ECET 2111

Circuits II

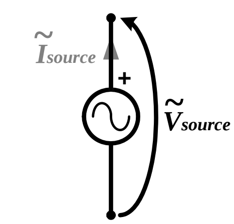
Dependent and Independent Sources

◆

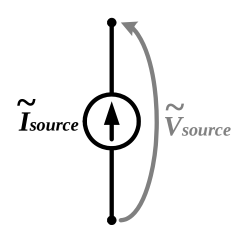
Mesh and Nodal Analysis

Ideal Sources

An <u>Ideal Voltage Source</u> is a source that maintains a constant voltage potential across its terminals independent of the current flowing through the source.



An <u>Ideal Current Source</u> is a source that maintains a constant current flow through the source independent of the voltage potential that is required to maintain that current.

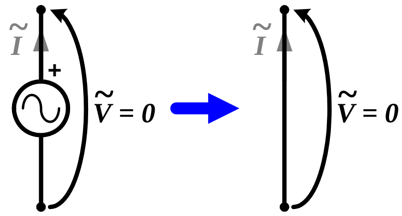


Ideal Sources "Turned-Off"

Since an Ideal Voltage Source maintains a constant voltage independent of current flow, if the source is "turned off" (set to zero volts), then the voltage across the source's terminals will be zero independent of the source's current.

This is equivalent to the characteristics of an "ideal wire"…

Thus, an Ideal Voltage Source that is "turned-off" acts like an ideal wire:

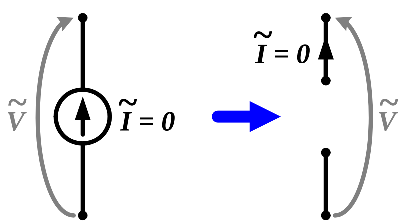


Ideal Sources "Turned-Off"

Since an Ideal Current Source maintains a constant current independent of the source voltage, if the source is "turned off" (set to zero volts), then the current flowing through the source will be zero independent of the voltage across its terminals.

This is equivalent to the characteristics of an "open-circuit"…

Thus, an Ideal Current Source that is "turned-off" acts like an open-circuit:



Independent vs Dependent Sources

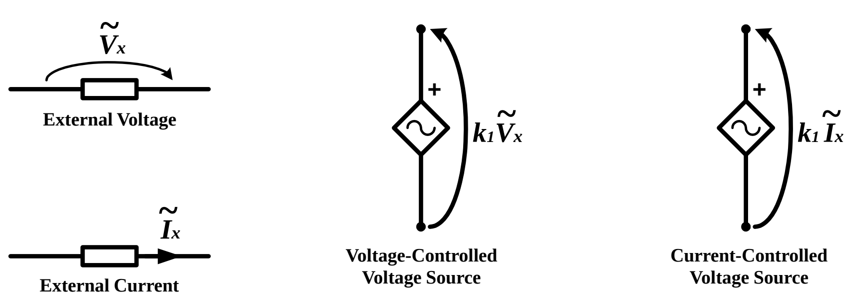
An **Independent Source** is a source that maintains its specified terminal characteristics (voltage or current) independent of the operation or characteristics of any other circuit element to which it is connected.

I.e. – it maintains a constant voltage or current whether or not it is connected within a circuit.

A **Dependent Source** (or Controlled Source) is a source whose terminal characteristics (voltage or current) are determined (or controlled) by an external voltage or current of the system in which it appears.

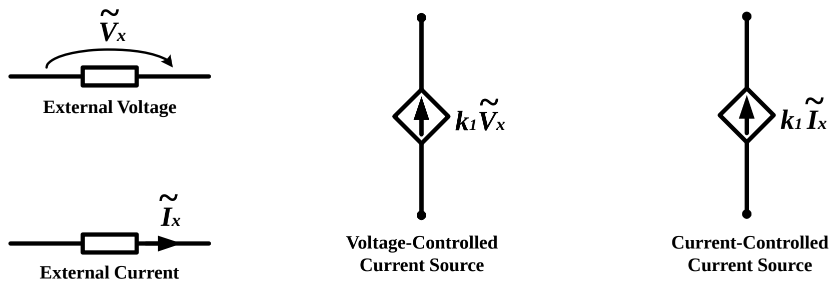
Dependent Voltage Source

A **Dependent Voltage Source** is a source whose voltage potential is determined (or controlled) by an external voltage or current of the system in which it appears.



