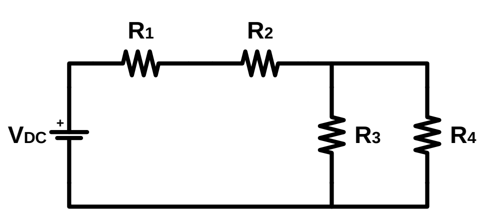
ECET 2111

Circuits II

Series-Parallel Circuits

Series-Parallel Circuits

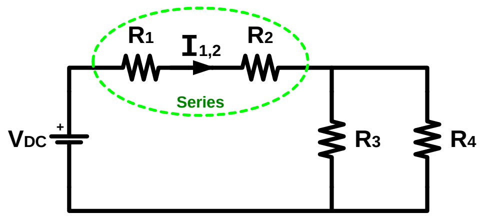
A <u>series-parallel circuit</u> is a circuit that is formed using a combination of series-connected and parallel-connected elements.



Series-Parallel Circuit Example #1

Series Elements

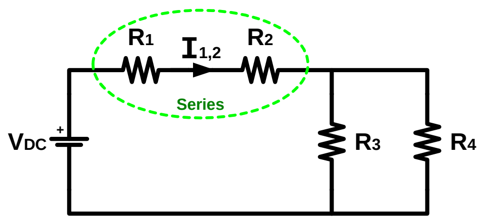
Two or more elements are connected in <u>series</u> if the current flowing through one of the elements must entirely flow through the other elements.



Series-Parallel Circuit Example #1

Series Elements

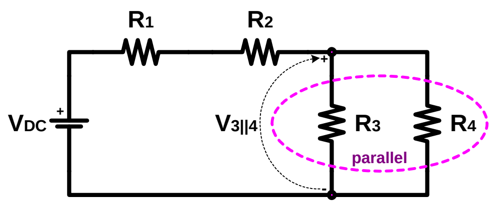
Note – the voltage source is also connected in series with resistors R_1 and R_2 since the current $I_{1,2}$ flows through all three elements.



Series-Parallel Circuit Example #1

Parallel Elements

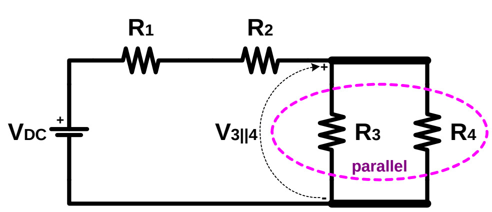
Two or more elements are connected in parallel if they are connected across the same two nodes (i.e. – the exact same voltage appears across each of the elements).



Series-Parallel Circuit Example #1

Parallel Elements

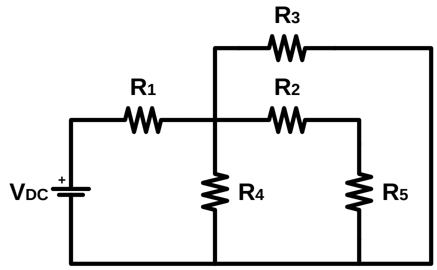
Note – although a node is a common "point" of connection for two or more circuit elements, a section of ideal wire can be considered equivalent to a node since there is no potential difference across the wire.



Series-Parallel Circuit Example #1

Series-Parallel Circuits

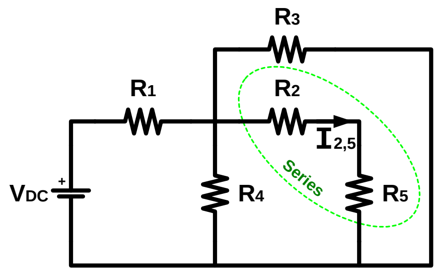
When a circuit contains both series and parallel connected elements, careful inspection is often required in order to identify the correct combinations.



Series-Parallel Circuit Example #2

Series-Parallel Circuits

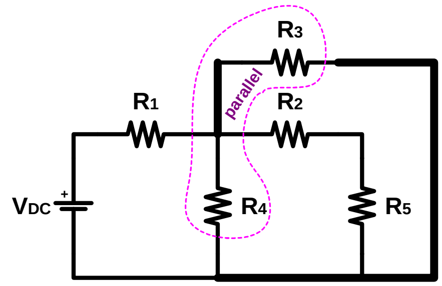
In the circuit shown below, resistors R_2 and R_5 are connected in <u>series</u> since the current $I_{2,5}$ must flow through both of the resistors.



