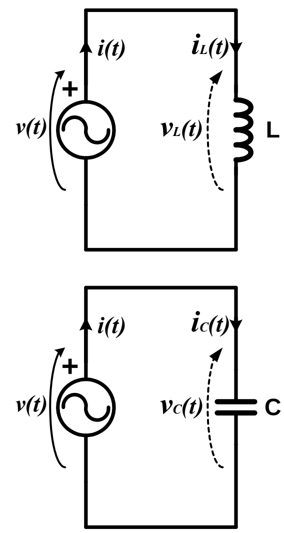
"***Volt-Amps-Reactive***".

$$Q_L = +V_L \cdot I_L \quad Vars$$

$$Q_C = -V_C \cdot I_C \quad Vars$$

Note that the <u>reactive power for an inductor is **positive**</u> while the <u>reactive power for a capacitor is **negative**</u>.

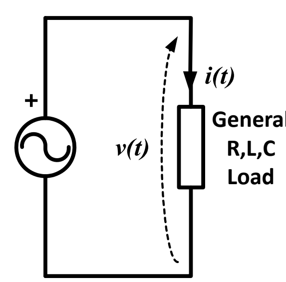
Thus, it is often stated that an inductor "consumes" reactive power while a capacitor "produces" reactive power.



AC Power – General Case

The previous results can be used to define the relevance of the **three terms** that appear in the **general AC power expression**:

$$p(t) = \boxed{V \cdot I \cdot \cos(\theta)}$$
$$- V \cdot I \cdot \cos(\theta) \cdot \cos(2 \cdot \omega \cdot t)$$
$$+ V \cdot I \cdot \sin(\theta) \cdot \sin(2 \cdot \omega \cdot t)$$

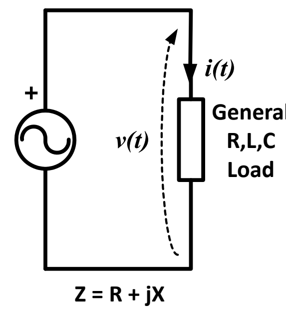


The <u>**first term**</u> is a **constant** that provides the **average** or <u>**Real Power**</u> that is consumed by the resistive portion of the load:

$$P = V \cdot I \cdot \cos(\theta)$$

AC Power – General Case

The previous results can be used to define the relevance of the **three terms** that appear in the **general AC power expression**:

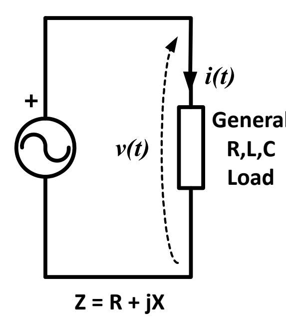


$$p(t) = V \cdot I \cdot \cos(\theta)$$
$$\boxed{- V \cdot I \cdot \cos(\theta) \cdot \cos(2 \cdot \omega \cdot t)}$$
$$+ V \cdot I \cdot \sin(\theta) \cdot \sin(2 \cdot \omega \cdot t)$$

The **second term** is a sinusoidal term that varies at 2x the source frequency and provides the fluctuation in the power being supplied to the resistive portion of the load.

AC Power – General Case

The previous results can be used to define the relevance of the **three terms** that appear in the **general AC power expression**:



$$p(t) = V \cdot I \cdot \cos(\theta)$$
$$- V \cdot I \cdot \cos(\theta) \cdot \cos(2 \cdot \omega \cdot t)$$
$$\boxed{+ V \cdot I \cdot \sin(\theta) \cdot \sin(2 \cdot \omega \cdot t)}$$

The **third term** is also purely sinusoidal, the magnitude of which provides the **Reactive Power** "consumed" by the reactive portion of the load.

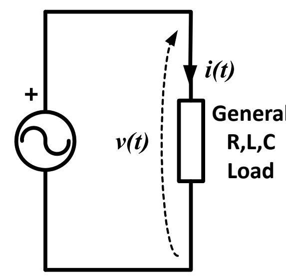
$$Q = V \cdot I \cdot \sin(\theta) \quad Vars$$

AC Power in Combination Circuits

Note that, if the load has **both** a resistive and a capacitive or an inductive component, then the **power angle *θ*** will fall somewhere in the range:

-90°≤ *θ* ≤+90°

resulting in the existence of all three terms in the general power expression.



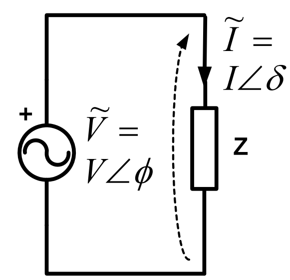
Thus, there will be **Real** and **Reactive Powers** flowing into the load, as defined by:

$$P = V \cdot I \cdot \cos(\theta)$$

$$Q = V \cdot I \cdot \sin(\theta)$$

Complex Power

The term **Complex Power** is used to characterize both the **Real Power** and the **Reactive Power** that an AC source is producing or that a complex load impedance (with a resistive component and/or an inductive or capacitive reactive component) is consuming**.**



$Z = R + jX$

Complex Power (***S***) is a complex number and is defined by:

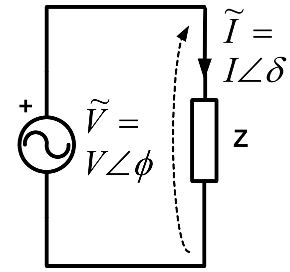
$$S = P + j\,Q$$

where: ***P*** is **Real Power**, and
Q is **Reactive Power**.

Complex Power

Complex Power (*S*):

$$S = P + j\,Q$$



$$Z = R + jX$$

may be solved directly from a circuit element's **phasor voltage** and **phasor current** as:

$$S = P + jQ = \tilde{V} \cdot \tilde{I}^* = (V\angle\phi)\cdot(I\angle -\delta)$$

$$= V \cdot I \angle(\phi - \delta) = V \cdot I \angle\theta$$

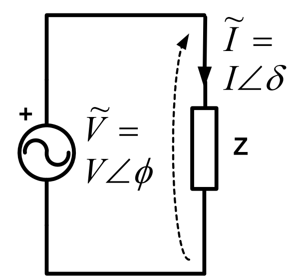
$$= \boxed{V \cdot I \cdot \cos\theta + j\,V \cdot I \cdot \sin\theta}$$

the **real portion** of which relates to **Real Power** and the **imaginary portion** of which relates to **Reactive Power**.

Complex Conjugate

Note that $\tilde{I}^*$ is the **complex conjugate** of $\tilde{I}$, and is defined as:

$$\tilde{I}^* = (I\angle\delta)^* = (I\angle -\delta)$$



$$Z = R + jX$$

The **complex conjugate** of a complex number expressed in **polar form** has the **same magnitude as the original number** but the **angle is negated**.

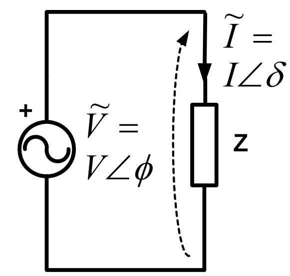
Apparent Power

Apparent Power (|***S***|) is defined to be the **magnitude of complex power**:

$$|S| = V \cdot I = \sqrt{P^2 + Q^2}$$

Note that **apparent power** is often specified as one of the "***ratings***" of a machine, such that:

$$|S|_{rated} = V_{rated} \cdot I_{rated}$$



$Z = R + jX$

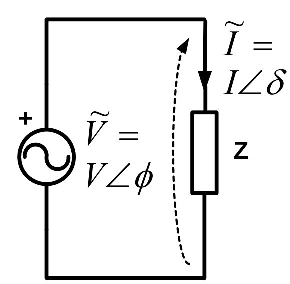
Power Factor

Power Factor (***pf***) provides a measure of the portion of the apparent power that relates to real power:

$$pf = \frac{P}{|S|}$$

Thus, **power factor** may be defined as:

$$pf = \frac{P}{|S|} = \frac{V \cdot I \cdot \cos\theta}{V \cdot I} = \boxed{\cos\theta}$$



$Z = R + jX$

Note that **power factor** is often specified as having a **leading** or **lagging** characteristic, which is based on the angle relationship between the phasor voltage and the phasor current

Power Factor

A **leading** power factor exists when the current waveform is "leading" the voltage, which occurs with a **capacitive** load impedance and results in a **negative** angle difference for $\boldsymbol{\theta}$:

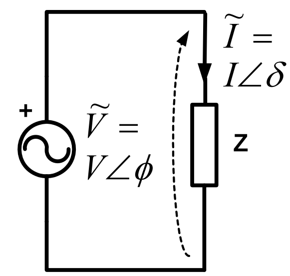
$$\theta = \phi - \delta$$

$$-90° \leq \theta < 0°$$

A **lagging** power factor exists when the current waveform is "lagging" the voltage, which occurs with an **inductive** load impedance and results in a **positive** angle difference for $\boldsymbol{\theta}$:

$$\theta = \phi - \delta$$

$$0° < \theta \leq +90°$$



$$Z = R + jX$$

Power Factor

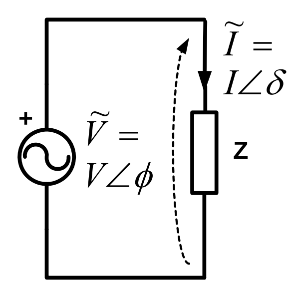
Note that the **angle** $\boldsymbol{\theta}$ for a **purely resistive load** has a **zero** value since the voltage and current waveforms are "***in-phase***", resulting in a power factor that which is <u>neither leading nor lagging</u>.