Dependent Current Source

A <u>Dependent Current Source</u> is a source whose current is determined (or controlled) by an external voltage or current of the system in which it appears.



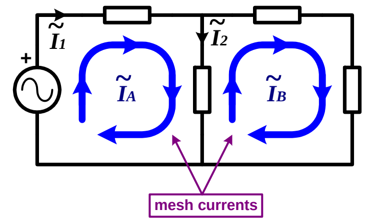
Mesh Analysis

Although many circuits can easily be solved by applying basic Ohm's Law theory such as with the Reduce-and-Return method for solving series-parallel circuits, these brute force methods of solution can become impractical for complex circuits or for circuits that contain multiple sources.

<u>Mesh Analysis</u> is an method of analysis that allows for the simultaneous solution of all of the <u>*currents*</u> within an electric circuit by specifying and solving a complete set of <u>*Kirchhoff's Voltage Law*</u> equations, the voltages of which are expressed in terms of a complete set of "*mesh currents*".

Mesh Analysis

<u>Mesh Currents</u> are theoretical currents that flow in closed-loops around the "*independent meshes*" in a circuit.

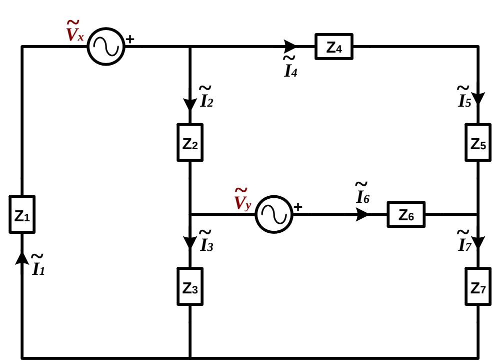


<u>Independent Meshes</u> are closed-loop paths in a circuit that do not contain any other closed-loop paths.

Although the mesh currents are theoretical currents, the <u>actual currents</u> that flow in the various branches of the circuit can easily be determined from the mesh currents, in-turn allowing for solution of any of the circuit voltages.

Mesh Analysis Example

Perform a <u>Mesh Analysis</u> of the following circuit and then define each of the circuit's branch currents in terms of the mesh currents.

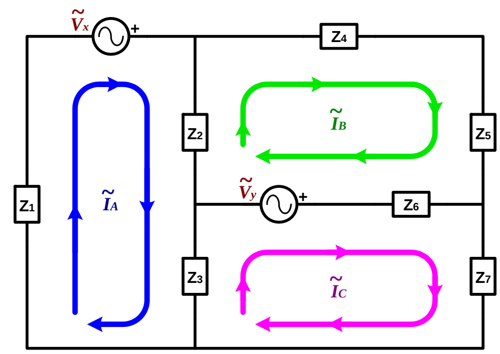


Mesh Analysis Example

Step 1: Define a complete set of mesh currents for the circuit, such that there is a mesh current flowing in each of the circuit's independent meshes.

Note – although it is not required, all of the mesh currents are typically drawn such that they flow around the loops in the same directions…

As a standard, a clock-wise direction will be chosen for all mesh currents.

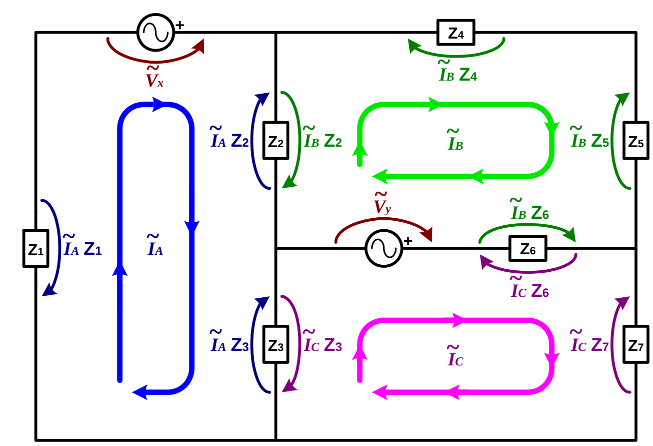


Mesh Analysis Example

Step 2: Define the voltage drops that will occur across each of the circuit impedances due to the mesh currents.