Series-Parallel Circuit Example #2

Series-Parallel Circuits

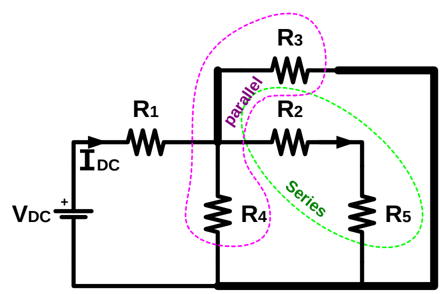
Careful inspection will also reveal that resistors R_3 and R_4 are connected in parallel since the ends of both resistors are connected to the same two "nodes".



Series-Parallel Circuit Example #2

Series-Parallel Circuits

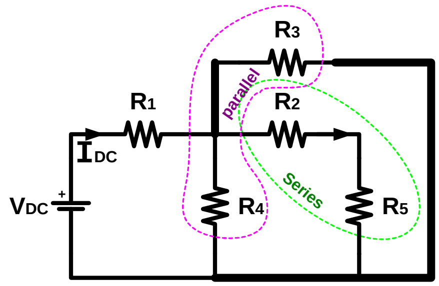
Although R_1 is connected in series with the battery, it is not connected in series or in parallel with any of the other resistors in the circuit.



Series-Parallel Circuit Example #2

Series-Parallel Circuits

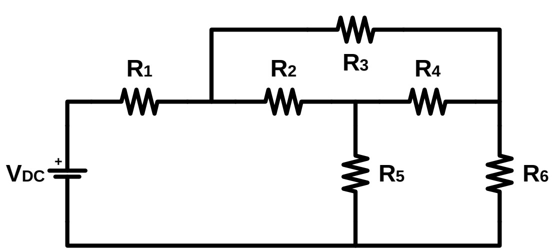
Note – that the series combination of resistors R_2 and R_5 may be considered as also being connected in parallel with resistors R_3 and R_4.



Series-Parallel Circuit Example #2

Series-Parallel Circuits

Take a moment to examine the following circuit in order to determine which resistors are connected in series and which are connected in parallel.

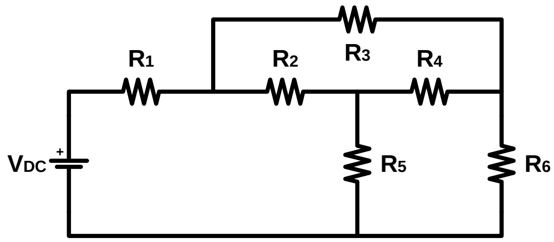


Series-Parallel Circuit Example #3

Series-Parallel Circuits

In this circuit, there are no series-connected or parallel-connected sets of resistors.

The techniques required to analyze this type of network will be presented later in the course.



Series-Parallel Circuit Example #3

The Reduce & Return Approach

Although series-parallel circuits may be analyzed by a variety of methods, one of the simplest methods is the:
Reduce and Return Approach.

With this method, the circuit is incrementally reduced in complexity by replacing sets of series or parallel connected elements with their series or parallel equivalents.

Each incremental reduction provides a simpler circuit containing fewer elements that can either be further reduced until only a trivial circuit remains.

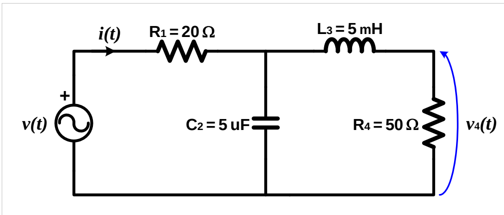
The Reduce & Return Approach

Once the original circuit is reduced down into a trivial circuit, that circuit can be analyzed in order to determine any unknown voltages or currents.

The initially-solved voltages or currents can then be utilized, step-by-step in reverse order back through the simplified circuits, facilitating the analysis of each incrementally more complex circuit until the desired unknown quantities specified in the original circuit are known.