This is referred to as a **unity power factor** since the value of **power factor** under this condition equals **one**.

$$\phi = \delta$$

$$\theta = \phi - \delta = 0°$$

$$\cos(\theta) = \cos(0°) = 1$$



$$Z = R + jX$$

Summary of Complex Power Equations

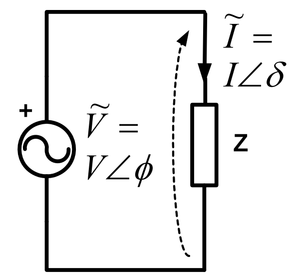
Complex Power (S): $S = P + jQ = \tilde{V} \cdot \tilde{I}^*$

Real Power (P): $P = V \cdot I \cdot \cos\theta$

Reactive Power (Q): $Q = V \cdot I \cdot \sin\theta$

Apparent Power (|S|): $|S| = V \cdot I = \sqrt{P^2 + Q^2}$

Power Factor (pf): $pf = \cos\theta$



$Z = R + jX$

Resonance

Resonance

Resonance is a condition that occurs within an R-L-C circuit when the reactive power "consumed" by the inductive elements is equal to the reactive power "produced" by the capacitive elements.

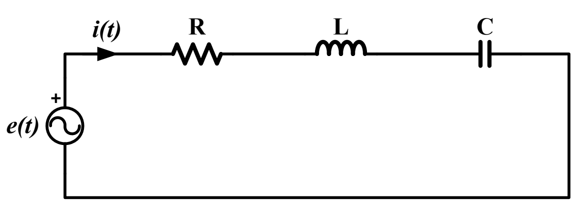
Note – if the reactive powers are equal, then the energy that the inductive elements are absorbing (or releasing) at any point in time will be equal to the energy that the capacitive elements are releasing (or absorbing) at that same instant.

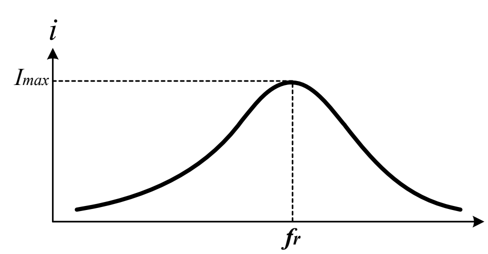
The resonant frequency, f_r, is the source frequency at which resonance occurs.

Series-Resonant Circuits

A Series-Resonant Circuit is an R-L-C circuit that contains a resistor, an inductor, and a capacitor that are connected in-series with each other.

In a series-resonant circuit, the current will be maximum when the source is operating at the resonant frequency.





Series-Resonance

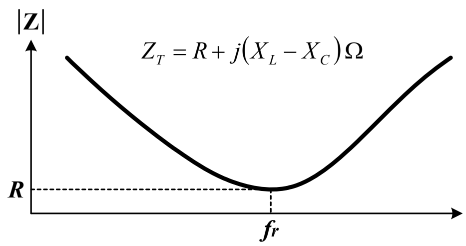
Resonance occurs in a series-resonant circuit when $X_L = X_C$.

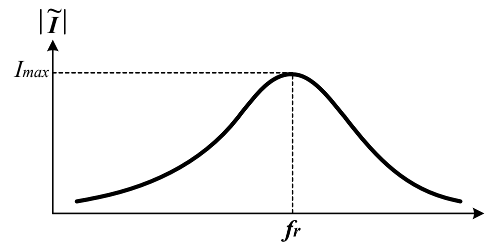
Thus, at resonance, the total series-impedance will be:

$$Z_T = R\,\Omega$$

the magnitude of which will be at its minimum value, resulting in a current that is in-phase with the source voltage and at its maximum possible magnitude, I_{max}.

T



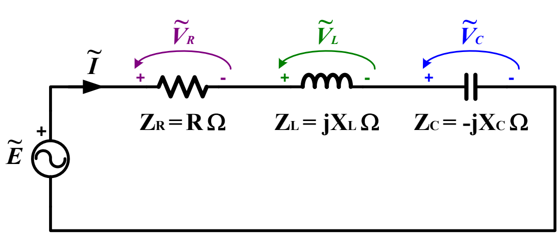


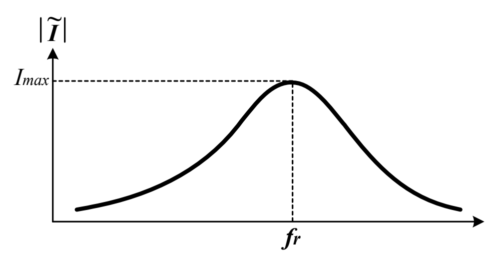
Series-Resonant Frequency

The <u>resonant frequency</u> is the frequency at which X_L equals X_C.

$$X_L = X_C \longrightarrow \omega \cdot L = \frac{1}{\omega \cdot C}$$

$$\omega = \sqrt{\frac{1}{L \cdot C}} \longrightarrow f_r = \frac{1}{2\pi}\sqrt{\frac{1}{L \cdot C}}$$





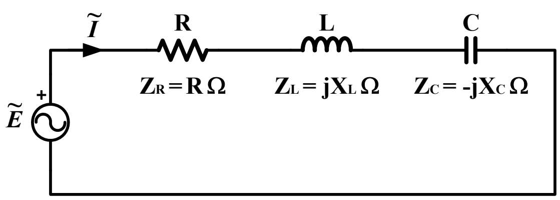
Series-Resonance Peak Current

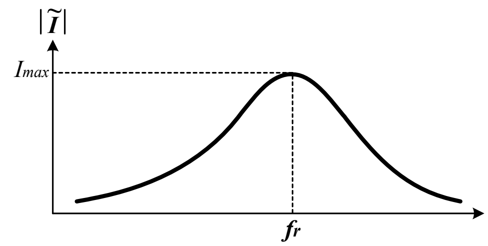
Since the total series-impedance at resonance is:

$$Z_T = R\,\Omega$$

if $\tilde{E} = E\angle\phi°$**, then the <u>maximum current</u> magnitude will be:**

$$I_{\max} = \left|\tilde{I}\right| = \left|\frac{\tilde{E}}{R}\right| = \frac{E}{R}$$





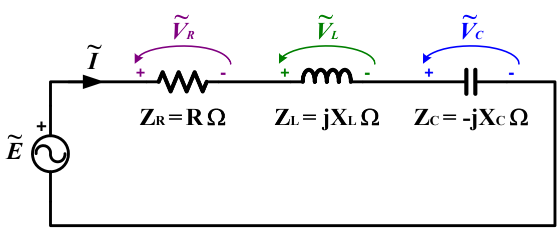
Quality Factor

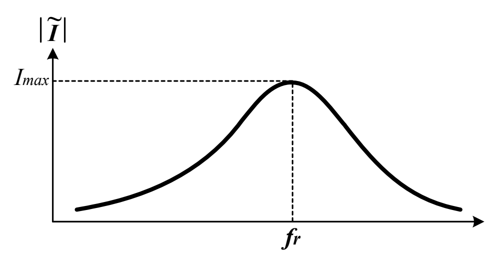
The <u>quality factor</u>, Q_s**, of a series-resonant circuit is defined by:**

$$Q_s = \frac{\text{Reactive Power}}{\text{Real Power}}$$

Thus:

$$Q_s = \frac{\omega \cdot L}{R} = \frac{1}{\omega \cdot R \cdot C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



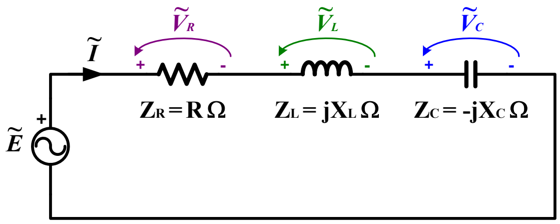


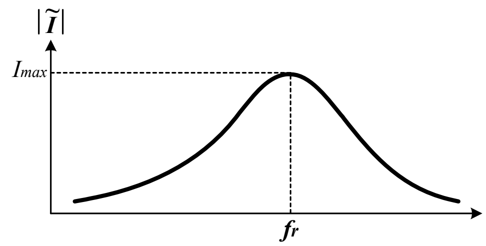
Quality Factor

Additionally, note that the magnitude of the inductor and capacitor voltages <u>at the resonant frequency</u> will be:

$$\left|\tilde{V}_L\right| = \left|\tilde{I} \cdot (jX_L)\right| = \frac{E}{R} \cdot X_L = E \cdot Q_s$$

$$\left|\tilde{V}_C\right| = \left|\tilde{I} \cdot (-jX_C)\right| = \frac{E}{R} \cdot X_C = E \cdot Q_s$$

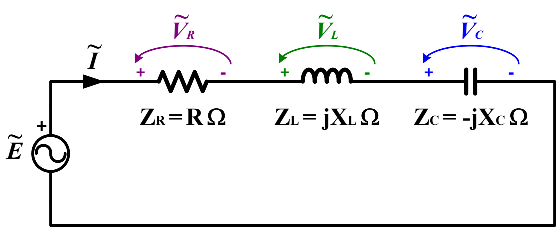


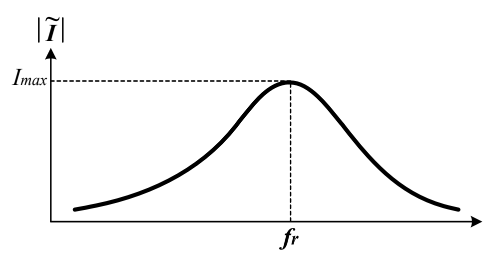


Selectivity

The term <u>Selectivity</u> is used to characterize the range of frequencies for which currents will pass through or flow in a series-resonant circuit.

Since the current magnitude decays as frequency varies away from the resonant frequency, arbitrary cutoffs are chosen in order to define the circuit's selectivity.



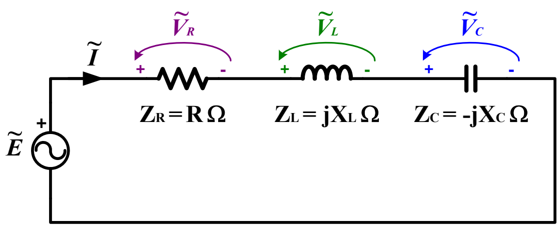


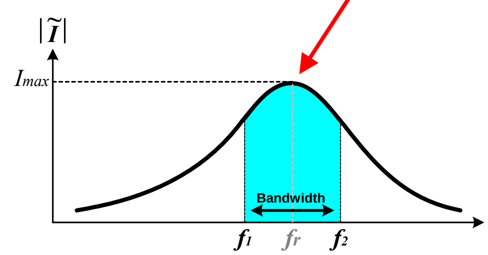
Bandwidth

The <u>Bandwidth</u> of a series-resonant circuit is the range of frequencies for which the real power delivered to the resistor is greater than or equal to one-half of the peak power value.

$$P_R \geq \frac{1}{2} P_{\max}$$

Note that peak power occurs at resonant frequency when the current is at a maximum.