An individual voltage drop must be defined across every impedance for each of the mesh currents that circulate through the impedance.

Thus, impedance Z_1 only experiences one voltage drop while impedance Z_2 experiences two.



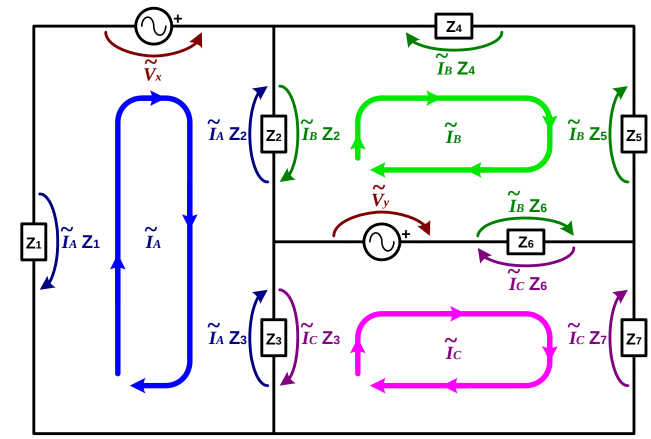
Mesh Analysis Example

Step 3: **Write a Kirchhoff's Voltage Law (KVL) equation for each mesh with the unknown voltages expressed in terms of the mesh currents.**

As a standard, the KVL equations:

$$\sum V_{rises} - \sum V_{drops} = 0$$

will be defined around each of the meshes in the opposite direction compared to the flow of the mesh currents.



Mesh Analysis Example

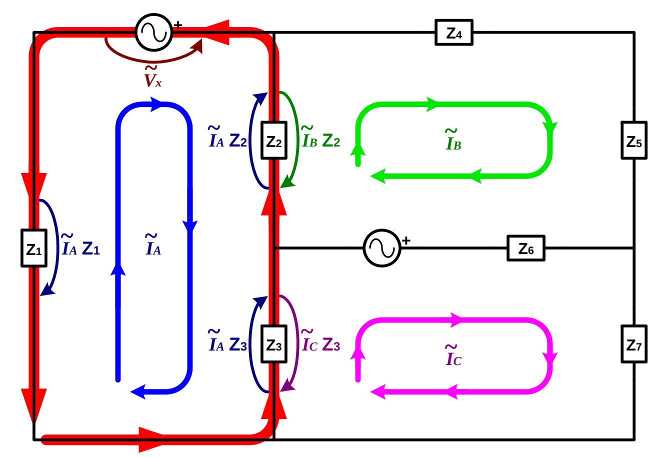
Step 3A: **The KVL equation for Mesh "A" is:**

$$\left(\tilde{I}_A \cdot Z_3 + \tilde{I}_A \cdot Z_2 + \tilde{I}_A \cdot Z_1\right) - \left(\tilde{I}_C \cdot Z_3 + \tilde{I}_B \cdot Z_2 + \tilde{V}_x\right) = 0$$

The KVL equations will all be rewritten in the standard form:

$$A \cdot \tilde{I}_A + B \cdot \tilde{I}_B + C \cdot \tilde{I}_C = K$$

once all of the equations have been defined.



Mesh Analysis Example

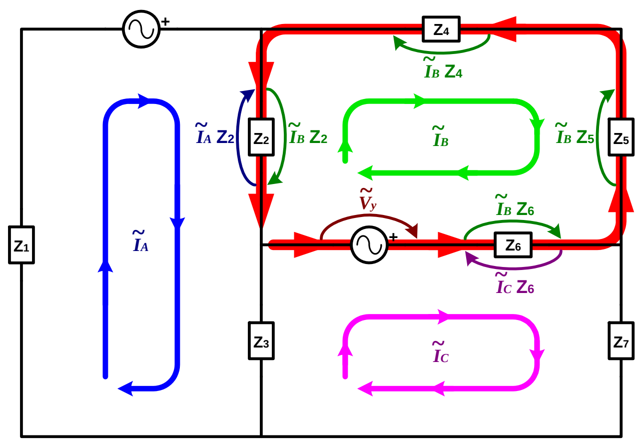
Step 3B: The KVL equation for Mesh "B" is:

$$\left(\tilde{I}_B \cdot Z_6 + \tilde{I}_B \cdot Z_5 + \tilde{I}_B \cdot Z_4 + \tilde{I}_B \cdot Z_2 + \tilde{V}_y\right) - \left(\tilde{I}_C \cdot Z_6 + \tilde{I}_A \cdot Z_2\right) = 0$$

The KVL equations will all be rewritten in the standard form:

$$A \cdot \tilde{I}_A + B \cdot \tilde{I}_B + C \cdot \tilde{I}_C = K$$

once all of the equations have been defined.



Mesh Analysis Example

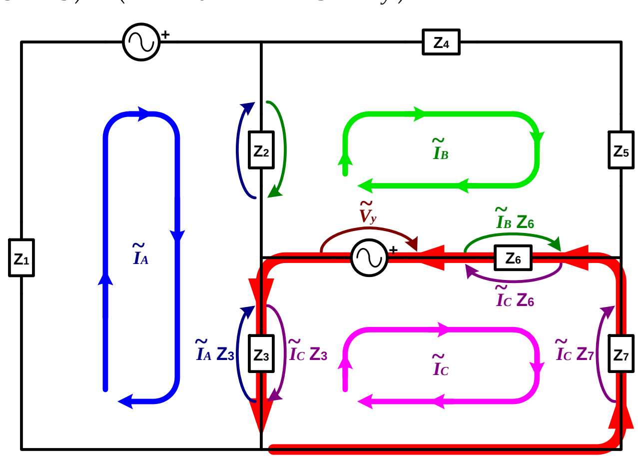
Step 3C: The KVL equation for Mesh "C" is:

$$\left(\tilde{I}_C \cdot Z_7 + \tilde{I}_C \cdot Z_6 + \tilde{I}_C \cdot Z_3\right) - \left(\tilde{I}_B \cdot Z_6 + \tilde{I}_A \cdot Z_3 + \tilde{V}_y\right) = 0$$

The KVL equations will all be rewritten in the standard form:

$$A \cdot \tilde{I}_A + B \cdot \tilde{I}_B + C \cdot \tilde{I}_C = K$$

once all of the equations have been defined.



Mesh Analysis Example

Step 4: **Rewrite the KVL equations:**

$$\left(\tilde{I}_A \cdot Z_3 + \tilde{I}_A \cdot Z_2 + \tilde{I}_A \cdot Z_1\right) - \left(\tilde{I}_C \cdot Z_3 + \tilde{I}_B \cdot Z_2 + \tilde{V}_x\right) = 0$$

$$\left(\tilde{I}_B \cdot Z_6 + \tilde{I}_B \cdot Z_5 + \tilde{I}_B \cdot Z_4 + \tilde{I}_B \cdot Z_2 + \tilde{V}_y\right) - \left(\tilde{I}_C \cdot Z_6 + \tilde{I}_A \cdot Z_2\right) = 0$$

$$\left(\tilde{I}_C \cdot Z_7 + \tilde{I}_C \cdot Z_6 + \tilde{I}_C \cdot Z_3\right) - \left(\tilde{I}_B \cdot Z_6 + \tilde{I}_A \cdot Z_3 + \tilde{V}_y\right) = 0$$

in standard form:

$$\left(Z_1 + Z_2 + Z_3\right) \cdot \tilde{I}_A - Z_2 \cdot \tilde{I}_B - Z_3 \cdot \tilde{I}_C = \tilde{V}_x$$

$$-Z_2 \cdot \tilde{I}_A + \left(Z_2 + Z_4 + Z_5 + Z_6\right) \cdot \tilde{I}_B - Z_6 \cdot \tilde{I}_C = -\tilde{V}_y$$

$$-Z_3 \cdot \tilde{I}_A - Z_6 \cdot \tilde{I}_B + \left(Z_3 + Z_6 + Z_7\right) \cdot \tilde{I}_C = \tilde{V}_y$$

Mesh Analysis Example

Step 4: **Given the KVL equations:**

$$\left(Z_1 + Z_2 + Z_3\right) \cdot \tilde{I}_A - Z_2 \cdot \tilde{I}_B - Z_3 \cdot \tilde{I}_C = \tilde{V}_x$$

$$-Z_2 \cdot \tilde{I}_A + \left(Z_2 + Z_4 + Z_5 + Z_6\right) \cdot \tilde{I}_B - Z_6 \cdot \tilde{I}_C = -\tilde{V}_y$$

$$-Z_3 \cdot \tilde{I}_A - Z_6 \cdot \tilde{I}_B + \left(Z_3 + Z_6 + Z_7\right) \cdot \tilde{I}_C = \tilde{V}_y$$

and:

$$\begin{aligned} Z_{AA} \cdot \tilde{I}_A &+ Z_{AB} \cdot \tilde{I}_B + Z_{AC} \cdot \tilde{I}_C = \tilde{V}_x \\ Z_{BA} \cdot \tilde{I}_A &+ Z_{BB} \cdot \tilde{I}_B + Z_{BC} \cdot \tilde{I}_C = -\tilde{V}_y \\ Z_{CA} \cdot \tilde{I}_A &+ Z_{CB} \cdot \tilde{I}_B + Z_{CC} \cdot \tilde{I}_C = \tilde{V}_y \end{aligned}$$

Mesh Analysis Example

Step 4: Given the KVL equations:

$$\begin{aligned}
(Z_1+Z_2+Z_3)\cdot\tilde{I}_A \quad & -Z_2\cdot\tilde{I}_B & -Z_3\cdot\tilde{I}_C &= \tilde{V}_x \\
-Z_2\cdot\tilde{I}_A \quad & +(Z_2+Z_4+Z_5+Z_6)\cdot\tilde{I}_B & -Z_6\cdot\tilde{I}_C &= -\tilde{V}_y \\
-Z_3\cdot\tilde{I}_A \quad & -Z_6\cdot\tilde{I}_B & +(Z_3+Z_6+Z_7)\cdot\tilde{I}_C &= \tilde{V}_y
\end{aligned}$$

and:

$$\begin{aligned}
Z_{AA} &= (Z_1+Z_2+Z_3) & Z_{AB} &= Z_{BA} = -Z_2 \\
Z_{BB} &= (Z_2+Z_4+Z_5+Z_6) & Z_{AC} &= Z_{CA} = -Z_3 \\
Z_{CC} &= (Z_3+Z_6+Z_7) & Z_{BC} &= Z_{CB} = -Z_6
\end{aligned}$$

Mesh Analysis Example

Step 5: Solve the KVL equations:

$$D = \begin{vmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{vmatrix} \qquad
\begin{aligned}
Z_{AA} &= (Z_1+Z_2+Z_3) & Z_{AB} &= Z_{BA} = -Z_2 \\
Z_{BB} &= (Z_2+Z_4+Z_5+Z_6) & Z_{AC} &= Z_{CA} = -Z_3 \\
Z_{CC} &= (Z_3+Z_6+Z_7) & Z_{BC} &= Z_{BC} = -Z_6
\end{aligned}$$

then:

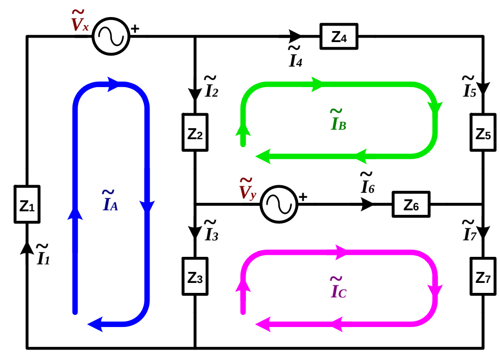
$$\tilde{I}_A = \frac{\begin{vmatrix} \tilde{V}_x & Z_{AB} & Z_{AC} \\ -\tilde{V}_y & Z_{BB} & Z_{BC} \\ \tilde{V}_y & Z_{CB} & Z_{CC} \end{vmatrix}}{D} \qquad
\tilde{I}_B = \frac{\begin{vmatrix} Z_{AA} & \tilde{V}_x & Z_{AC} \\ Z_{BA} & -\tilde{V}_y & Z_{BC} \\ Z_{CA} & \tilde{V}_y & Z_{CC} \end{vmatrix}}{D} \qquad
\tilde{I}_C = \frac{\begin{vmatrix} Z_{AA} & Z_{AB} & \tilde{V}_x \\ Z_{BA} & Z_{BB} & -\tilde{V}_y \\ Z_{CA} & Z_{CB} & \tilde{V}_y \end{vmatrix}}{D}$$

Mesh Analysis Example

Step 6: Define the branch currents in terms of the mesh currents.

$$\tilde{I}_1 = \tilde{I}_A \qquad \tilde{I}_3 = \tilde{I}_A - \tilde{I}_C \qquad \tilde{I}_6 = -\tilde{I}_B + \tilde{I}_C$$
$$\tilde{I}_2 = \tilde{I}_A - \tilde{I}_B \qquad \tilde{I}_4 = \tilde{I}_5 = \tilde{I}_B \qquad \tilde{I}_7 = \tilde{I}_C$$

The current flowing in any branch in the circuit is equal to the sum of the mesh currents that flow through the branch in the same direction as the desired current minus the sum of the mesh currents that flow through the branch in the opposite direction.



Nodal Analysis

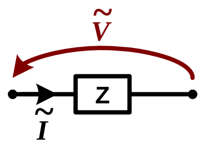
Nodal Analysis is an method of analysis that allows for the simultaneous solution of all of the *node voltages* within an electric circuit by specifying and solving a complete set of *Kirchhoff's Current Law* equations, the currents of which are expressed in terms of a complete set of "*node voltages*".

Once the node voltages are determined, the actual voltages across the circuit's impedances can easily be determined, in-turn allowing for solution of any of the circuit currents.

Nodal Analysis

Expressing branch currents in terms of the node voltages:

Given an impedance through which a current $\tilde{I}$ flows:



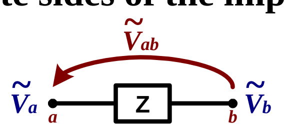
Then a voltage rise $\tilde{V}$ must exist across the resistor, defined in the opposite direction compared to that of the current flow, where:

$$\tilde{I} = \frac{\tilde{V}}{Z}$$

Nodal Analysis

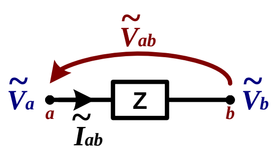
Expressing branch currents in terms of the node voltages:

If the voltage rise across an impedance is expressed as the difference in potential of the node voltages on the opposite sides of the impedance:



$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b$$

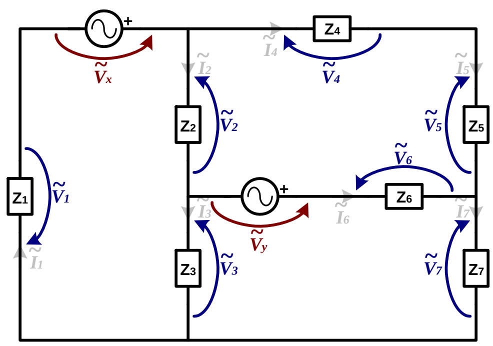
then the current flowing through the impedance can be expressed in terms of the node voltages:



$$\tilde{I} = \frac{\tilde{V}_a - \tilde{V}_b}{Z}$$

Nodal Analysis Example

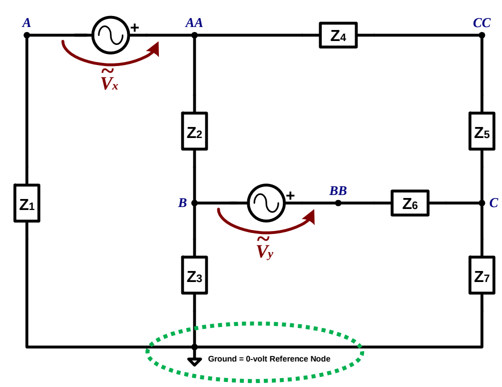
Perform a <u>Nodal Analysis</u> of the following circuit and then define voltage across each of the circuit's impedances in terms of the node voltages.



Nodal Analysis Example

<u>Step 1</u>: Choose a ground (reference) node and then identify and determine whether the other nodes in the circuit have "known" or "unknown" voltages.

Note – it is often useful to choose the node connected to the largest number of branches and/or the node connected to the largest number of sources as the ground node.



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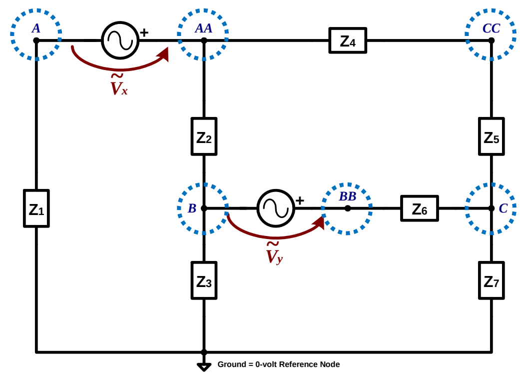
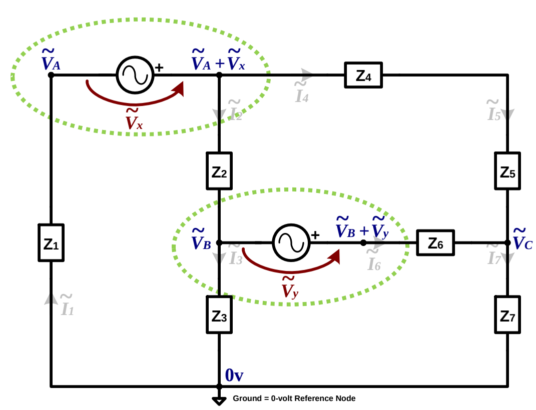


Nodal Analysis Example

Step 2: Specify the actual values of any nodes that have "known" voltages and assign a variable to the remaining nodes with "unknown" voltages.

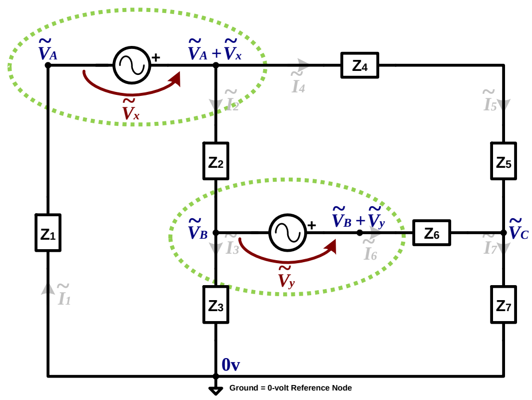
Note – if a voltage source exists between two unknown nodes, the a variable can be assigned to one of the nodes, after which the other node's voltage can be defined based on the assigned variable and the source value.



Nodal Analysis Example

<u>Step 3</u>: Write a Kirchhoff's Current Law (KCL) equation with the unknown branch currents expressed in terms of the node voltages for each node that has an unknown voltage.

Note – if a voltage source exists between two unknown nodes, the nodes and the connecting source should be grouped together and considered a "super node" for this analysis.



Nodal Analysis Example

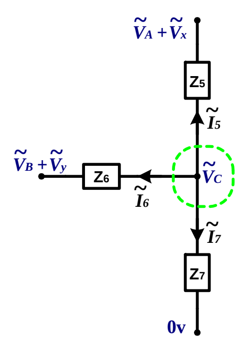
<u>Step 3a</u>: Write a KCL equation for node "C":

If all of the branch currents are defined to be "exiting" the nodes, then the KCL equations will simplify to:

$$\sum \tilde{I}_{exiting} = 0$$

Thus, for node "C", if the three branch currents are defined to be exiting the node, the resultant KCL equation is:

$$\tilde{I}_5 + \tilde{I}_6 + \tilde{I}_7 = 0$$

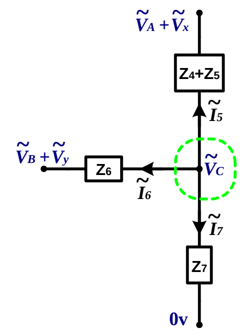


Nodal Analysis Example

<u>Step 3a</u>: **Write a KCL equation for node "C":**

The three branch currents can be expressed in terms of the node voltages as follows:

$$\tilde{I}_5 = \frac{\tilde{V}_C - \left(\tilde{V}_A + \tilde{V}_x\right)}{Z_4 + Z_5} \qquad \tilde{I}_6 = \frac{\tilde{V}_C - \left(\tilde{V}_B + \tilde{V}_y\right)}{Z_6} \qquad \tilde{I}_7 = \frac{\tilde{V}_C - 0}{Z_7}$$



Thus, the KCL equation for node "C" is:

$$\frac{\tilde{V}_C - \left(\tilde{V}_A + \tilde{V}_x\right)}{Z_4 + Z_5} + \frac{\tilde{V}_C - \left(\tilde{V}_B + \tilde{V}_y\right)}{Z_6} + \frac{\tilde{V}_C - 0}{Z_7} = 0$$

Nodal Analysis Example

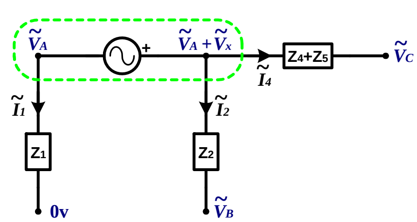
<u>Step 3b</u>: **Write a KCL equation for node "A":**

The branch currents for node "A" can be expressed in terms of the node voltages as follows:

$$\tilde{I}_1 = \frac{\tilde{V}_A - 0}{Z_1} \qquad \tilde{I}_2 = \frac{\left(\tilde{V}_A + \tilde{V}_x\right) - \tilde{V}_B}{Z_2} \qquad \tilde{I}_4 = \frac{\left(\tilde{V}_A + \tilde{V}_x\right) - \tilde{V}_C}{Z_4 + Z_5}$$

Thus, the KCL equation for node "A" is:

$$\frac{\tilde{V}_A - 0}{Z_1} + \frac{\left(\tilde{V}_A + \tilde{V}_x\right) - \tilde{V}_B}{Z_2} + \frac{\left(\tilde{V}_A + \tilde{V}_x\right) - \tilde{V}_C}{Z_4 + Z_5} = 0$$

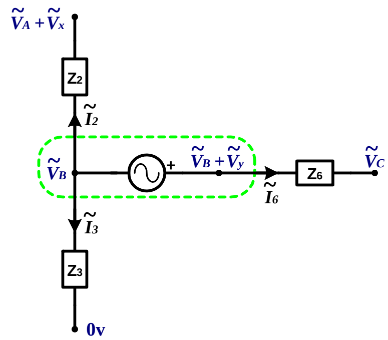


Nodal Analysis Example

<u>Step 3c</u>: Write a KCL equation for node "B":

The branch currents for node "B" can be expressed in terms of the node voltages as follows:

$$\tilde{I}_2 = \frac{\tilde{V}_B - \left(\tilde{V}_A + \tilde{V}_x\right)}{Z_2} \qquad \tilde{I}_3 = \frac{\tilde{V}_B - 0}{Z_3} \qquad \tilde{I}_6 = \frac{\left(\tilde{V}_B + \tilde{V}_y\right) - \tilde{V}_C}{Z_6}$$



Thus, the node "B" KCL equation is:

$$\frac{\tilde{V}_B - \left(\tilde{V}_A + \tilde{V}_x\right)}{Z_2} + \frac{\tilde{V}_B - 0}{Z_3} + \frac{\left(\tilde{V}_B + \tilde{V}_y\right) - \tilde{V}_C}{Z_6} = 0$$

Nodal Analysis Example

<u>Step 4</u>: Solve the set of node equations:

$$\frac{\tilde{V}_A - 0}{Z_1} + \frac{\left(\tilde{V}_A + \tilde{V}_x\right) - \tilde{V}_B}{Z_2} + \frac{\left(\tilde{V}_A + \tilde{V}_x\right) - \tilde{V}_C}{Z_4 + Z_5} = 0$$

$$\frac{\tilde{V}_B - \left(\tilde{V}_A + \tilde{V}_x\right)}{Z_2} + \frac{\tilde{V}_B - 0}{Z_3} + \frac{\left(\tilde{V}_B + \tilde{V}_y\right) - \tilde{V}_C}{Z_6} = 0$$

$$\frac{\tilde{V}_C - \left(\tilde{V}_A + \tilde{V}_x\right)}{Z_4 + Z_5} + \frac{\tilde{V}_C - \left(\tilde{V}_B + \tilde{V}_y\right)}{Z_6} + \frac{\tilde{V}_C - 0}{Z_7} = 0$$