Reduce & Return Example

Use the Reduce and Return Approach to solve for the voltage $v_4(t)$ as specified in the following figure:



Circuit #1

where: $v(t) = 24 \cdot \sin(\omega \cdot t + 30°)$ volts

$\omega = 1000$ rad/sec

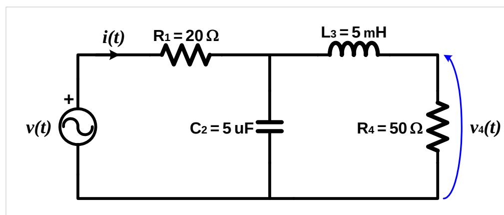
Reduce & Return Example

Step 1 – Express the source voltage by its <u>phasor value</u> and all circuit elements as <u>impedances</u>.

$$v(t) = 24 \cdot \sin(\omega \cdot t + 30°) \text{ volts}$$
$$\omega = 1000 \text{ rad/sec}$$
$$\tilde{V} = 24\angle 30° = 24e^{j\frac{\pi}{6}} \text{ volts}$$

$$Z_1 = R_1 = 20\,\Omega$$
$$Z_2 = -j\left(\frac{1}{1000 \cdot 5x10^{-6}}\right) = -j200\,\Omega$$
$$Z_3 = j\left(1000 \cdot 50x10^{-3}\right) = +j50\,\Omega$$
$$Z_4 = R_4 = 50\,\Omega$$

$$v(t) = 24 \cdot \sin(\omega \cdot t + 30°) \text{ volts}$$
$$\omega = 1000 \text{ rad/sec}$$



Circuit #1

$$Z_R = R$$
$$Z_L = j(\omega \cdot L)$$
$$Z_C = -j\left(\frac{1}{\omega \cdot C}\right)$$

Reduce & Return Example

Step 1 – Express the source voltage by its phasor value and all circuit elements as impedances.

$$v(t) = 24 \cdot \sin(\omega \cdot t + 30°) \text{ volts}$$
$$\omega = 1000 \text{ rad/sec}$$
$$\tilde{V} = 24\angle 30° = 24e^{j\frac{\pi}{6}} \text{ volts}$$

$$Z_1 = R_1 = 20\,\Omega$$
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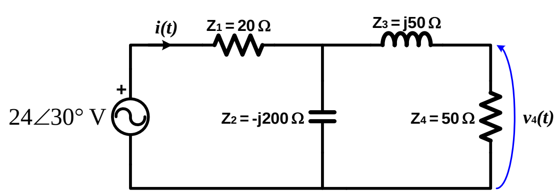


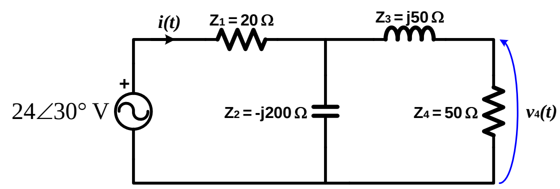
Circuit #1

Reduce & Return Example

Step 2 – Reduce the Original Circuit

Identify a set of either series-connected or parallel-connected impedances and replace them with a single equivalent impedance.

Note – if multiple series or parallel combinations exists, the ones furthest from the source are typically reduced first.



Circuit #1

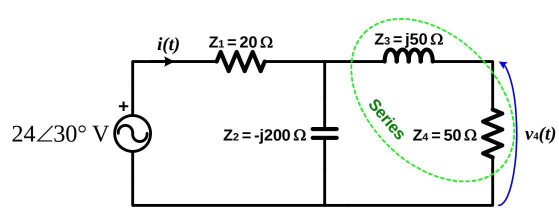
Reduce & Return Example

Step 2 – Reduce the Original Circuit

Impedances Z_3 and Z_4 are connected in series.

The series-equivalent resistance $Z_{3,4}$ that may be used to replace the series Z_3 and Z_4 combination is:

$$Z_{3,4} = Z_3 + Z_4 = 50 + j50\,\Omega$$

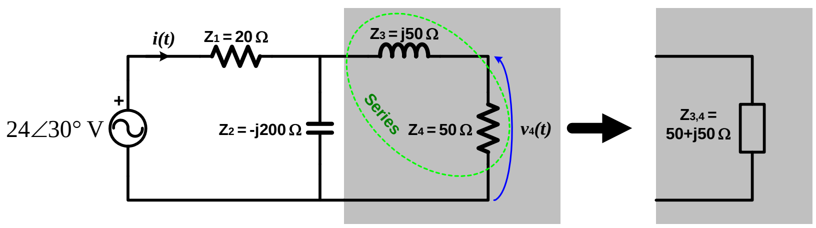


Circuit #1

Reduce & Return Example

Step 2 – Reduce the Original Circuit

When replacing a combination of impedances with a single equivalent impedance, the rest of the circuit remains unchanged.