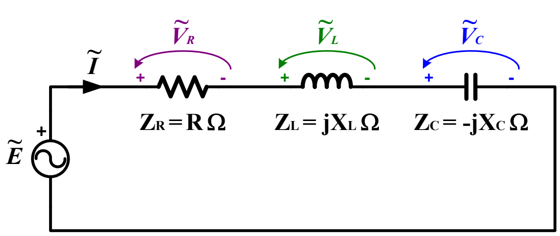
Half-Power Cutoff Frequencies

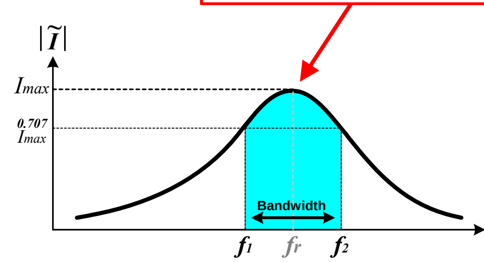
The <u>Half-Power Cutoff Frequencies</u> are the frequencies at which the real power delivered to the resistor is equal to one half of the peak power value.

Note that one-half peak power occurs whenever:

$$\left|\tilde{I}\right| = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$

Note that peak power occurs at resonant frequency when the current is at a maximum.

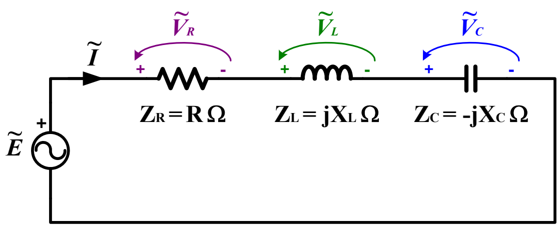


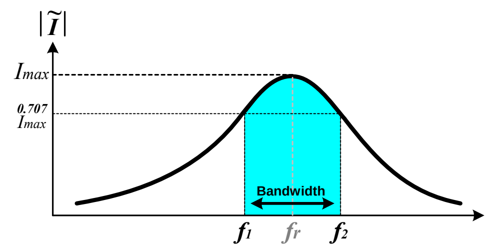


Half-Power Cutoff Frequencies

The <u>lower</u> and <u>upper</u> half-power cutoff frequencies, f_1 and f_2, can be calculated from:

$$f_1 = \frac{1}{2\pi}\left[\frac{-R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] \text{ Hz} \qquad f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] = \text{ Hz}$$

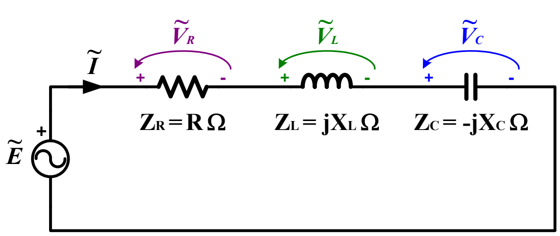


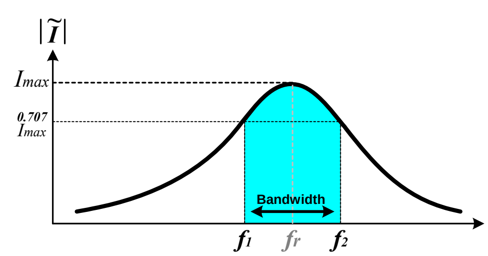


Half-Power Cutoff Frequencies

Note that the <u>resonant frequency</u>, f_r, is directly related to the lower and upper half-power cutoff frequencies, f_1 and f_2:

$$f_r = \sqrt{f_1 \cdot f_2}$$





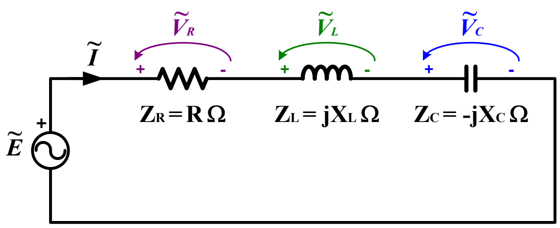
Bandwidth

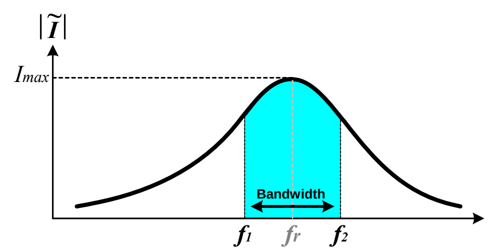
<u>Bandwidth</u> can be defined in terms of the cutoff frequencies as:

$$BW = f_2 - f_1$$

Additionally, <u>bandwidth</u> is equal to:

$$BW = \frac{f_r}{Q_S}$$





Filters

Decibels & Logarithms

Decibel – a unit defined by a logarithmic expression that is commonly used to define the levels of a variety of parameters including voltage gain, field strength, energy, and sound pressure.

Logarithm – a quantity representing the power to which a fixed number (the base) must be raised to produce a given number.

Given: $A = B^x$

Then: $x = \log_B A$

Logarithms

Commonly used logarithms include:

$x = \log_{10} A$ $\qquad A = 10^x$

$x = \log_e A$ $\qquad A = e^x$

Notes: $\log_e A = 2.303 \cdot \log_{10} A$

$\log_e A \equiv \ln A$

Properties of Logarithms

- The Log of one (1) is always equal to zero (0).

$$\log_{10} 1 = 0 \qquad \log_e 1 = 0 \qquad \log_n 1 = 0$$

- If (A>1) then the Log of A is positive.

$$\log_{10} 2000 = 3.3 \qquad \log_e 5 = 1.61$$

- If (A<1) then the Log of A is negative.

$$\log_{10} 0.5 = -0.3 \qquad \log_e 0.1 = -2.3$$

- Additional properties include:

$$\log_n a \cdot b = \log_n a + \log_n b \qquad \log_n \frac{a}{b} = \log_n a - \log_n b \qquad \log_n a^b = b \cdot \log_n a$$

Bels & Decibels

Power Gain

Bel (***B***) – a base unit defined as a logarithmic ratio of powers:

$$B = \log_{10} \frac{P_2}{P_1}$$

Decibel (***dB***) – a logarithmic ratio of powers that is commonly utilized in order to define the **gain** (increase) in power P_2 compared to power P_1.

$$dB = 10 \cdot B = 10 \cdot \log_{10} \frac{P_2}{P_1}$$

Properties of Decibels

- If $P_2 = P_1$, then the decibel **gain is zero**.

$$10 \cdot \log_{10} \frac{5mW}{5mW} = 10 \cdot \log_{10} 1 = 10 \cdot 0 = 0 \text{ dB}$$

- If $P_2 > P_1$, then the decibel **gain is positive**.

$$10 \cdot \log_{10} \frac{20mW}{5mW} = 10 \cdot \log_{10} 4 = 10 \cdot 0.6 = +6 \text{ dB}$$

- If $P_2 < P_1$, then the decibel **gain is negative**.

$$10 \cdot \log_{10} \frac{1mW}{100mW} = 10 \cdot \log_{10} 0.01 = 10 \cdot (-2) = -20 \text{ dB}$$

Properties of Decibels

- If $P_2 = 2^n \cdot P_1$, then the decibel gain is $n \cdot (+3\text{dB})$.

$$P_1 = 1\,mW \qquad P_1 = 32\,mW = 2^5 \cdot 1\,mW \qquad 10 \cdot \log_{10} \frac{32mW}{1mW} = +15 \text{ dB} = 5 \cdot (+3 \text{ dB})$$

- If $P_2 = ½^n \cdot P_1$, then the decibel gain is $n \cdot (-3\text{dB})$.

$$P_1 = 200\,W \qquad P_1 = 25\,W = \left(\tfrac{1}{2}\right)^3 \cdot 200\,W \qquad 10 \cdot \log_{10} \frac{25W}{200W} = -9 \text{ dB} = 3 \cdot (-3 \text{ dB})$$

- If $P_2 = 10^n \cdot P_1$, then the decibel gain is $n \cdot (+10\text{dB})$.
- If $P_2 = 10^{-n} \cdot P_1$, then the decibel gain is $n \cdot (-10\text{dB})$

dBm

<u>dBm</u> – a specific value of power, relating to a power P_2 (mW), but expressed in terms of the decibel gain of P_2 compared to a reference power of 1mW.

$$dBm = 10 \cdot \log_{10} \frac{P_2}{1\,\text{mW}}$$

For example – convert a power of **+6dBm** to a **mW** value:

$$+6\ dBm = 10 \cdot \log_{10} \frac{P_2}{1\,\text{mW}}$$

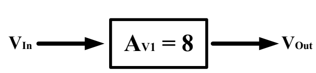
$$P_2 = 1\,\text{mW} \cdot 10^{\frac{+6}{10}} = 1\,\text{mW} \cdot 4 = \boxed{4\,\text{mW}}$$

Voltage Gain

<u>Voltage Gain</u> (A_V) – a ratio of voltages that is commonly utilized in order to define the **gain** (increase) in voltage V_{Out} compared to voltage V_{In}.

$$A_V = \frac{V_{Out}}{V_{In}}$$

For example – is an amplifier has a voltage gain $A_V = 8$, then:



$$V_{Out} = A_V \cdot V_{In} = 8 \cdot V_{In}$$

dBv

dBv – a logarithmic ratio of voltages, expressed in terms of decibels, that is commonly utilized in order to define the **gain** in the power supplied to a resistive load ***R*** by voltage $\boldsymbol{V_2}$ compared to the power supplied to the same resistive load ***R*** by voltage $\boldsymbol{V_1}$.

