ECET 2111

Circuits II

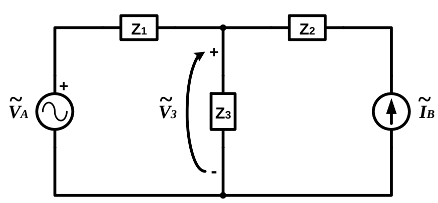
Network Theorems

Superposition Theorem

<u>Superposition Theorem</u>:

Given a linear network that contains multiple sources…

Any voltage (or current) in the network may be determined by solving for that voltage (or current) with each of the sources individually "turned-on" (and all of the other sources "turned-off "), and then summing the individual solutions.

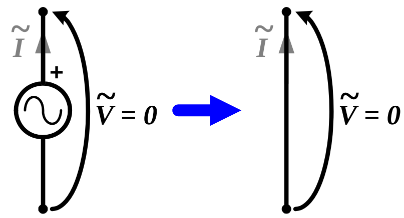


Ideal Sources "Turned-Off"

Since an <u>Ideal Voltage Source</u> maintains a constant voltage independent of current flow, if the source is "<u>turned off</u>" (set to zero volts), then the voltage across the source's terminals will be zero independent of the source's current.

This is equivalent to the characteristics of an "ideal wire"…

Thus, an Ideal Voltage Source that is "turned-off" acts like an ideal wire:

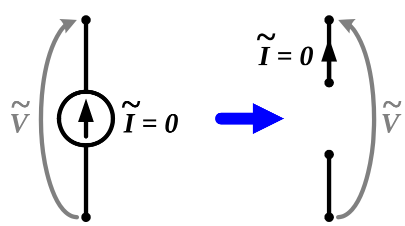


Ideal Sources "Turned-Off"

Since an <u>Ideal Current Source</u> maintains a constant current independent of the source voltage, if the source is "turned off" (set to zero volts), then the current flowing through the source will be zero independent of the voltage across its terminals.

This is equivalent to the characteristics of an "open-circuit"…

Thus, an Ideal Current Source that is "turned-off" acts like an open-circuit:

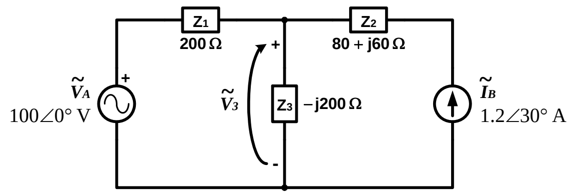


Superposition Theorem Example

<u>Superposition Theorem Example</u>:

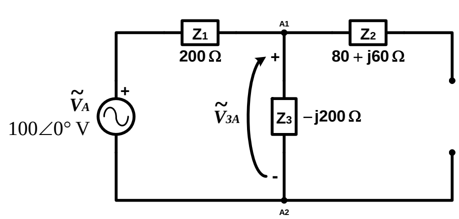
Given the following circuit containing two sources…

Determine the voltage V_3 using the superposition theorem.

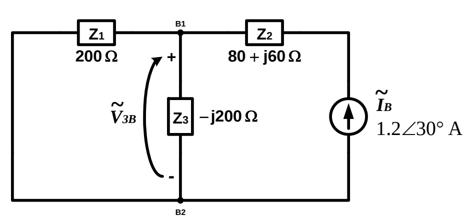


Superposition Theorem Example

In order to determine the voltage V_3 using the superposition theorem, the circuit must be analyzed with each of the sources individually “turned-on” one at a time:



Source A “turned-on”

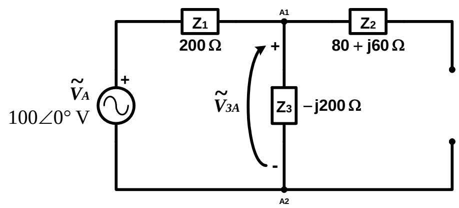


Source B “turned-on”

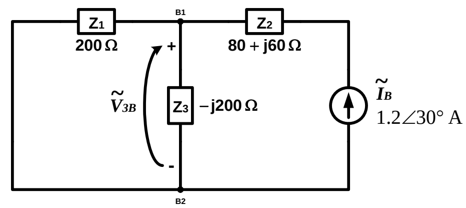
Superposition Theorem Example

After the contributions of the individually-sourced circuits to the voltage V_3 are determined, the results can be summed in order to determine its actual value.

$$\tilde{V}_3 = \tilde{V}_{3A} + \tilde{V}_{3B}$$



Source A "turned-on"



