ECET 2111

Circuits II

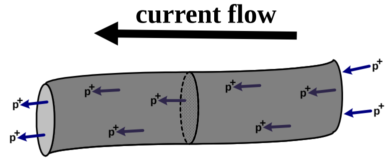
Basic Electric Circuits

♦

DC Review

Current

In terms of electric circuits, current can be defined as a measure of the rate at which positive charge flows through a conductor or a circuit component.

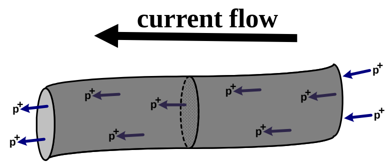


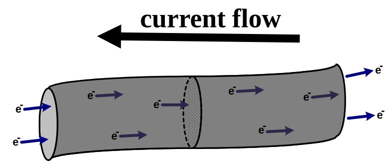
The standard unit for current is an Ampere, which equals to one coulomb of charge crossing a surface per second.

1 coulomb = $6.242x10^{18}$

Current

Although the direction of current flow is defined in terms of positive charge flow, it is important to remember that it is actually <u>electrons</u> that flow through the circuit.





Although this may cause some confusion from a physics perspective, it does not present a problem when analyzing an electric circuit since classical electric circuit theory is based on the concept of positive charge flow and thus will still provide the correct results.

Voltage

Voltage is often referred to as the "potential difference" because it relates to the difference in electric potential energy per unit of charge that exists at different locations.

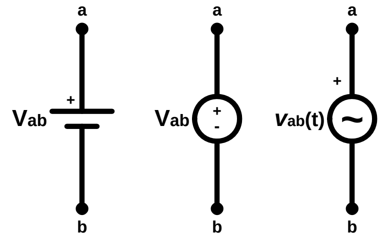
In terms of electric circuits, <u>voltage</u> can be thought of a measure of the (potential) force developed by any circuit element that tries to create or oppose the flow of current.

The standard unit for voltage is <u>Volts</u>.

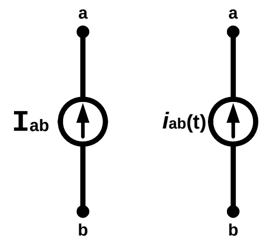
Technically, a <u>1-Volt</u> potential difference exists between points *a* and *b* if <u>1 Joule</u> of energy is required to move <u>1 Coulomb</u> of charge from point *b* to point *a*.

Ideal Sources

An <u>ideal voltage source</u> is a device that maintains a constant voltage potential across its terminals independent of the current flowing through the source.



An <u>ideal current source</u> is a device that maintains a constant current flow through the source independent of the voltage potential that is required to create across its terminals in order to maintain that current.



Electric Loads – Resistors

The primary electric load used in DC circuits is a <u>Resistor</u>.

A resistor is a device that provides an oppositional force to the flow of current that is linearly proportional to the amount of current that is flowing through the resistor.

The ability of the resistor to oppose the flow of current is characterized by its <u>resistance</u>, *R*.

<u>Resistance</u> of defined as the ratio of the oppositional force (voltage) provided by the resistor over the amount of current that is flowing through the resistor.

Ohm's Law – Resistors

Thus, for an ideal resistive load, resistance is defined as:

$$R = \frac{V_R}{I_R}\,(\Omega)$$

where: V_R **is the oppositional voltage provided by the resistor, and**
I_R **is the current flowing through the resistor.**

This relationship is referred to as <u>Ohm's Law</u>, and is typically expressed in the form:

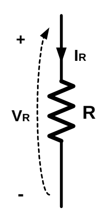
$$V_R = I_R \cdot R$$

Note that the standard units for resistance are Ohms (Ω):

$$Ohms = \frac{Volts}{Amps}$$

Resistors

The symbol for an ideal voltage source is:



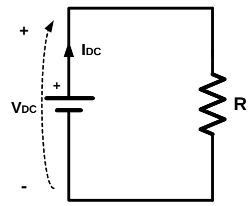
where: V_R **is the voltage developed by the resistor, and**
I_R **is the current flowing through the resistor.**

$$V_R = I_R \cdot R$$

Note that for an ideal resistor, resistance is constant.