$$dB = 10 \cdot \log_{10} \frac{P_2}{P_1} = 10 \cdot \log_{10} \frac{\frac{V_2^2}{R}}{\frac{V_1^2}{R}} = 10 \cdot \log_{10} \frac{V_2^2}{V_1^2} = 10 \cdot \log_{10} \left(\frac{V_2}{V_1} \right)^2 = 20 \cdot \log_{10} \frac{V_2}{V_1}$$

$$dB_V = 20 \cdot \log_{10} \frac{V_2}{V_1}$$

Filters

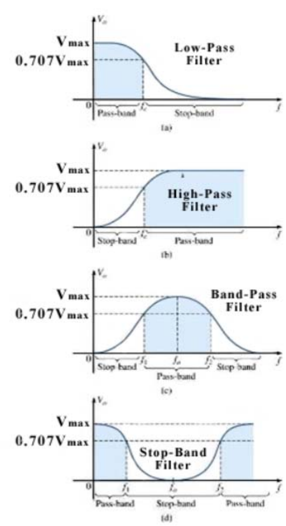
There are four primary categories of filters:

Low-Pass Filters

High-Pass Filters

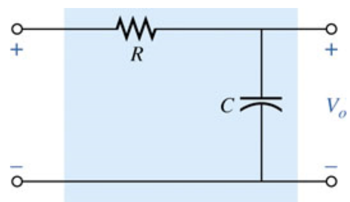
Band-Pass Filters

Stop-Band Filters



R-C Low-Pass Filter

The following combination of elements can be utilized to create a **R-C Low-Pass Filter**:

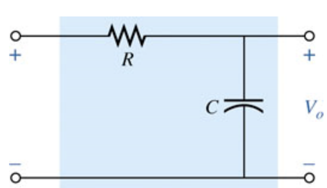


$$V_{out} = V_{in} \cdot \frac{-jX_C}{R - jX_C}$$

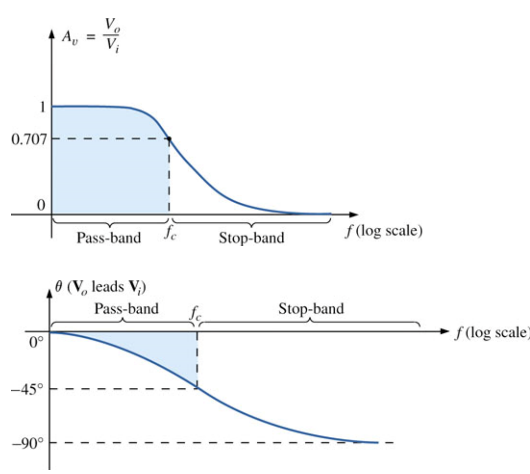
$$A_V = \frac{V_{out}}{V_{in}} = \frac{-jX_C}{R - jX_C}$$

R-C Low-Pass Filter

The following plots show the **voltage gain** and **phase response** of the low-pass filter:

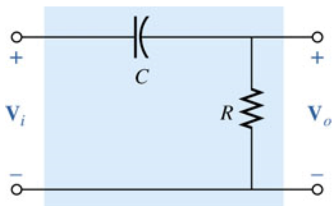


$$f_c = \frac{1}{2 \cdot \pi \cdot R \cdot C}$$



R-C High-Pass Filter

The following combination of elements can be utilized to create a **R-C High-Pass Filter**:

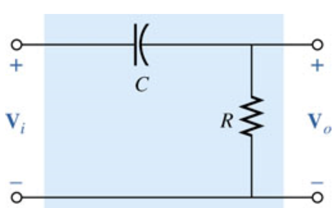


$$V_{out} = V_{in} \cdot \frac{R}{R - jX_C}$$

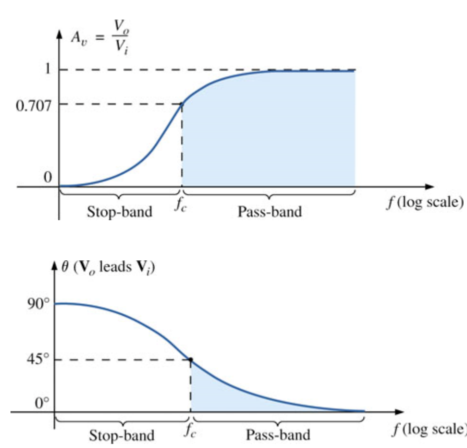
$$A_V = \frac{V_{out}}{V_{in}} = \frac{R}{R - jX_C}$$

R-C High-Pass Filter

The following plots show the **voltage gain** and **phase response** of the high-pass filter:

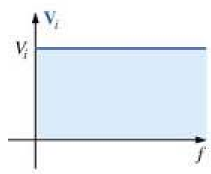


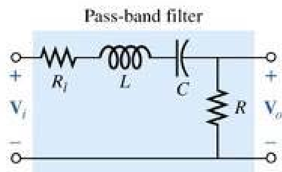
$$f_c = \frac{1}{2 \cdot \pi \cdot R \cdot C}$$

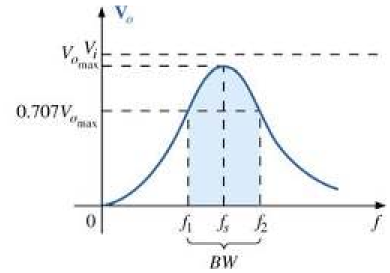


Band-Pass Filter

The following combination of elements can be utilized to create a series-resonant **Band-Pass Filter**:







$$f_s = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

$$V_{out} = V_{in} \cdot \frac{R}{R + R_L + jX_L - jX_C} \qquad A_V = \frac{V_{out}}{V_{in}} = \frac{R}{R + R_L + jX_L - jX_C}$$

Bode Plots

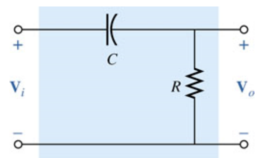
Bode Plots are the curves obtained for the magnitude and phase response (versus frequency) of a system.

Idealized Bode Plots utilize straight-line segments to efficiently estimate the frequency response of a system.

There is a quick technique for sketching the frequency response of a system on a decibel scale that provides a good method for comparing the expected decibel levels at different frequencies.

R-C High-Pass Filter

Lets look back at the **R-C High-Pass Filter**.



$$f_c = \frac{1}{2\pi \cdot R \cdot C}$$

The formula for the voltage gain can be rewritten as:

$$A_V = \frac{V_{out}}{V_{in}} = \frac{R}{R - jX_C}$$

$$= \frac{1}{1 - j\dfrac{1}{2\pi \cdot f \cdot R \cdot C}}$$

$$= \frac{1}{1 - j\dfrac{f_c}{f}}$$

R-C High-Pass Filter

Given the voltage gain for a **R-C High-Pass Filter**:

$$A_V = \frac{V_{out}}{V_{in}} = \frac{1}{1 - j\dfrac{f_c}{f}}$$

the **magnitude** of the voltage gain can be expressed as:

$$|A_V| = \frac{1}{\sqrt{1 + \left(\dfrac{f_c}{f}\right)^2}} = \frac{1}{\left(1 + \left(\dfrac{f_c}{f}\right)^2\right)^{\frac{1}{2}}}$$

R-C High-Pass Filter

If **voltage gain** is expressed in **decibels**, then:

$$\left|A_V\right|_{dB} = -10\log_{10}\left(1+\left(\frac{f_c}{f}\right)^2\right)$$

and when $\boldsymbol{f << f_c}$,

$$1+\left(\frac{f_c}{f}\right)^2 \cong \left(\frac{f_c}{f}\right)^2$$

thus:

$$\left|A_V\right|_{dB(f<<f_c)} = -10\log_{10}\left(\frac{f_c}{f}\right)^2 = -20\log_{10}\left(\frac{f_c}{f}\right) = +20\log_{10}\left(\frac{f}{f_c}\right)$$

Bode Plots and High-Pass Filters

Note that, given the **decibel voltage gain** function ($\boldsymbol{f << f_c}$):

$$\left|A_V\right|_{dB(f<<f_c)} = +20\log_{10}\left(\frac{f}{f_c}\right)$$

For every decrease in the frequency by a factor of 0.5 (one octave), there will be a 6dB decrease in the gain, and

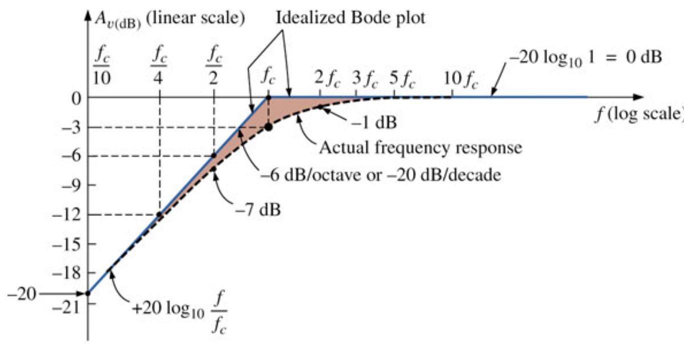
For every decrease in the frequency by a factor of 0.1 (one decade), there will be a 20dB decrease in the gain.

Thus, an **Idealized Bode Plot** can be drawn for the gain function because the dB change per octave or decade is constant.