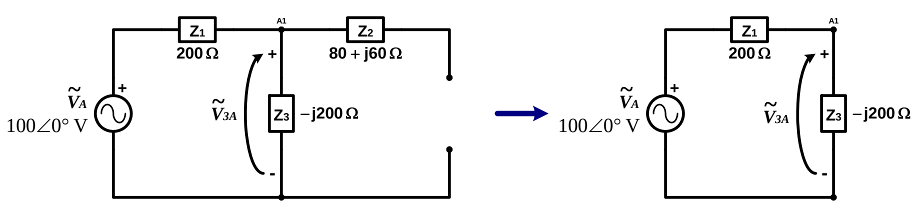
Source B "turned-on"

Superposition Theorem Example

Solve for V_{3A}:

Since a current source reacts like an open-circuit when it is turned-off, the only current path is through Z_1 and Z_3, allowing V_{3A} to be determined using a voltage divider:

$$\tilde{V}_{3A} = 100\angle 0° \cdot \left(\frac{-j200\Omega}{200\Omega - j200\Omega} \right) = 70.7\angle -45° \text{ V}$$



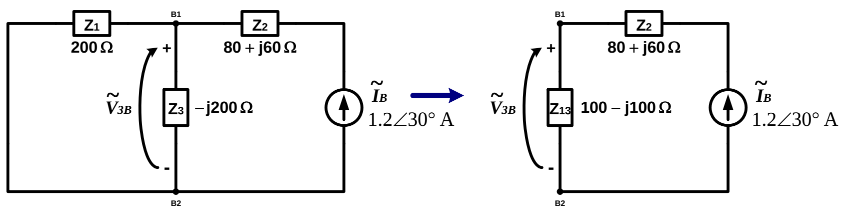
Source A "turned-on"

Superposition Theorem Example

Solve for V_{3B}:

Since a voltage source reacts like a short-circuit when it is turned-off, source "A" can be replaced by an ideal wire, resulting in the parallel connection of Z_1 and Z_3.

$$200\ \Omega \parallel -j200\ \Omega = \left(\frac{1}{200} + \frac{1}{-j200} \right)^{-1} = 100 - j100\ \Omega$$



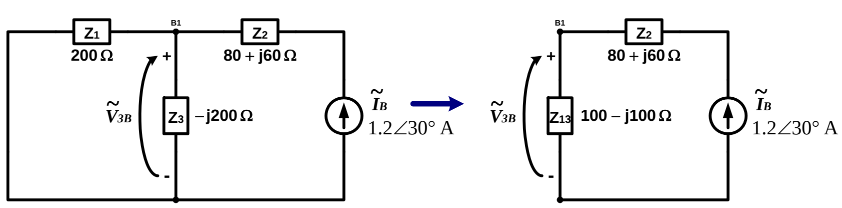
Source B "turned-on"

Superposition Theorem Example

Solve for V_{3B}:

Once Z_1 and Z_3 are replaced by the parallel-equivalent Z_{13}, V_{3B} can be determined using Ohm's Law:

$$\tilde{V}_{3B} = 1.2\angle 30° \cdot (100\Omega - j100\Omega) = 169.7\angle -15°\ \text{V}$$



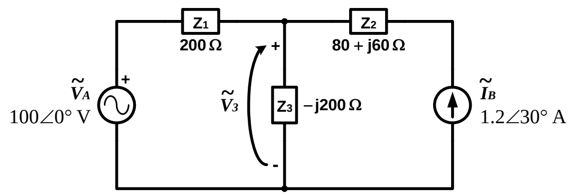
Source B "turned-on"

Superposition Theorem Example

Determine V_3:

Now that V_{3A} and V_{3B} have been solved, the actual voltage V_3 can be determined by summing the individual results:

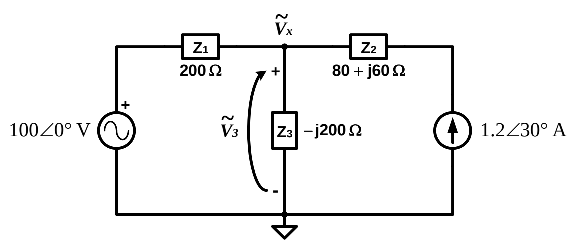
$$\tilde{V}_3 = \tilde{V}_{3A} + \tilde{V}_{3B} = 70.7\angle -45° + 169.7\angle -15° = 233.6\angle -23.7° \text{ V}$$



Superposition Theorem Example

Check the Superposition results using <u>Nodal Analysis</u>:

If the bottom node is chosen as ground and the top node is assigned the node-voltage variable V_x, then the voltage rise from ground to node "x" is equivalent to V_3.