Simple Electric Circuit

A simple electric circuit can be constructed by connecting a single voltage source to a single resistor.

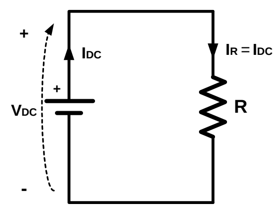


The voltage source provides a force that tries to "push" current out if its positive terminal. Since initially there is no force (voltage) opposing the flow of current, current will quickly begin to flow out of the source.

Simple Electric Circuit

But, the only path that the current can take to get back to the source is through the resistor.

$$I_{DC} = I_R$$

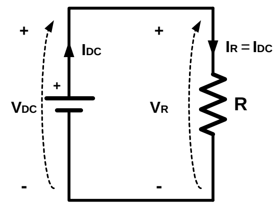


Note – if all of the current flowing through one element in a circuit also flows through another element in the circuit, then those two elements are ***"connected in series"*** with each other.

Simple Electric Circuit

But, the only path that the current can take to get back to the source is through the resistor.

$$I_{DC} = I_R$$

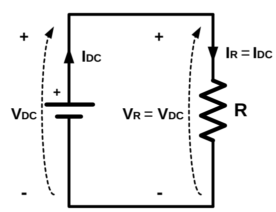


As soon as current begins to flow through the resistor, the resistor will develop a voltage (force), V_R, that opposes the flow of current.

Simple Electric Circuit

As is often the case in physical systems, for steady-state operation to occur, the forces must be balanced.

$$I_{DC} = I_R$$



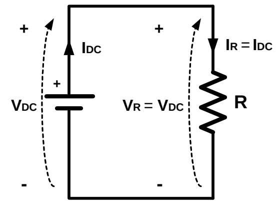
$$V_{DC} = V_R$$

Thus, the current will (instantaneously) increase in magnitude until the resistor voltage equals the source voltage, at which point the current magnitude will remain constant and steady-state operation is achieved.

Simple Electric Circuit

As is often the case in physical systems, for steady-state operation to occur, the forces must be balanced.

$$I_{DC} = I_R$$



$$V_{DC} = V_R$$

The steady-state current magnitude can be solved by applying the above relationships and Ohm's Law as follows:

$$I_{DC} = I_R = \frac{V_R}{R} = \frac{V_{DC}}{R}$$

Electric Power

and

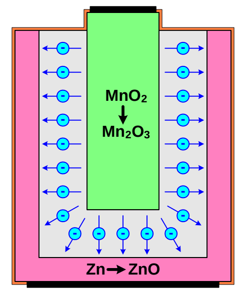
Energy

Energy

Energy, which can be defined as the ability of a system to perform work, is a property of objects which can be transferred to other objects or converted into different forms but cannot be created or destroyed.

For example: The energy that is released during the chemical reaction that occurs in a battery is transferred to the electrons that are deposited on the anode, in-turn providing them with the potential to flow externally from the anode back to the cathode.

This energy is referred to as "electric energy"



Power

Power is defined as the rate at which work is performed or the rate at which energy is converted from one form to another.

Thus, during the steady-state operation of a system, the amount of work performed (energy) equals:

$$Energy = Power \cdot Time$$

Electric Power

When a battery (electric source) is connected to a resistor, the battery provides the force necessary to push current through the resistor. But, it requires energy to move the current against this oppositional force.

The electric energy utilized to make this process happen is the energy that was released during the battery's chemical reaction and transferred to the electrons that were deposited on the anode of the battery.

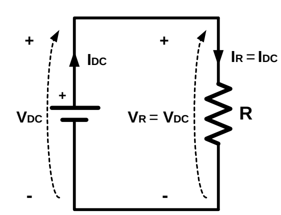
The rate at which this energy is utilized is referred to as <u>Electric Power</u>.

Electric Power

As current is pushed through the resistor, the electric energy associated with the moving charge is converted to heat.