Bode Plots and High-Pass Filters

The Bode Plot for the **decibel voltage gain** function is:

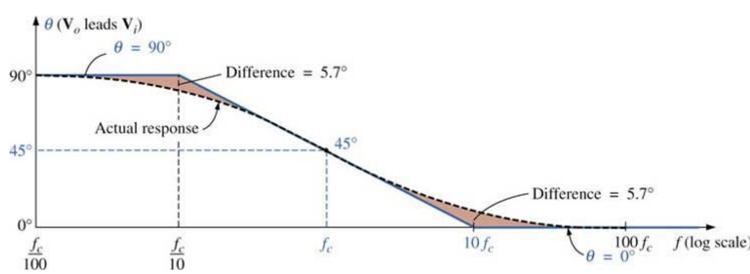
$$\left|A_V\right|_{dB(f<<f_c)} = +20\log_{10}\left(\frac{f}{f_c}\right)$$



Bode Plots and High-Pass Filters

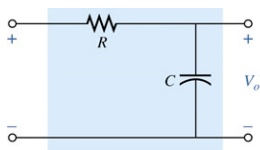
Additionally, the phase response may be drawn as:

$$\theta = \tan^{-1}\left(\frac{f_c}{f}\right)$$



Bode-Plots and Low-Pass Filters

Given an **R-C Low-Pass Filter**, the **decibel voltage gain** ($f >> f_c$) can be written as:

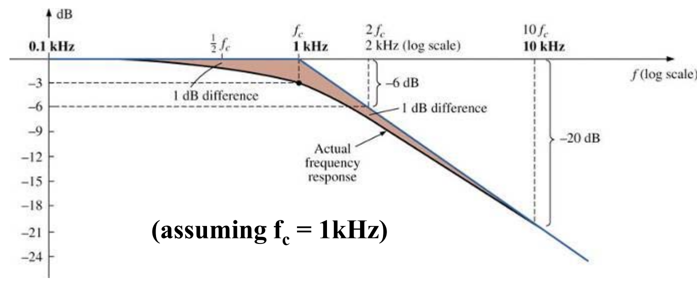


$$f_c = \frac{1}{2 \cdot \pi \cdot R \cdot C}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\frac{f_c}{f}}$$

$$|A_V|_{dB(f>>f_c)} = -20\log_{10}\left(\frac{f}{f_c}\right)$$

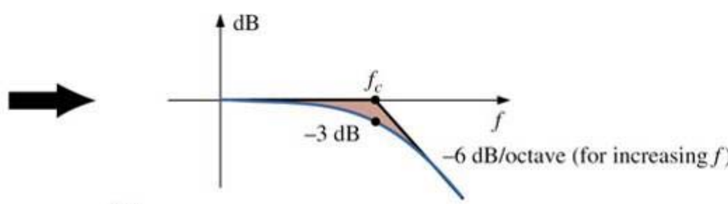
resulting in the following **Bode Plot**:



(assuming f_c = 1kHz)

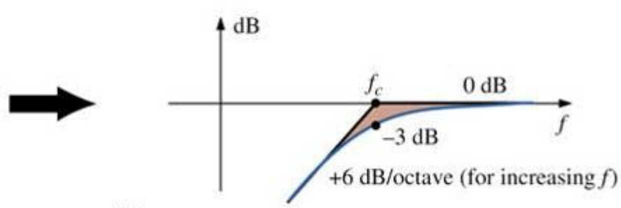
Sketching Bode Plots

Low-pass: $\frac{1}{1 + j\frac{f}{f_c}}$



(a)

High-pass: $\frac{1}{1 + j\frac{f_c}{f}}$



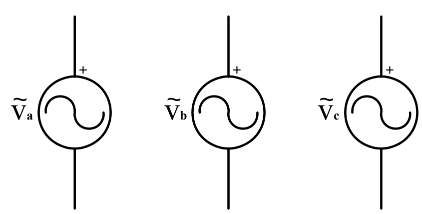
(b)

Three-Phase Systems

Three-Phase AC Voltage Sources

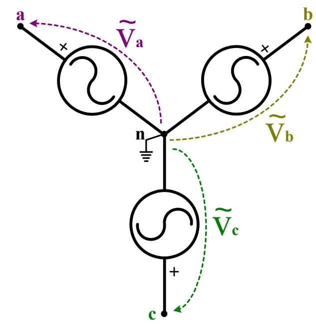
A ***three-phase*** (3Φ) ***AC voltage*** source is a composite source that can be modeled using three single-phase AC voltage sources that are connected together to function as one complete unit.

Note that the three single-phase AC voltage sources must be connected together in a symmetrical fashion.



Wye-connected Three-Phase Source

The three sources are typically connected together in a "***Wye***" (**Y**) format such that the reference terminals of the three supplies are tied to a common point of connection.



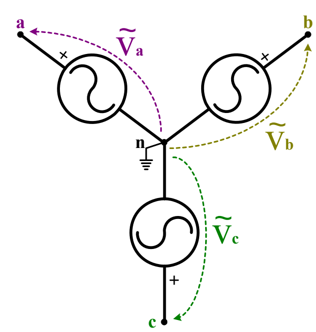
The common point of connection is referred to as the "***neutral point***".

(node **n** in the figure)

Note that the neutral point is often grounded in order to provide a zero-volt reference for the source.

Phase Voltages

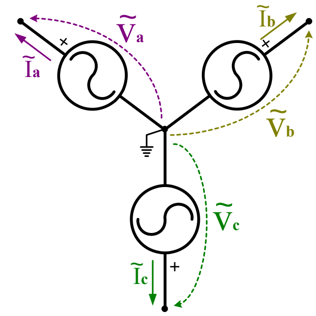
The voltages $\tilde{V}_a$, $\tilde{V}_b$, and $\tilde{V}_c$ are referred to as "***phase voltages***" because they correspond to the voltage across each individual phase of the wye-connected source.



The phase voltages are sometimes referred to as "***line-to-neutral voltages***", and as such may be expressed as $\tilde{V}_{an}$, $\tilde{V}_{bn}$, and $\tilde{V}_{cn}$.

Phase Currents

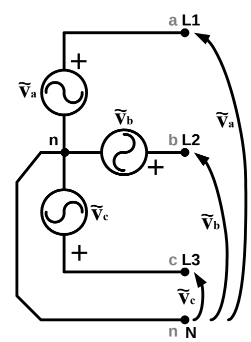
Similarly, the currents $\tilde{I}_a$, $\tilde{I}_b$, and $\tilde{I}_c$ are referred to as "***phase currents***" because they correspond to the current flowing through each individual phase of the wye-connected source.



Balanced Three-Phase Voltage Source

A "***balanced***" 3Φ source is a source whose phase voltages have <u>equal magnitudes</u> and <u>phase angles that are separated by 120º</u>.

Note that, despite slight magnitude differences that might exist between the three individual phases, most practical 3Φ sources are assumed to be balanced.

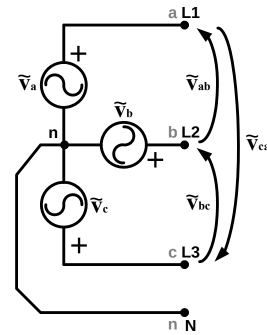


Line Voltages

A second set of voltages can also be defined for the 3Φ source in terms of the voltage rise between each pair of terminals:

a-b, **b-c**, and **c-a**.

The voltages $\tilde{V}_{ab}$, $\tilde{V}_{bc}$ and $\tilde{V}_{ca}$ are referred to as "***line voltages***" because they are the voltages between any pair of line terminals.



Line Voltages

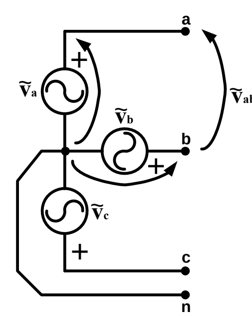
The ***line voltages*** for a balanced 3Φ source are closely related to the source's phase voltages.

The same logic can be used to express all three line voltages in terms of their respective phase voltages:

$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b$$

$$\tilde{V}_{bc} = \tilde{V}_b - \tilde{V}_c$$

$$\tilde{V}_{ca} = \tilde{V}_c - \tilde{V}_a$$



Line Voltages

A complete analysis of a 3Φ source having the phase voltages:

$$\tilde{V}_a = V\angle\phi°$$

$$\tilde{V}_b = V\angle\phi - 120°$$

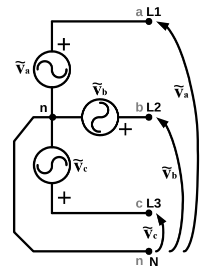
$$\tilde{V}_c = V\angle\phi - 240°$$

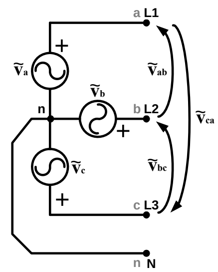
will result in the following set of line voltages:

$$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$$

$$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$$

$$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$$



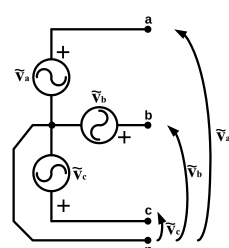


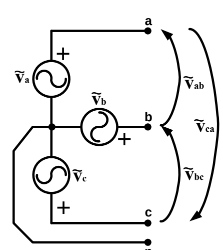
Line Voltages

Note that the line voltages have <u>equal magnitudes</u> and a <u>120º phase separation</u> between each pair;

Thus, the line voltages maintain the same **balanced** relationship as the phase voltages:

<u>Phase Voltages</u>	<u>Line Voltages</u>
$\tilde{V}_a = V\angle\phi$	$\tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$
$\tilde{V}_b = V\angle\phi - 120°$	$\tilde{V}_{bc} = \sqrt{3}\cdot V\angle\phi - 90°$
$\tilde{V}_c = V\angle\phi - 240°$	$\tilde{V}_{ca} = \sqrt{3}\cdot V\angle\phi - 210°$





Phase ↔ Line Voltage Relationship

A comparison of the phase and line voltages:

$$\tilde{V}_a = V\angle\phi° \quad \tilde{V}_{ab} = \sqrt{3}\cdot V\angle\phi + 30°$$

reveals that the line voltages are:

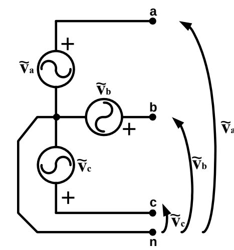
- $\sqrt{3}$x greater in magnitude, and
- 30° greater in phase angle

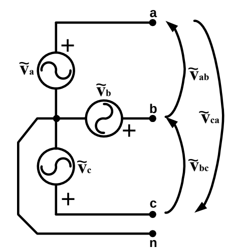
compared to the phase voltages.

$$\tilde{V}_{ab} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_a$$

$$\tilde{V}_{bc} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_b$$

$$\tilde{V}_{ca} = (\sqrt{3}\angle 30°)\cdot\tilde{V}_c$$



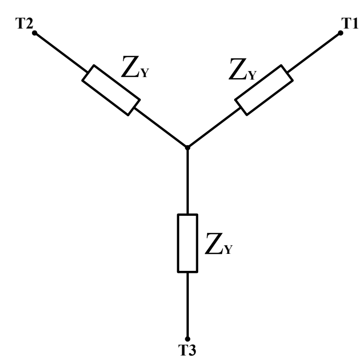


Wye-connected Three-Phase Loads

A ***wye-connected***, three-phase load is constructed by connecting one end of the three individual loads to form a common (neutral) node.