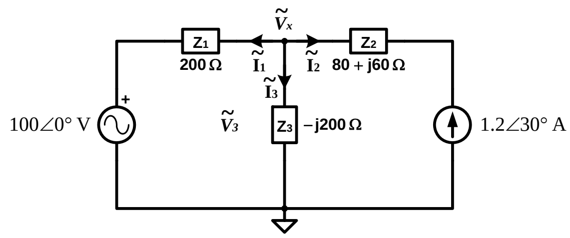
Superposition Theorem Example

Check the Superposition results using <u>Nodal Analysis</u> :

Write the KVL equation for node "x":

$$\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 0$$

$$\frac{\tilde{V}_x - 100\angle 0°}{200} - 1.2\angle 30° + \frac{\tilde{V}_x - 0}{-\text{j}200} = 0$$



Superposition Theorem Example

Check the Superposition results using <u>Nodal Analysis</u> :

Determine V_x from the node equation: $\frac{\tilde{V}_x - 100\angle 0°}{200} - 1.2\angle 30° + \frac{\tilde{V}_x - 0}{-\text{j}200} = 0$

$$\frac{\tilde{V}_x}{200} + \frac{-100\angle 0°}{200} + \frac{\tilde{V}_x}{-\text{j}200} = 1.2\angle 30°$$

$$\tilde{V}_x\left(\frac{1}{200} + \frac{1}{-\text{j}200}\right) = 1.2\angle 30° + \frac{100\angle 0°}{200}$$

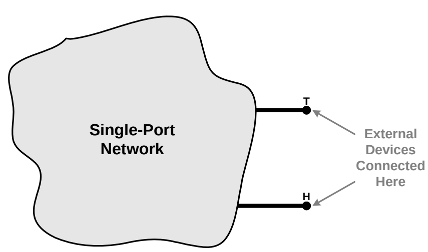
$$\tilde{V}_x(0.005 + \text{j}0.005) = 1.652\angle -21.29°$$

$$\tilde{V}_x(0.005 + \text{j}0.005) = \frac{1.652\angle -21.29°}{0.005 + \text{j}0.005} = \boxed{223.6\angle -23.7°\ \text{V}}$$

Thevenin's Theorem

Thevenin's Theorem

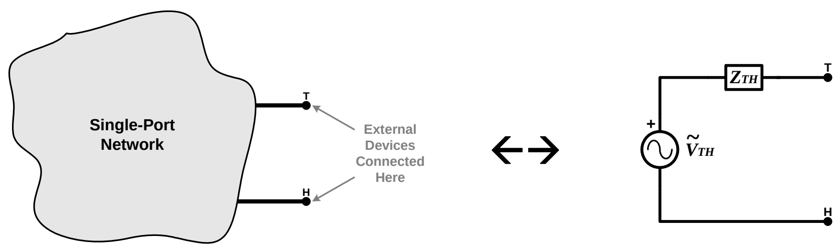
<u>**Thevenin's theorem**</u> **provides a method for modeling the response of a complex single-port network by replacing the complex network with a simple "equivalent" single-port network that will have the same response as the original network to any externally-connected devices.**



Thevenin's Theorem

<u>Thevenin's theorem</u>: (For steady-state AC circuits)

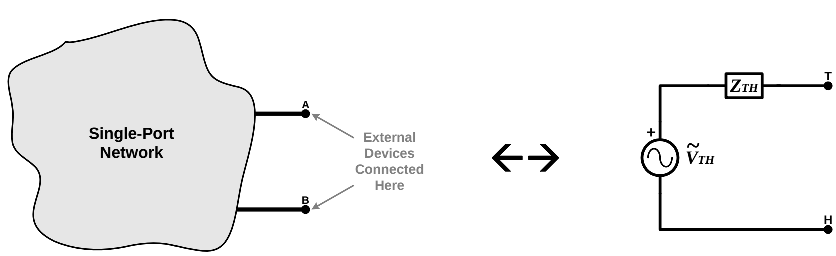
Any linear, single-port network can be replaced by a simple equivalent network that consists of a single voltage source, V_{TH}, in series with a single impedance, Z_{TH}, without affecting the operation of any devices that are externally connected to the network's terminals.



Thevenin's Theorem

The simple equivalent network is often referred to as <u>Thevenin's Equivalent Circuit</u>, where:

- **V_{TH} is Thevenin's Equivalent Voltage, and**
- **Z_{TH} is Thevenin's Equivalent Impedance.**



Thevenin's Theorem

The values for V_{TH} and Z_{TH} can be determined as follows:

The value for V_{TH} is obtained by determining the open-circuited terminal voltage of the single-port network with all sources in the network "turned-on".