The rate, P_R, at which the resistor converts the electric energy to heat, often referred to as the rate at which the resistor "consumes" electric power, is defined by:

$$P_R = V_R \cdot I_R \ (Watts)$$



Electric Power

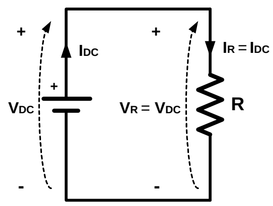
Note that the power "consumed" by the resistor:

$$P_R = V_R \cdot I_R \ (Watts)$$

must equal to the power "produced" by the source:

$$P_{source} = V_{DC} \cdot I_{DC} \ (Watts)$$

in order to maintain an energy balance in the system.



Electric Power

The standard units for electric power is <u>Watts</u>, such that:

$$1\,Watt = 1\frac{Joule}{Second}$$

Because a Joule is a tiny amount of energy, electric power is often specified in <u>kiloWatt·hours</u> (kWh), such that 1 kWh is equivalent to 1000 W of power being consumed for a 1 hour period of time.

Note that:

$$1\,kWh = 1000\,W \cdot hours = 3{,}600{,}000\,W \cdot sec = 3{,}600{,}000\ J$$

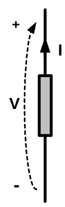
Energy Production/Consumption

In general terms:

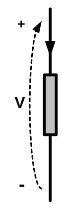
A circuit element "produces" electric energy (power) if the voltage rise across the element is in the same direction as the current flowing through the element, and

A circuit element "consumes" electric energy (power) if the voltage rise across the element is in the opposite direction compared to the current flowing through the element.

Circuit element produces electric energy/power →



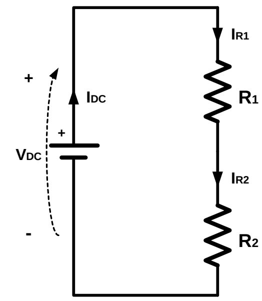
Circuit element consumes electric energy/power →



Series & Parallel Connected Resistors

Series-Connected Resistors

Two (or more) circuit elements are said to be connected "In-Series" if the current that flows through one of the elements must entirely flow through the other element(s).



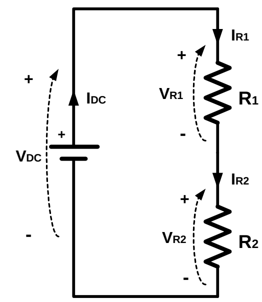
Thus, the two resistors shown in the circuit to the right are "in-series" with each other since the current flowing through resistor R_1 must also flow through resistor R_2.

Additionally, a similar analysis will show that the resistors are also connected "in-series" with the voltage source since:

$$I_{DC} = I_{R1} = I_{R2}$$

Series-Connected Resistors

In terms of the individual resistors, the voltages across the resistors will adhere to Ohm's Law:



$$V_{R1} = I_{R1} \cdot R_1 \qquad V_{R2} = I_{R2} \cdot R_2$$

Since the current flowing through the resistors is equal to the source current:

$$I_{DC} = I_{R1} = I_{R2}$$

the resistor voltages can be rewritten as:

$$V_{R1} = I_{DC} \cdot R_1 \qquad V_{R2} = I_{DC} \cdot R_2$$

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL) is a force-balance equation that states:

"The sum of the "voltage rises" must equal to the sum of the "voltage drops" defined in a continuous direction around any closed-loop path in an electric circuit."

where: **a "voltage rise" relates to an increase in voltage potential across a circuit element in the direction of summation, and**

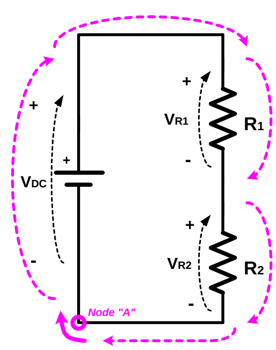
a "voltage drop" relates to a decrease in voltage potential across a circuit element in the direction of summation.

$$\sum V_{rises} = \sum V_{drops}$$

Series-Connected Resistors

There is only one closed-loop path around which current can flow in the given series circuit.

If Kirchhoff's Voltage Law is applied to the series circuit such that the voltages are summed in a CW direction around the closed-loop path beginning at node *"A"*, the following voltage relationship can be defined:



$$V_{DC} = V_{R1} + V_{R2}$$

I.e. – the force (voltage) provided by the source that tries to push current around the closed-loop path must equal to the sum of the resistor forces (voltages) that oppose the flow of current.

Series-Connected Resistors

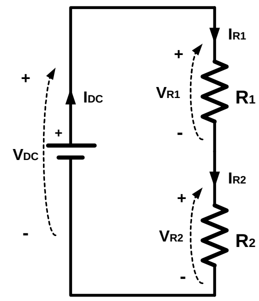
Based on the KVL equation:

$$V_{DC} = V_{R1} + V_{R2}$$

along with the Ohm's Law equations:

$$V_{R1} = I_{DC} \cdot R_1 \qquad V_{R2} = I_{DC} \cdot R_2$$

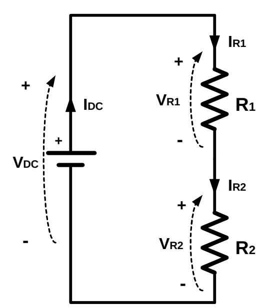
for the series circuit, the relationship between the source voltage and current can be defined by:



$$\begin{aligned} V_{DC} &= V_{R1} + V_{R2} \\ &= I_{DC} \cdot R_1 + I_{DC} \cdot R_2 \\ &= I_{DC} \cdot (R_1 + R_2) \end{aligned}$$

Series-Connected Resistors

Thus, given a circuit containing two series-connected resistors, the current that will flow in the circuit is:



$$I_{DC} = \frac{V_{DC}}{(R_1 + R_2)}$$

It can be seen from this expression that the total resistance provided by the series-connected resistors is equal to the sum of the individual resistances.

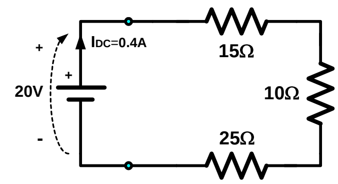
This result can be expanded for a set of "*N*" series-connected resistors, such that:

$$R_{series(total)} = R_1 + R_2 + \cdots + R_N$$

Series-Resistor Example

Given the circuit containing three series-connected resistors :

a) Determine the <u>source current</u> I_{DC}.



$$I_{DC} = \frac{V_{DC}}{(R_1 + R_2 + R_3)} = \frac{20\text{V}}{15\Omega + 10\Omega + 25\Omega}$$

$$= \frac{20\text{V}}{50\Omega} = 0.4\text{A}$$

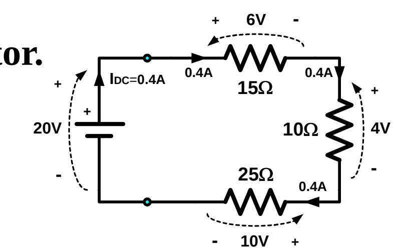
b) Determine the total <u>electric power</u> "produced" by the source.

$$P_{source} = V_{DC} \cdot I_{DC} = (20\text{V}) \cdot (0.4\text{A}) = 8\text{W}$$

Series-Resistor Example

Given the circuit containing three series-connected resistors :

c) Determine the <u>voltage</u> across each resistor.



$$I_{DC} = I_{15\Omega} = I_{10\Omega} = I_{25\Omega} = 0.4\text{A}$$

$$V_{15\Omega} = I_{15\Omega} \cdot 15\Omega = (0.4\text{A}) \cdot (15\Omega) = 6\text{V}$$

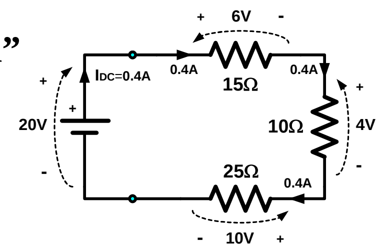
$$V_{10\Omega} = (0.4\text{A}) \cdot (10\Omega) = 4\text{V}$$

$$V_{25\Omega} = (0.4\text{A}) \cdot (25\Omega) = 10\text{V}$$

Series-Resistor Example

Given the circuit containing three series-connected resistors :

d) Determine the <u>electric power</u> "consumed" by each resistor.



$$P_{15\Omega} = V_{15\Omega} \cdot I_{15\Omega} = (6\text{V}) \cdot (0.4\text{A}) = 2.4\text{W}$$

$$P_{10\Omega} = (4\text{V}) \cdot (0.4\text{A}) = 1.6\text{W}$$

$$P_{25\Omega} = (10\text{V}) \cdot (0.4\text{A}) = 4\text{W}$$

e) Determine the <u>total power</u> "consumed" by the resistors.

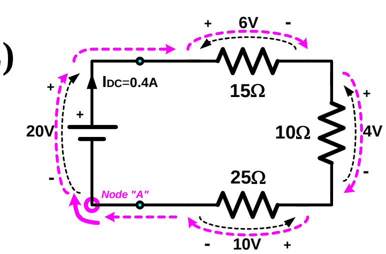
$$P_{R(total)} = P_{15\Omega} + P_{10\Omega} + P_{25\Omega} = 2.4\text{W} + 1.6\text{W} + 4\text{W} = 8\text{W}$$

$$P_{R(total)} = 8\text{W} = P_{source} \quad \Rightarrow$$ **The total power "produced" by the source equals the total power "consumed" by the resistors**

Series-Resistor Example

Given the circuit containing three series-connected resistors :

f) Confirm Kirchhoff's Voltage Law (KVL) using the results from this circuit.



$$\sum V_{rises} = \sum V_{drops}$$

KVL: CW Direction Summation

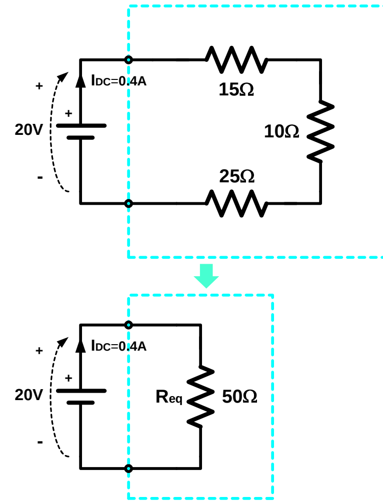
$$20\text{V}_{\text{rise}} = 6\text{V}_{\text{drop}} + 4\text{V}_{\text{drop}} + 10\text{V}_{\text{drop}}$$

$$= 20\text{V}_{\text{drop(total)}}$$

Series-Resistor Example

Given the circuit containing three series-connected resistors :

g) Determine the value of an <u>equivalent resistance</u> that can be used in place of the three resistors without affecting the overall operation of the source.



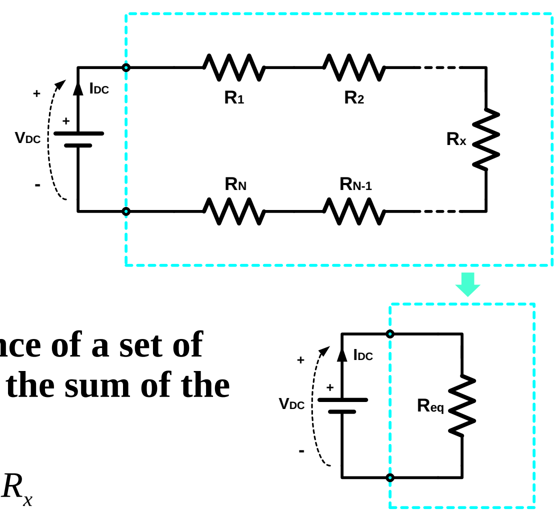
$$R_{eq} = \frac{V_{DC}}{I_{DC}} = \frac{20\text{V}}{0.4\text{A}} = 50\Omega$$

$$\begin{aligned} R_{eq} &= 50\Omega \\ &= 15\Omega + 10\Omega + 25\Omega \\ &= R_{series(total)} \end{aligned}$$

Series-Equivalent Resistance

Given a circuit containing "N" series-connected resistors:

A single "equivalent resistance" can be used in place of the set of series resistors without affecting the overall operation of the source, provided that:



$$R_{eq(series)} = R_1 + R_2 + \cdots + R_N$$

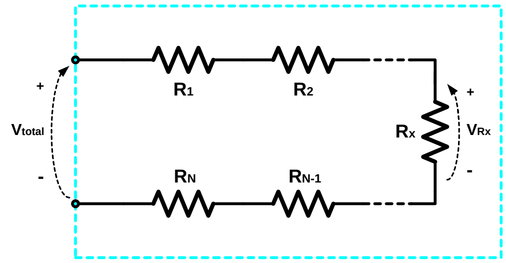
I.e. – the series-equivalent resistance of a set of series-connected resistors is the sum of the individual resistances.

$$R_{eq(series)} = \sum_{x=1}^{N} R_x$$

Voltage-Divider Equation

Given a set of series-connected resistors across which the total voltage, V_{total}, is known,

The voltage, V_{Rx}, across resistor R_x can be determined using the <u>voltage-divider</u> equation:

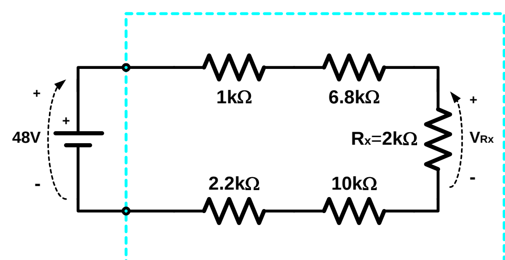


$$V_{Rx} = V_{total} \cdot \frac{R_x}{R_{eq(series)}}$$

Voltage-Divider Equation Example

Given the set of series-connected resistors shown in the figure across which a 48V source is connected,

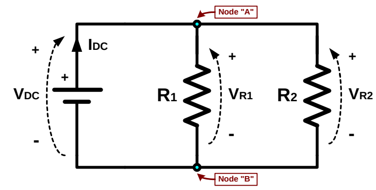
Determine the voltage, V_{Rx}, across the 2kΩ resistor using the <u>voltage-divider</u> equation.



$$R_{eq(series)} = 1\text{k}\Omega + 6.8\text{k}\Omega + 2\text{k}\Omega + 10\text{k}\Omega + 2.2\text{k}\Omega = 22\text{k}\Omega$$

$$V_{Rx} = V_{total} \cdot \frac{R_x}{R_{eq(series)}} = 48 \cdot \frac{2\text{k}\Omega}{22\text{k}\Omega} = 4.36\text{V}$$

Parallel-Connected Resistors



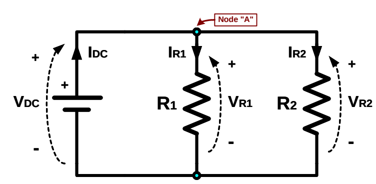
Two (or more) circuit elements are said to be connected "**In-Parallel**" if the elements are connected across the same two nodes in the circuit.

Thus, the two resistors in the above circuit are connected "in-parallel" with each other and with the source since they are all connected across nodes "A" and "B".

By applying Kirchhoff's Voltage Law around each closed-loop path in the circuit, it can be proven that:

$$V_{DC} = V_{R1} = V_{R2}$$

Parallel-Connected Resistors



In terms of the individual resistors, the currents flowing through each resistor will adhere to Ohm's Law:

$$I_{R1} = \frac{V_{R1}}{R_1} \qquad I_{R2} = \frac{V_{R2}}{R_2}$$

Since the voltage across each of the resistors is equal to the source voltage:

$$V_{DC} = V_{R1} = V_{R2}$$

the resistor currents can be rewritten as:

$$I_{R1} = \frac{V_{DC}}{R_1} \qquad I_{R2} = \frac{V_{DC}}{R_2}$$

Kirchhoff's Current Law

<u>Kirchhoff's Current Law</u> (KCL) is a mass-balance equation that states:

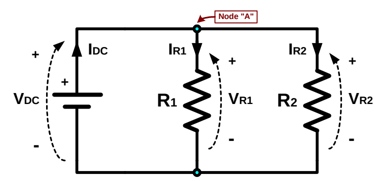
"The sum of the currents defined entering a node must equal to the sum the currents defined exiting the node for any fully-defined node in an electric circuit."

where: a "<u>fully-defined node</u>" is a node for which a current is defined in each branch connected to that node.

$$\sum I_{entering} = \sum I_{exiting}$$

Parallel-Connected Resistors

If Kirchhoff's Current Law is applied to node "A", it can be proven that:



$$I_{DC} = I_{R1} + I_{R2}$$

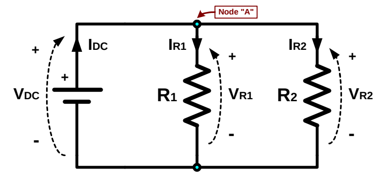
I.e. – the total current produced by the source must equal to the sum of the currents flowing through the parallel resistors.

The KCL and Ohm's Law equations can be used to define a relationship between the source voltage and current:

$$I_{DC} = I_{R1} + I_{R2} = \frac{V_{DC}}{R_1} + \frac{V_{DC}}{R_2} = V_{DC} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_{DC}}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}}$$

Parallel-Connected Resistors

Thus, given a circuit containing two parallel-connected resistors, the current that will flow from the source is:



$$I_{DC} = \frac{V_{DC}}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}$$

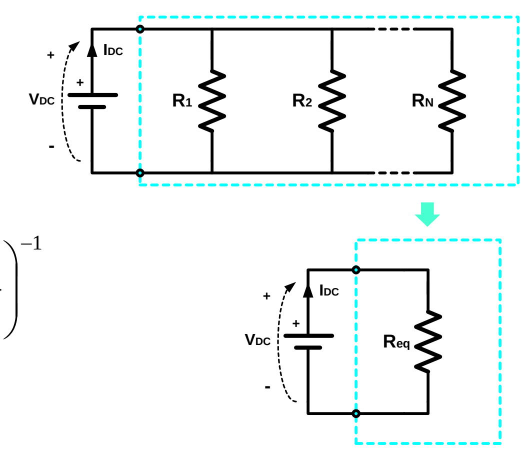
It can be seen from this expression that the total resistance provided by the parallel-connected resistors is equal to the inverse of the sum of the inverses of each resistor, which can be expanded for "N" parallel-connected resistors to:

$$R_{parallel(total)} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1}$$

Parallel-Equivalent Resistance

Given a circuit containing "N" parallel-connected resistors:

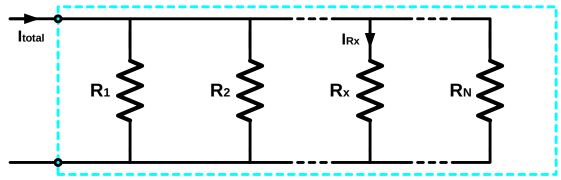
A single "equivalent resistance" can be used in place of the set of parallel resistors without affecting the overall operation of the source, provided that:



$$R_{eq(parallel)} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1}$$

Current-Divider Equation

Given a set of parallel-connected resistors through which the total current, I_{total}, is known,



The current, I_{Rx}, through resistor R_x can be determined using the <u>current-divider</u> equation:

$$I_{Rx} = I_{total} \cdot \frac{R_{eq(parallel)}}{R_x}$$

Basic Circuit Theory

The fundamental concepts or building blocks that form the foundation of basic circuit theory are:

- **Ohm's Law**
- **Series-connected Resistors**
- **Kirchhoff's Voltage Law (KVL)**
- **Parallel-connected Resistors**
- **Kirchhoff's Current Law (KCL)**
- **Series & Parallel Equivalent Resistances**
- **Voltage Divider & Current Divider Equations**