The opposite end of the three individual loads provide the terminals for connection to a 3Φ system.

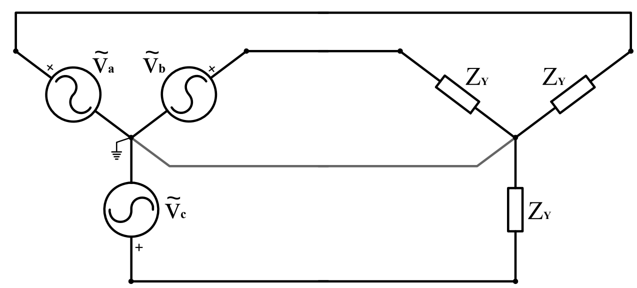
These terminals are often defined as load terminals **T1**, **T2**, and **T3**.



Wye-connected Loads in 3Φ Systems

Three wires or "***lines***" are used to connect the source terminals to the terminals of the Y-connected load.

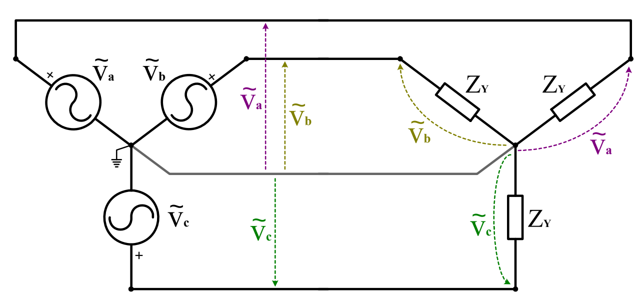
A "***neutral wire***" can be added to connect the grounded neutral-point of the source to the center-point of the load, holding both neutral points at a zero-volt potential.



Wye-connected Loads in 3Φ Systems

Note that the voltage potential present on each line (w.r.t. the neutral wire) is equal to the ***phase voltage*** of the source's phase to which the line is connected.

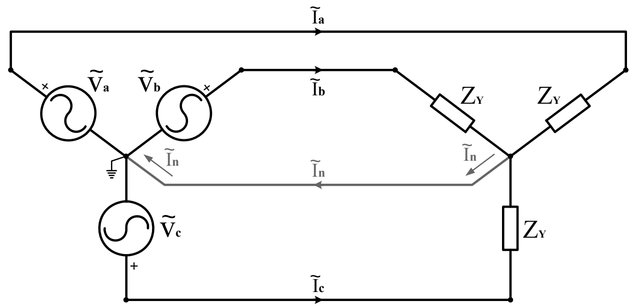
Thus, the four-wire connection results in the presence of a ***phase voltage*** <u>across each phase of the load</u>.



Wye-connected Load Currents

A set of ***line currents*** ($\tilde{I}_a$, $\tilde{I}_b$ and $\tilde{I}_c$) can be defined that flow from each phase of the source, down the lines and into the individual phases of the load.

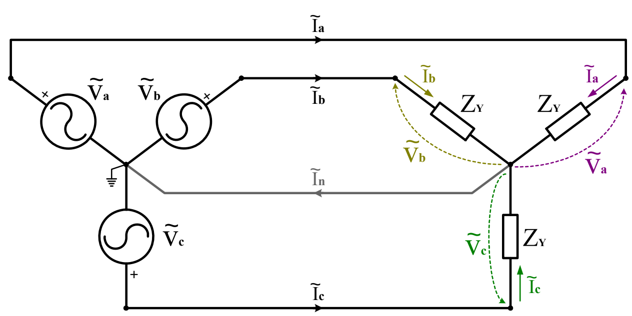
A ***neutral current*** ($\tilde{I}_n$) can also be defined that flows in the neutral wire from the load back to the source.



Wye-connected Load Currents

Since the phase voltages of the load and source are equal, the **line currents** can each be solved independently by applying Ohm's Law at each load.

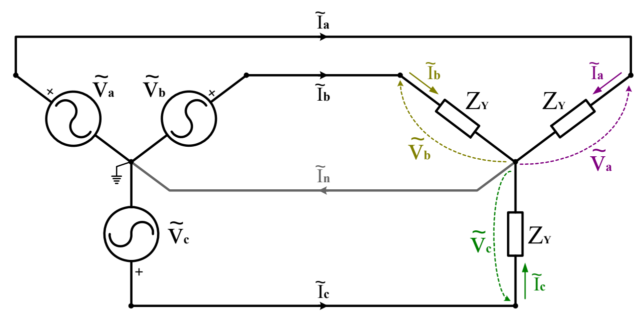
$$\tilde{I}_a = \frac{\tilde{V}_a}{Z_Y} \qquad \tilde{I}_b = \frac{\tilde{V}_b}{Z_Y} \qquad \tilde{I}_c = \frac{\tilde{V}_c}{Z_Y}$$



Wye-connected Load Currents

Furthermore, if the source voltages are balanced and the load impedances are all equal, then the **line currents** will also be balanced.

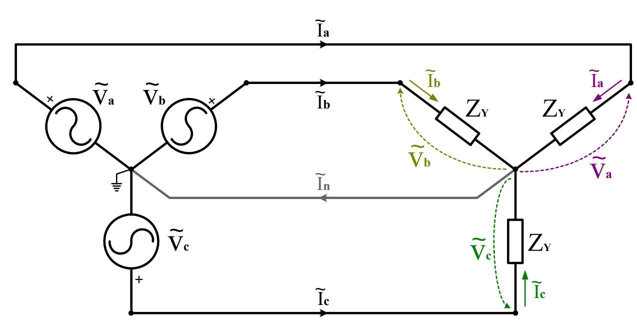
$$\tilde{I}_a = I\angle\delta \qquad \tilde{I}_b = I\angle\delta - 120° \qquad \tilde{I}_c = I\angle\delta - 240°$$



Complex Power in 3Φ Systems

The **total complex power** produced or consumed by a 3Φ source or load is equal to the sum of the complex powers produced or consumed by each of the source's or load's three individual phases.

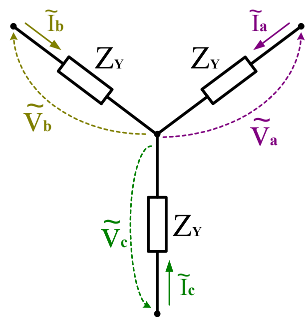
$$S_{3\Phi} = S_a + S_b + S_c$$



Complex Power in Y-connected Loads

In the case of a 3Φ, Y-connected load, the **complex powers** consumed by each of the load's three individual phases are:

$$S_a = \tilde{V}_a \cdot \tilde{I}_a^* \qquad S_b = \tilde{V}_b \cdot \tilde{I}_b^* \qquad S_c = \tilde{V}_c \cdot \tilde{I}_c^*$$



Complex Power in Y-connected Loads

Thus, the **total complex power** consumed by a balanced, 3Φ, Y-connected load will be equal to **3x** the power consumed by any individual phase:

$$S_{3\Phi} = S_a + S_b + S_c = 3 \cdot S_{1\Phi}$$

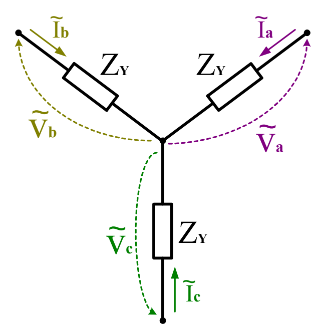
allowing the total complex power to be expressed in terms of a single phase:

$$S_{3\Phi} = 3 \cdot \tilde{V}_a \cdot \tilde{I}_a^* = 3 \cdot V \cdot I \angle \phi - \delta$$

where:

$$\tilde{V}_a = V \angle \phi$$

$$\tilde{I}_a = I \angle \delta$$

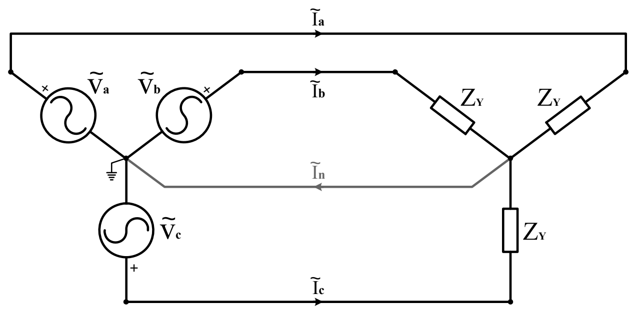


Neutral Current in 3Φ Systems

In a balanced system, the **neutral current** $\tilde{I}_n$ will be:

$$\tilde{I}_n = \tilde{I}_a + \tilde{I}_b + \tilde{I}_c = I\angle\delta + I\angle(\delta - 120°) + I\angle(\delta - 240°) = 0$$

If the line currents are balanced, then they will sum to zero
→ **no current will flow in the neutral wire**.



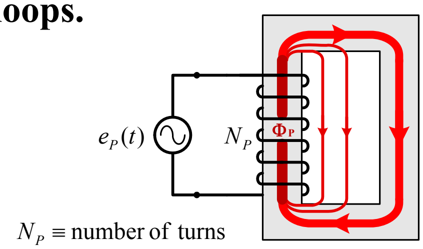
Transformers

AC-Supplied Coil

If a coil is supplied by an AC-source, $e_P(t)$, then a time-varying <u>magnetic flux</u>, Φ_P, will be created, as defined by:

$$e_P(t) = N_P \cdot \frac{d\Phi_P(t)}{dt} \quad \text{(Faraday's Law)}$$

the field lines of which will pass through the center of the coil and then back around the outside in order to form closed-loops.

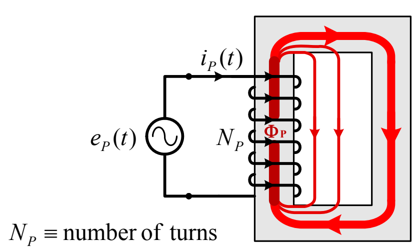


Self-Inductance

<u>Self-inductance</u>, L_P, can be defined as:

$$L_P = N_P \cdot \frac{d\Phi_P(t)}{di_P(t)}$$

such that L_P is proportional to the rate of change in the flux created by the coil over the rate of change in the current flowing through the coil.

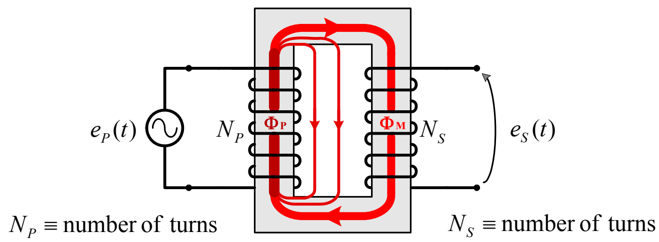


Mutually-Linked Coils

If a second coil is placed such that some of the flux passes through the coil, then a voltage will be induced across the second coil, also defined by:

$$e_S(t) = N_S \cdot \frac{d\Phi_M(t)}{dt} \quad \text{(Faraday's Law)}$$

where: $\boldsymbol{\Phi_M}$ **is the flux that passes through the second coil.**

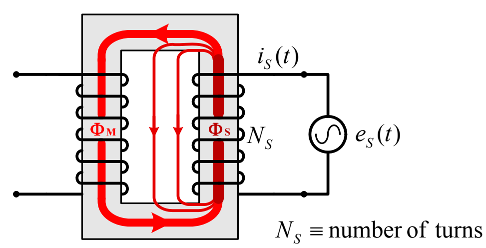


Self-Inductance

Note that, if an AC source is connected to the second coil, a self-inductance, L_S, can also be defined for that coil:

$$L_S = N_S \cdot \frac{d\Phi_S(t)}{di_S(t)} \quad \textbf{(Henries)}$$

such that L_S is proportional to the rate of change in the flux created by that coil over the rate of change that coil's current.

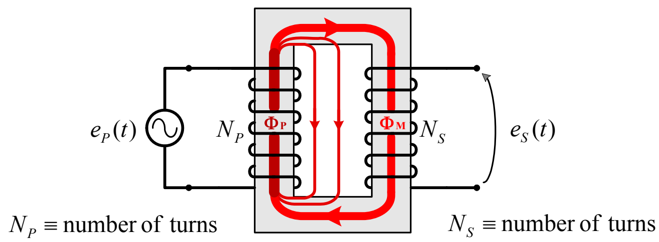


Coupling Factor

A <u>coupling-factor</u>, k, can be defined as:

$$k = \frac{\Phi_M(t)}{\Phi_P(t)}$$

such that k is the ratio of the flux created by the first coil that passes through the second coil over the total flux created by the first coil.

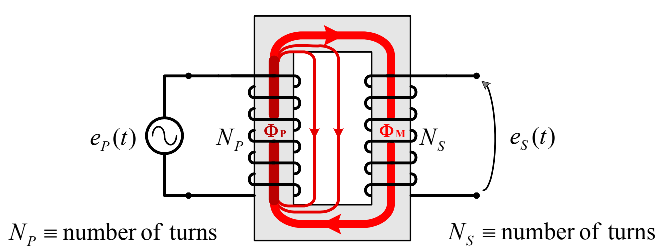


Mutual Inductance

Furthermore, a <u>mutual inductance</u>, M, can be defined as:

$$M = N_S \cdot \frac{d\Phi_M(t)}{di_P(t)} \quad \textbf{(Henries)}$$

such that M is proportional to the rate of change in the flux that passes through the second coil over the rate of change in the current flowing through the first coil.



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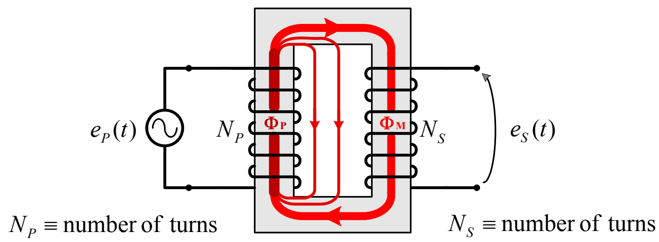
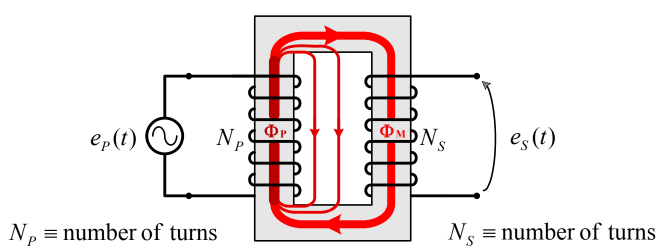


Mutual Inductance

It also turns out that the **mutual inductance**, M, can be expressed in terms of the self-inductances of the coils as:

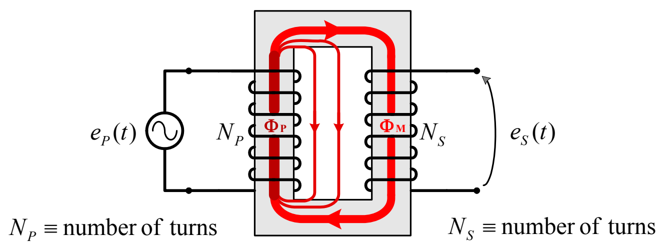
$$M = k \cdot \sqrt{L_P \cdot L_S} \quad \textbf{(Henries)}$$



Mutually-Linked Coils

Note that the voltage, $e_S(t)$, induced across the second coil by the <u>mutually-linked flux</u> created by the first coil can be expressed in terms of the mutual inductance as follows:

$$e_S(t) = M \cdot \frac{di_P(t)}{dt} \qquad \textbf{(volts)}$$

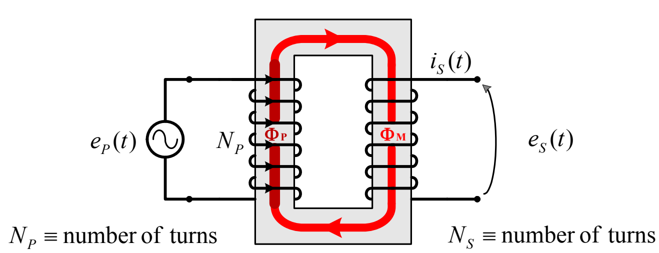


Iron-Core Transformers

An "ideal" iron-core transformer consists of two coils that are mutually-linked by an iron core that provides an "ideal" closed-loop path for the flux created by the first coil.

If all of the flux stays within the iron core, then all of the flux created by the first coil will pass through the second coil:

$$\Phi_M(t) = \Phi_P(t) = \Phi_S(t)$$

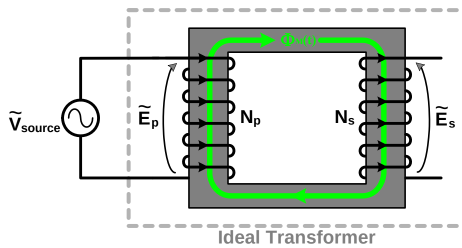


Mutually Linked Coils

If the magnetic core is assumed to be ideal, then the total flux created by the sourced coil will pass through the second coil.

Since a time-varying flux passes through the second coil, a voltage will be induced across that coil, also defined by:

$$\tilde{E}_s = N_s \cdot \frac{d\Phi_M(t)}{dt}$$

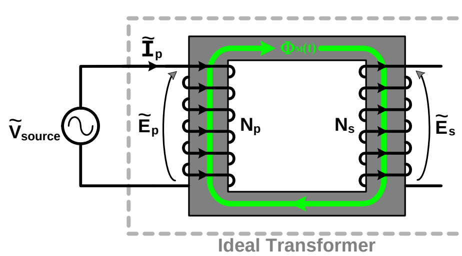


Mutually Linked Coils

If the total flux passes through both coils, then the rate of change, $\frac{d\Phi(t)}{dt}$, of the flux through the coils must be the same.

The following relationship may be derived by solving for $\frac{d\Phi(t)}{dt}$ in both coils and equating the results:

$$\frac{\tilde{E}_s}{N_s} = \frac{d\Phi_M(t)}{dt} = \frac{\tilde{E}_p}{N_p}$$

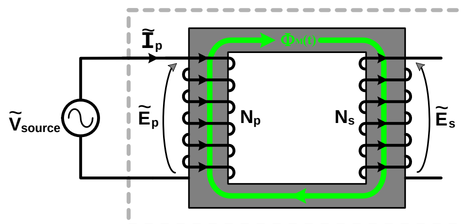


Voltage Relationship

The relationship between the two coil voltages is typically expressed as a ratio of the voltages, which equals to the ratio of their respective number of turns.