The value for Z_{TH} is obtained by determining the resistance "seen" looking into the terminals of the original single-port network with all of the independent sources contained within the network "turned-off".

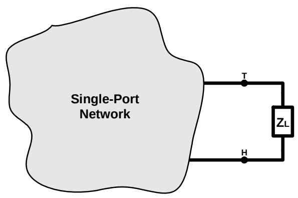
Maximum Power Transfer Theorem

Maximum Power Transfer Theorem

The <u>Maximum Power Transfer Theorem</u> states that, in order to transfer maximum power from a linear, single-port, AC network to a single, <u>arbitrary impedance-type load</u> where:

$$Z_L = R_L + jX_L$$

the value of the impedance should be set equal to the complex conjugate of the single-port network's Thevenin Equivalent Impedance.



If: $Z_{TH} = R_{TH} + jX_{TH}$

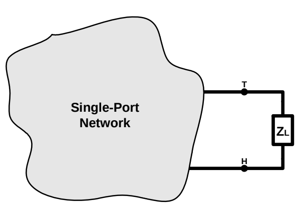
Then set: $Z_L = Z_{TH}^* = R_{TH} - jX_{TH}$

Maximum Power Transfer Theorem

Additionally, the <u>Maximum Power Transfer Theorem</u> states that, in order to transfer maximum power from the single-port network to an arbitrary, <u>purely-resistive load</u> where:

$$Z_L = R_L$$

the value of the load resistance should be set equal to the magnitude of the Thevenin Equivalent Impedance.

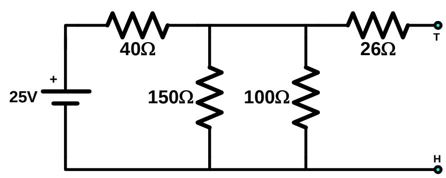


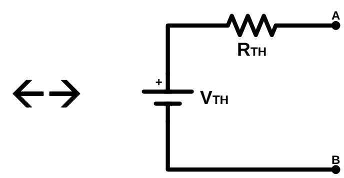
If: $Z_{TH} = R_{TH} + jX_{TH}$

Then set: $R_L = \left|Z_{TH}\right| = \sqrt{R_{TH}^2 + X_{TH}^2}$

Thevenin's Theorem Example

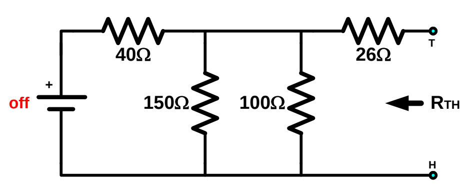
Determine the <u>Thevenin's Equivalent Circuit</u> for the following network with respect to terminals "T" and "H":





Thevenin's Theorem Example

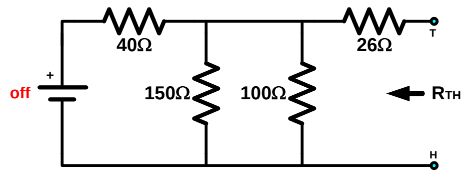
Determine <u>Thevenin's Resistance</u>, R_{TH}:



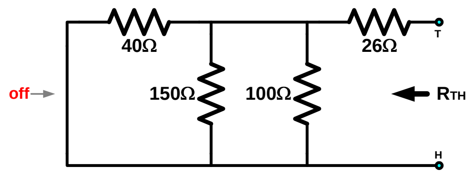
The value for R_{TH} is obtained by determining the resistance looking into terminals "T" and "H" while all sources are "turned-off".

Thevenin’s Theorem Example

Determine <u>Thevenin’s Resistance</u>, R_{TH}:



The value for R_{TH} is obtained by determining the resistance looking into terminals “T” and “H” while all sources are “turned-off”.

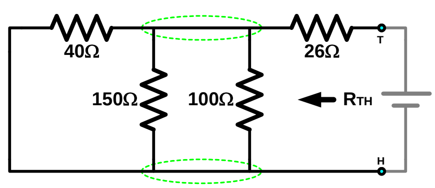


1st – “Turn-off” all sources.

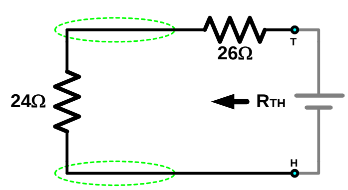
Remember that an ideal voltage source, when “turned-off”, looks like an ideal wire.

Thevenin’s Theorem Example

Determine <u>Thevenin’s Resistance</u>, R_{TH}:



2nd – Place a “test” source across terminals “T” and “H”, and reduce the network connected to the “test” source down to a single equivalent resistance.