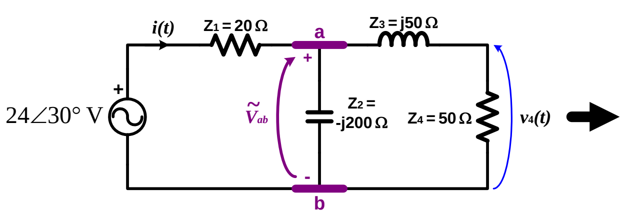
Circuit #1

Reduce & Return Example

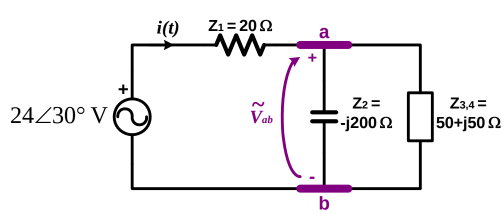
Step 2 – Reduce the Original Circuit

Note that $v_4(t)$ does not exist in the reduced circuit because Z_4 is no longer shown as a discrete element.

But, V_{ab}, the voltage across the new load $Z_{3,4}$ exists both in the original circuit and in the reduced circuit.



Circuit #1



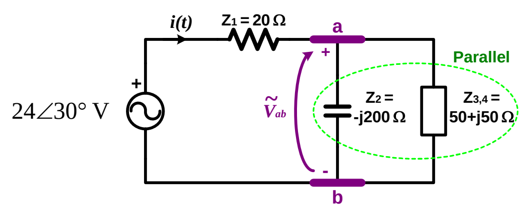
Circuit #2

Reduce & Return Example

Step 3 – Continue Reducing the Circuit as Needed

Identify another set of series or parallel impedances and replace them with a single equivalent impedance.

The impedance, $Z_{3,4}$, is connected in parallel with Z_2.



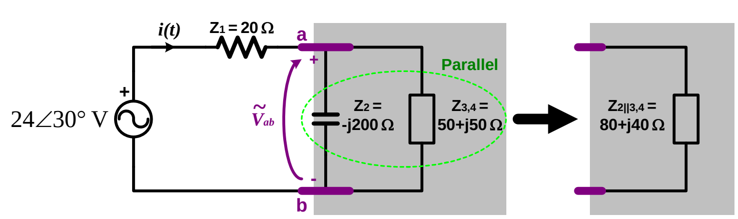
Circuit #2

Reduce & Return Example

Step 3 – Continue Reducing the Circuit as Needed

Since $Z_{3,4}$ is connected in parallel with Z_2, the two impedances can be replaced with a single equivalent impedance, $Z_{2||3,4}$, having the value:

$$Z_{2||3,4} = \left(\frac{1}{Z_2} + \frac{1}{Z_{3,4}}\right)^{-1} = \left(\frac{1}{-j200} + \frac{1}{50+j50}\right)^{-1} = 80 + j40\,\Omega$$



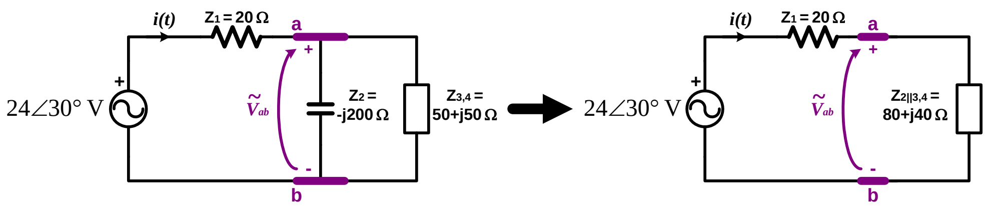
Circuit #2

Reduce & Return Example

Step 3 – Continue Reducing the Circuit as Needed

Note that V_{ab} does exist in the reduced circuit because nodes “a” and “b” still remain.

Although this is not the voltage that we are being tasked to solve, it will prove useful for the overall solution of this problem.

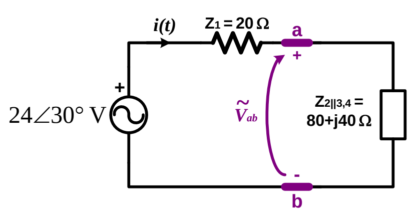


Circuit #2 Circuit #3

Reduce & Return Example

Step 4 – Continue Reducing the Circuit as Needed

Although the circuit can be reduced one more time by combining the series combination of Z_1 and $Z_{2||3,4}$, the circuit is of a sufficiently-trivial nature to allow for a simple solution of any unknown voltages or currents.



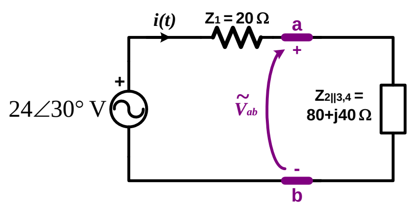
Circuit #3

Reduce & Return Example

Step 5 – Analyze the Trivial Circuit

Since Z_1 is in-series with $Z_{2||3,4}$, a voltage divider equation can be used to determine the voltage V_{ab}:

$$\tilde{V}_{ab} = 24\angle 30° \cdot \left(\frac{Z_{2||3,4}}{Z_1 + Z_{2||3,4}} \right) = 24\angle 30° \cdot \left(\frac{80 + j40}{20 + 80 + j40} \right) = 19.9\angle 34.8° \, volts$$



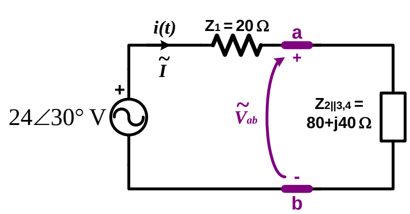
Circuit #3

Reduce & Return Example

Step 5 – Analyze the Trivial Circuit

Additionally, if the source-current was needed, it could be calculated as follows:

$$\tilde{I} = \left(\frac{24\angle 30°}{Z_1 + Z_{2||3,4}} \right) = \left(\frac{24\angle 30°}{100 + j40} \right) = 0.223\angle 8.20° \, amps$$

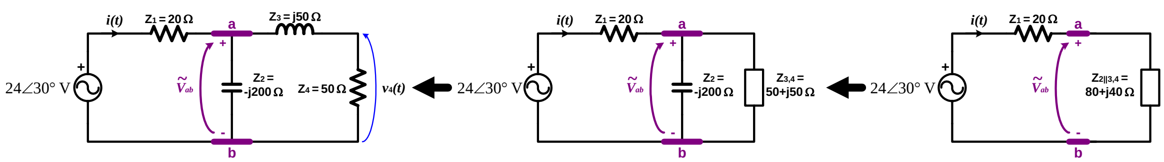


Circuit #3

Reduce & Return Example

Step 6 – Return Back Through the Reduced Circuits

Utilize the results that were obtained by analyzing the trivial circuit in order to determine any quantities in the incrementally more complex circuits that can, in-turn, be used to the solve the originally-tasked circuit parameter $v_4(t)$.



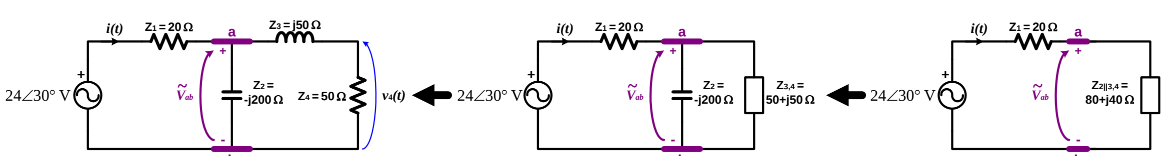
Circuit #1 Circuit #2 Circuit #3

Reduce & Return Example

Step 6 – Return Back Through the Reduced Circuits

Since the voltage V_{ab} exists in all three versions of the circuit, we can immediately return back to the original circuit to complete our solution.

Note that this will not always be the case. In many Reduce-and-Return problems, the incrementally-reduced circuit may also need to be analyzed.



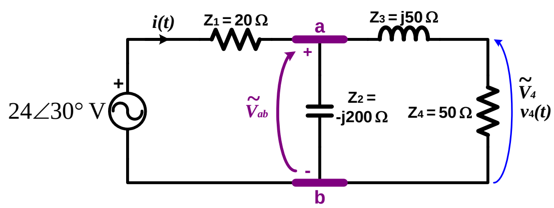
Circuit #1 Circuit #2 Circuit #3

Reduce & Return Example

Step 7 – Determine the Desired Circuit Parameter

In the original circuit, V_{ab} is the total voltage across the series-connected impedances Z_3 and Z_4. Thus, V_4 can be determined by utilizing a voltage divider equation:

$$\tilde{V}_4 = \tilde{V}_{ab} \cdot \left(\frac{Z_4}{Z_3 + Z_4} \right) = 19.9\angle 34.8° \cdot \left(\frac{50}{50 + j50} \right) = 14.1\angle -10.2° \, volts$$



Circuit #1

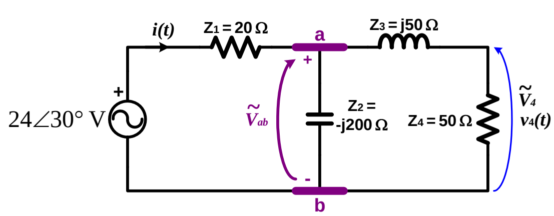
Reduce & Return Example

Step 7 – Determine the Desired Circuit Parameter

Finally, the phasor value of V_4 can be converted back into a time function as follows:

$$\tilde{V}_4 = 14.1\angle -10.2° \, volts$$

$$v_4(t) = 14.1 \cdot \sin(\omega \cdot t - 10.2°) \text{ volts}$$

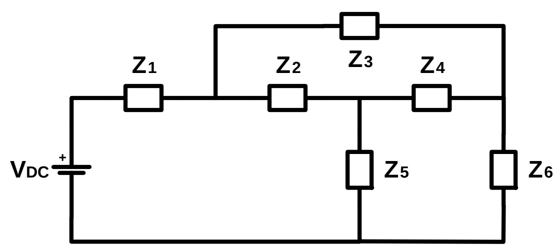


Circuit #1

Δ – Y or Π – T Conversions

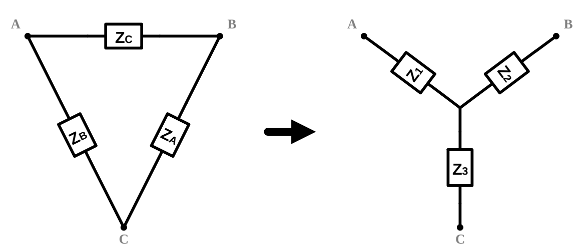
As mentioned previously, sometimes circuits are constructed such that no elements are connected either in-series or in-parallel with each other, thus preventing use of the reduce-and-return method of analysis.

In such cases, a Δ–Y or a Π–T conversion can often be utilized to remedy the situation.



Δ → Y or Π → T Conversion

If there impedances within a circuit are connected together in a Δ (or Π) format, they can be replaced by a set of impedances connected together in a Y (or T) format without affecting the operation of the rest of the circuit, provided that the new impedance values are calculated by:



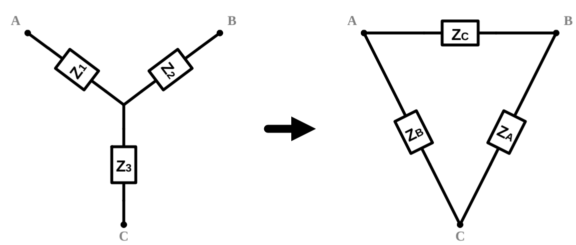
$$Z_1 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

$$Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

$$Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

Y → Δ or T → Π Conversion

If there impedances within a circuit are connected together in a Y (or T) format, they can be replaced by a set of impedances connected together in a Δ (or Π) format without affecting the operation of the rest of the circuit, provided that the new impedance values are calculated by:



$$Z_A = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_2}{Z_1}$$

$$Z_B = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_2}{Z_2}$$

$$Z_C = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_2}{Z_3}$$

Δ – Y or Π – T Conversions

In the circuit shown below, impedances Z_4, Z_5, and Z_6 are connected together in a Δ (or Π) format. Thus, they can be replaced by a set of impedances connected together in a Y (or T) format, which would allow the circuit to be reduced.