(I.e. – the "turns ratio" of the transformer).

$$\frac{\widetilde{E}_p}{N_p} = \frac{\widetilde{E}_s}{N_s} \quad \Rightarrow \quad \boxed{\frac{\widetilde{E}_p}{\widetilde{E}_s} = \frac{N_p}{N_s}}$$



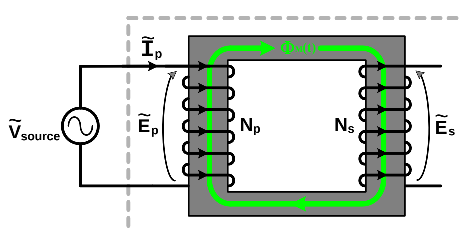
Ideal Transformer

Turns-Ratio

The ratio relationship, referred to as the <u>turns ratio</u> (*a*):

$$a = \frac{\widetilde{E}_p}{\widetilde{E}_s} = \frac{N_p}{N_s}$$

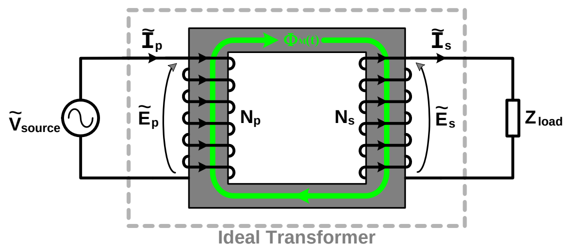
defines the basic operation of an ideal transformer in terms of the primary and secondary voltages.



Ideal Transformer

Determining the Polarity Relationship

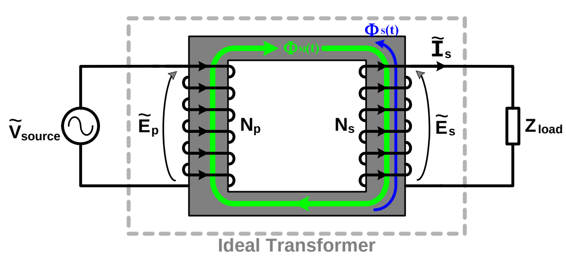
If a load is connected to the second winding, then a current will flow out of the secondary winding and through the load due to the induced voltage.



Secondary Current Effects

But, the existence of a counter-flux produced by the current that is flowing in the second coil would tend to decrease the overall flux within the magnetic core, in-turn decreasing the total flux passing through the primary coil.

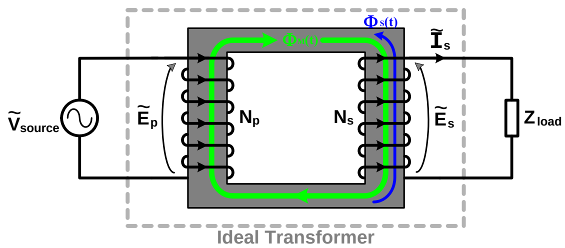
$$\Phi_{Net} = \Phi_M - \Phi_S$$



Secondary Current Effects

Assuming that the source is ideal, this presents a problem because Faraday's Law does not allow for a change in the flux passing through the primary coil unless the supply voltage changes accordingly.

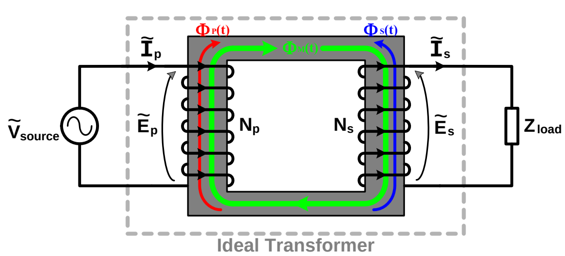
$$\tilde{E}_P = N_P \cdot \frac{d\Phi_{Net}(t)}{dt}$$



Secondary Current Effects

Thus, the existence of the secondary current's counter-flux requires that a additional (primary) current be drawn into the primary winding.

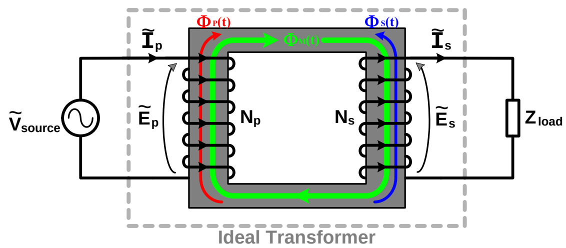
The primary current will, in-turn, create an additional flux component, Φ_p, within the core that is equal in magnitude but opposite in direction compared to the secondary flux Φ_s.



Primary Current

Since the primary and secondary fluxes are equal in magnitude but opposite in direction, they will cancel, leaving the net flux in the core the same as defined by Faraday's Law applied to the primary winding:

$$\Phi_{Net} = \Phi_M - \Phi_S + \Phi_P = \Phi_M$$



Primary/Secondary Current Ratio

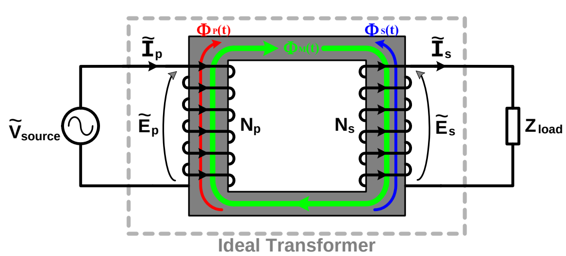
Based on the MMF relationship applied to both coils:

$$N \cdot i(t) = \Phi(t) \cdot \Re$$

the ratio of the primary and secondary currents must be:

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$

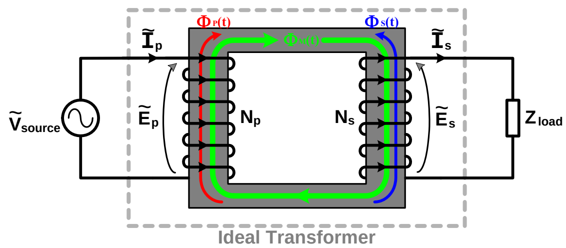
in order for their fluxes to cancel.



Overall Operation of Ideal Transformer

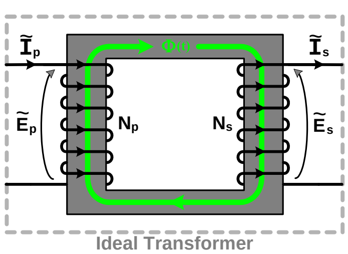
Thus, the overall operation of the ideal transformer that supplies a single load can be defined by the following set of equations:

turns ratio $a = \dfrac{N_p}{N_s}$ $\quad a = \dfrac{\tilde{E}_p}{\tilde{E}_s}$ $\quad \dfrac{\tilde{I}_p}{\tilde{I}_s} = \dfrac{1}{a}$ $\quad \tilde{I}_s = \dfrac{\tilde{E}_s}{Z_{load}}$



Ideal Transformer Equivalent Circuit

The following equivalent circuit will be used to represent an ideal transformer:



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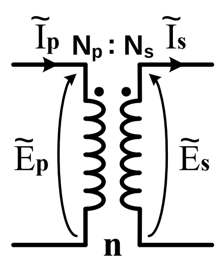


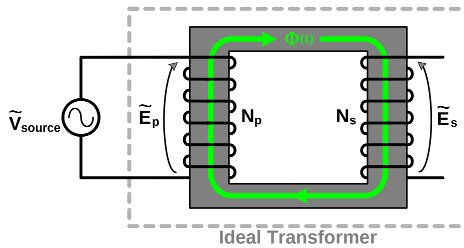
$a = \dfrac{N_p}{N_s}$ $\quad \dfrac{\tilde{E}_p}{\tilde{E}_s} = a$ $\quad \dfrac{\tilde{I}_p}{\tilde{I}_s} = \dfrac{1}{a}$ $\quad a =$ turns ratio

Ideal Transformer Definitions

Primary Winding ≡ the winding that creates the mutually-linked flux (I.e. – the sourced winding).

Secondary Winding ≡ the winding across which a voltage is induced (I.e. – the load winding).

Note – the primary & secondary winding designations can also be defined in terms of the power flow direction (I.e. – the source & load connections)

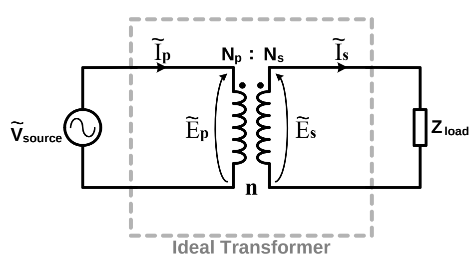


Ideal Transformer Definitions

High-Voltage Winding ≡ the winding with the larger voltage magnitude. (I.e. – the coil with the larger number of turns)

Low-Voltage Winding ≡ the winding with the smaller voltage magnitude. (I.e. – the coil with the smaller number of turns)

Note – the high-voltage winding will have the larger number of turns while the low-voltage winding will have the smaller number of turns.



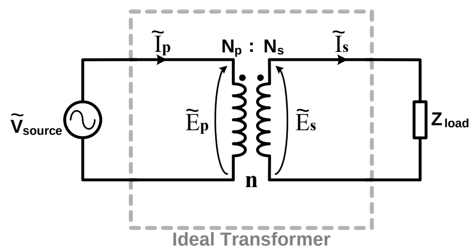
Ideal Transformer Definitions

Step-Up Transformer ≡ a transformer whose <u>voltage</u> increases from primary to secondary winding.

Step-Down Transformer ≡ a transformer whose <u>voltage</u> decreases from primary to secondary winding.

Notes: **A step-up transformer's turns ratio will be less than one (*a*<1).**

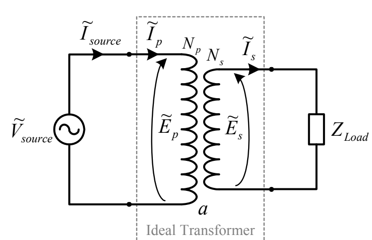
A step-down transformer's turns ratio will be greater than one (*a*>1).



Input Impedance

Thus, the input impedance of an ideal transformer is equal to its turns-ratio squared times the impedance of its connected load:

$$Z_{in} = a^2 \cdot Z_{Load} = Z'_{Load}$$



$$a = \frac{N_p}{N_s}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a$$

$$\frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$