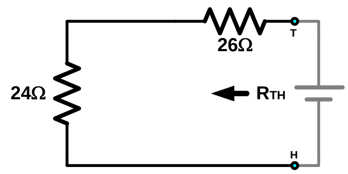


Replace 40Ω||150Ω||100Ω with single equivalent resistance:

$$R_{eq} = \left(\frac{1}{40} + \frac{1}{150} + \frac{1}{100} \right)^{-1} = 24\Omega$$

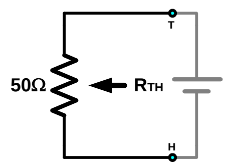
Thevenin's Theorem Example

Determine <u>Thevenin's Resistance</u>, R_{TH}:



Finally – Replace the 24Ω and 26Ω series-connected resistors with a single equivalent resistance:

$$R_{eq} = 24 + 26 = 50\Omega$$

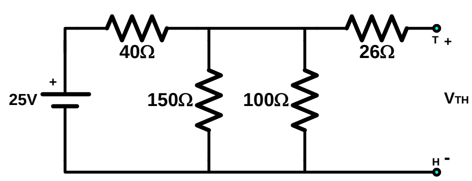


The resistance seen "looking" into terminals "T" and "H" of the network with all sources "turned-off" is:

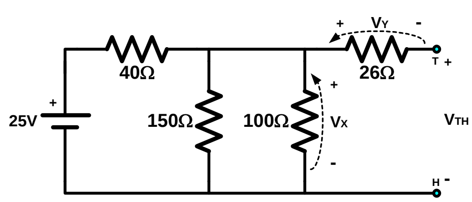
$$R_{TH} = 50\Omega$$

Thevenin's Theorem Example

Determine <u>Thevenin's Voltage</u>, V_{TH}:



The value for V_{TH} is obtained by determining the voltage across terminals "T" and "H" while the terminals are open-circuited.

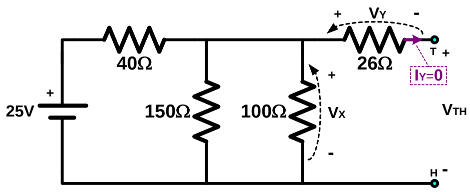


Based on KVL, V_{TH} can be defined in terms of the resistor voltages V_X and V_Y:

$$V_{TH} = V_X - V_Y$$

Thevenin’s Theorem Example

Determine Thevenin’s Voltage, V_{TH}:



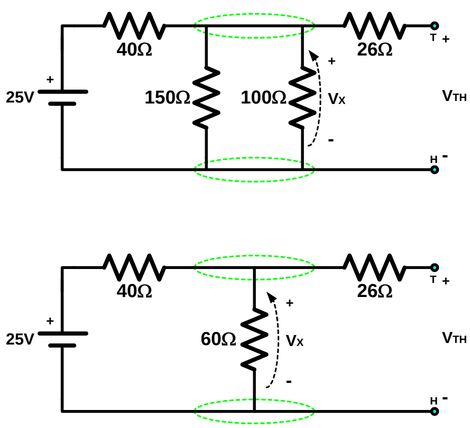
Since I_Y must equal zero due to the open-circuit between terminals “T” and “H”, then V_Y must also equal zero, and:

$$V_{TH} = V_X$$

Thus, the Thevenin’s Voltage can be determined by solving for the voltage, V_X, across the 100Ω resistor.

Thevenin’s Theorem Example

Determine Thevenin’s Voltage, V_{TH}:



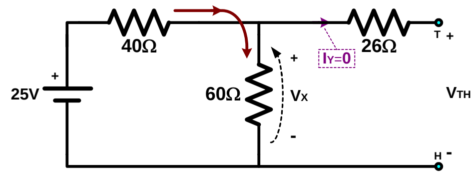
1st – Reduce the circuit by replacing the 150Ω||100Ω resistors with a single equivalent resistance.

$$R_{eq} = \left(\frac{1}{150} + \frac{1}{100}\right)^{-1} = 60\Omega$$

Note that the voltage across the 60Ω resistor will equal V_X in the previous circuit.

Thevenin's Theorem Example

Determine Thevenin's Voltage, V_{TH}:



Since no current can flow through the 26Ω resistor, all of the current that flows through the 40Ω resistor must flow through the 60Ω resistor.

Thus, the two resistors are operationally connected in-series, allowing V_X to be solved using a voltage divider:

$$V_{TH} = V_X = 25 \cdot \left(\frac{60}{40 + 60} \right) = 15\,\text{V}$$

Thevenin's Theorem Example

The Thevenin's Equivalent Circuit for the following network with respect to terminals "T" and "H" is: