ECET 3500

3Φ Squirrel Cage Induction Machines

†

Introduction

1

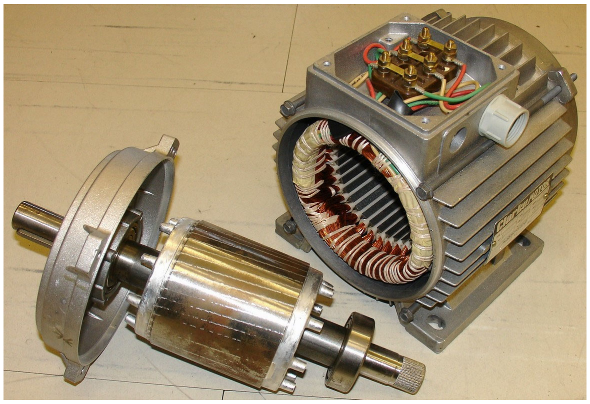
Three-Phase Induction Machines

The Three-Phase (3Φ) Induction Machine is a rotational device that, when supplied with a 3Φ balanced voltage, can operate as either a motor or a generator.

Although different versions of the induction machine exist, this presentation will cover the 3Φ "Squirrel-Cage" Induction Machine since this type is routinely used in industry as motors due to their extreme durability, simple operation, and ease of speed control when supplied by a Variable Frequency Drive (VFD).

2

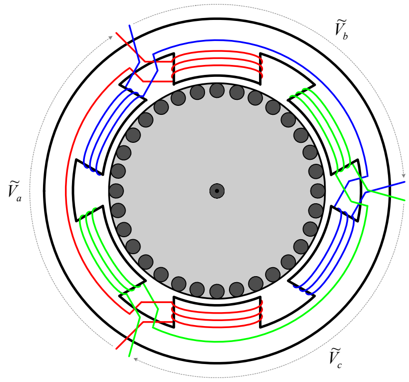
Three-Phase Induction Machines



The Three-Phase (3Φ) Induction Machine consists of a <u>stator</u> (stationary portion) and a <u>rotor</u> that are separated by a small air-gap.

3

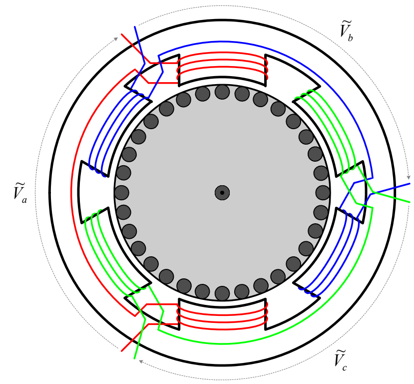
Induction Machine Construction



The following presentation covers the construction and operation of a <u>conceptual</u>, 3Φ, 2-pole, Squirrel-Cage, Induction machine.

4

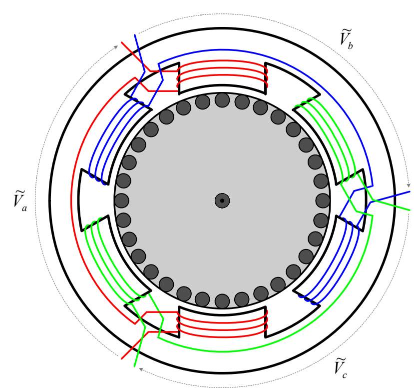
Induction Machine Construction



Note – although the construction of an actual Induction machine may vary from the conceptual machine shown, the operational mechanisms and characteristics will be similar to those presented.

5

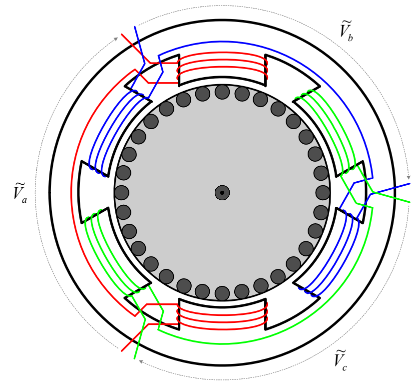
Induction Machine Construction



The <u>stator</u> is the stationary (outer) portion of the machine. It provides the primary magnetic field required for operation. This field will be referred to as the "stator field".

6

Induction Machine Construction



The <u>rotor</u> is the rotational (center) portion of the machine. It that provides the mechanism for energy conversion (elec→mech or mech→elec) as it interacts with the stator field.

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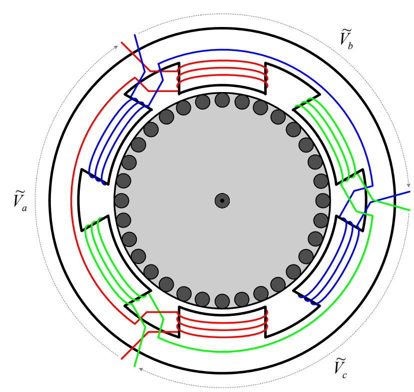
Induction Machine Construction



Although they are magnetically coupled together, the stator and the rotor will initially be considered individually, after which their mutual operation will be discussed.

8

Rotor Construction

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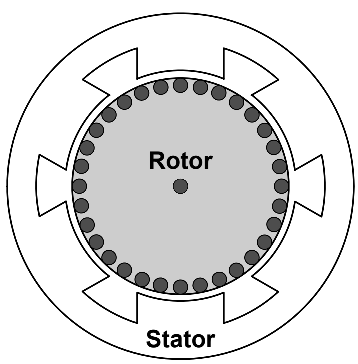
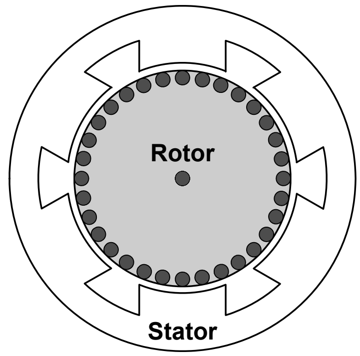
Squirrel-Cage Rotor



The construction of the rotor determines the type of induction machine. This presentation will focus on "Squirrel-Cage" type rotors.

10

Squirrel-Cage Rotor



A "Squirrel-Cage" rotor is constructed with a set of conductive bars, the ends of which are all shorted together by a pair of conductive rings.

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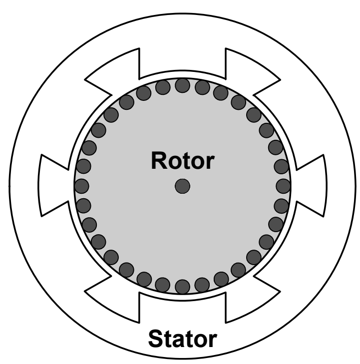
Squirrel-Cage Rotor



The conductive bars are embedded just under the surface of a cylindrical rotor, allowing for only a small air-gap between the rotor and the stator.

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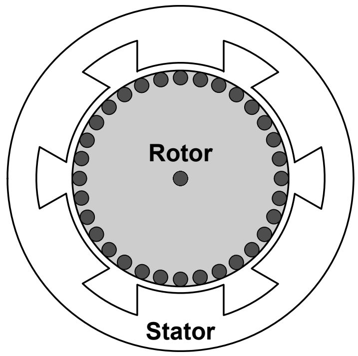
Squirrel-Cage Rotor



Note – The rotor itself is constructed using laminated sheets of steel. The laminations provide insulation between the sheets, preventing currents from flowing length-wise through the rotor unless they actually travel through the embedded conductive bars.

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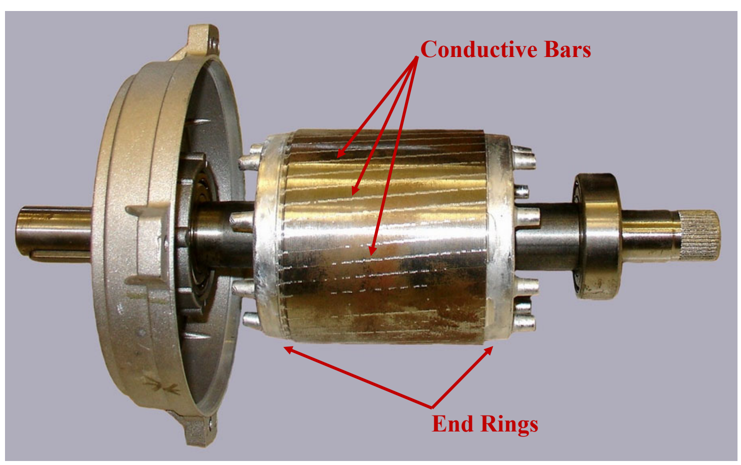
Squirrel-Cage Rotor



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Stator Construction

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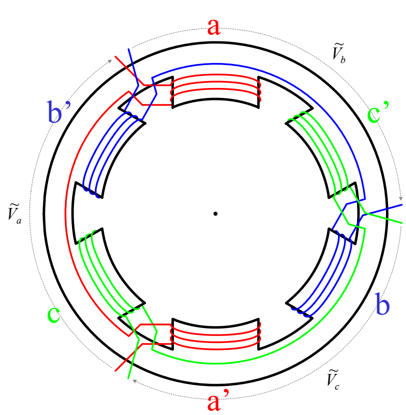
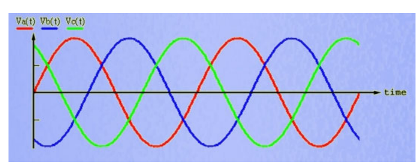
3Φ Induction Machine Stator





The stator of the machine has three symmetrically-placed windings to which a balanced, 3Φ voltage source is connected.

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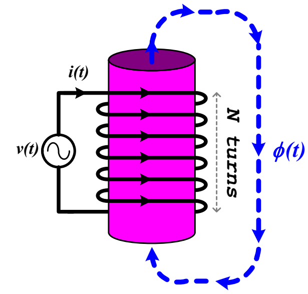
Review: AC-Supplied Coils

If a field coil is supplied by an AC source whose voltage is:

$$v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$$

then the coil will develop a time-varying flux that will pass through the center of the coil, the value of which can be determined by applying Faraday's Law:



$$v(t) = N_p \cdot \frac{d\Phi_M(t)}{dt}$$

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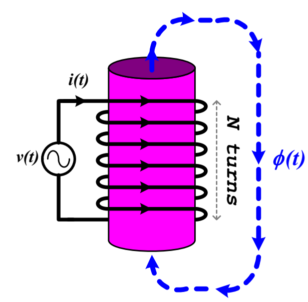
Review: AC-Supplied Coils

Given the coil voltage:

$$v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$$

the resultant flux $\Phi(t)$ will be:

$$\Phi(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)$$



Furthermore, the magnetizing current drawn into the coil can be determined from the relationship:

$$i(t) = \Phi(t) \cdot \frac{\Re}{N}$$

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Review: AC-Supplied Coils

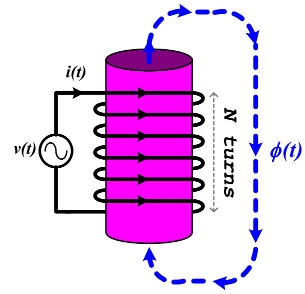
Thus, given the coil voltage v(t) and flux $\Phi(t)$:

$$v(t) = \sqrt{2} \cdot V \cdot \cos(\omega \cdot t)$$

$$\Phi(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)$$

The <u>magnetizing current</u> drawn into the coil will be:

$$i(t) = \sqrt{2} \cdot \frac{V \cdot \Re}{\omega \cdot N^2} \cdot \cos(\omega \cdot t - 90°)$$



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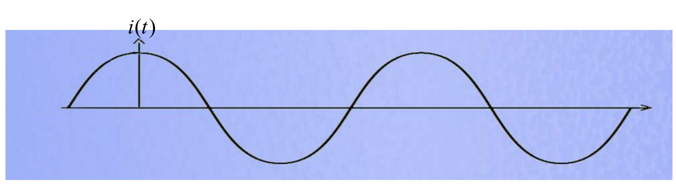
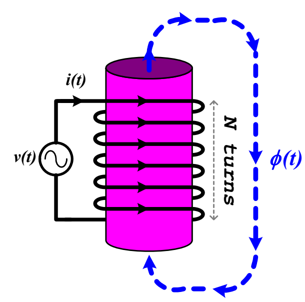
Review: AC-Supplied Coils

Note that both the flux $\Phi(t)$ and the current $i(t)$ vary sinusoidally and that they are <u>in-phase</u> with each other:

$$\Phi(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)$$

$$i(t) = \sqrt{2} \cdot \frac{V \cdot \Re}{\omega \cdot N^2} \cdot \cos(\omega \cdot t - 90°)$$





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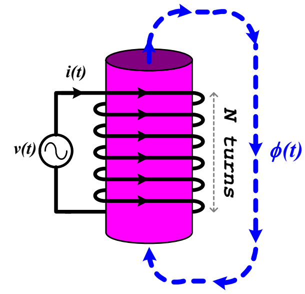
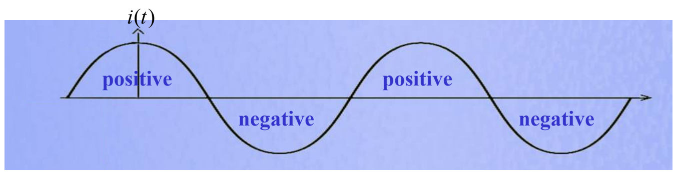
Review: AC-Supplied Coils

Thus, the flux will be positive whenever the coil-current is positive and it will be negative when the current is negative.

$$\Phi(t) = \sqrt{2} \cdot \frac{V}{\omega \cdot N} \cdot \cos(\omega \cdot t - 90°)$$

$$i(t) = \sqrt{2} \cdot \frac{V \cdot \Re}{\omega \cdot N^2} \cdot \cos(\omega \cdot t - 90°)$$





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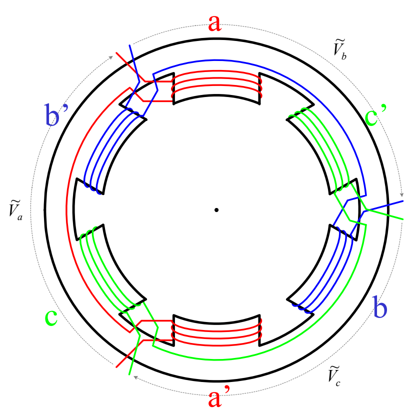
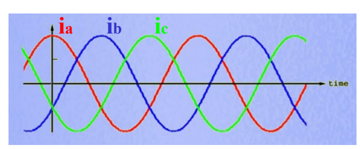
3Φ Stator Windings





The same principle can be applied to the each of the three symmetrically-placed stator windings in the 3Φ induction machine.

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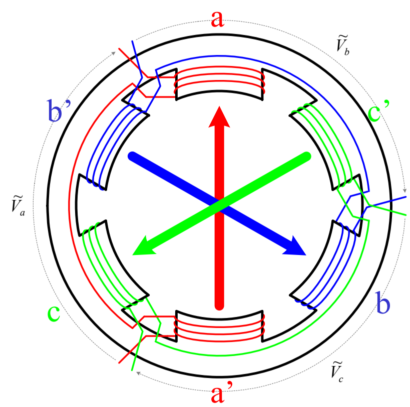
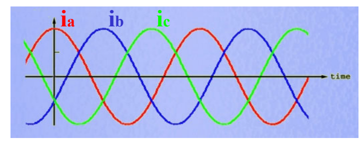
3Φ Stator Windings





Each winding induces a magnetic field that passes through the machine's "rotor region", the magnitude of which varies proportionally with that winding's instantaneous current.

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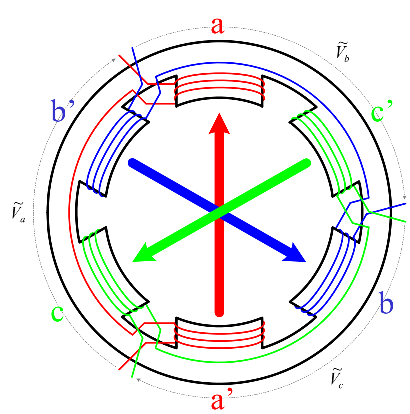
3Φ Stator Windings



Positive Field Directions

"Positive" instantaneous winding currents result in fields that point in the "positive" directions (as shown in the figure).

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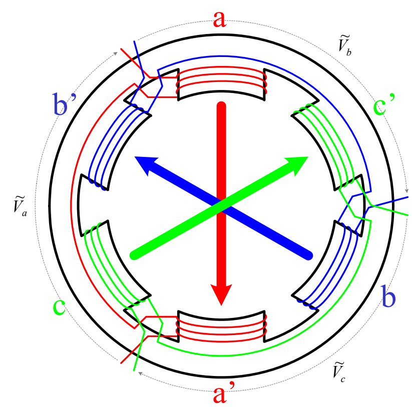
3Φ Stator Windings

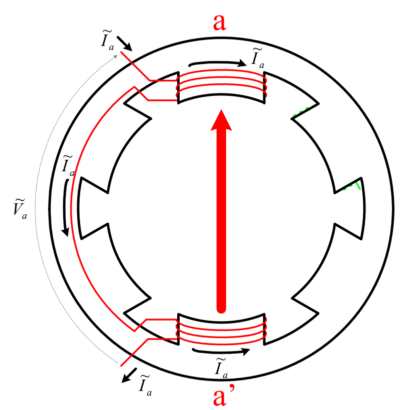


Negative Field Directions

"Negative" instantaneous winding currents result in fields that point in the "negative" directions (as shown in the figure).

25

3Φ Stator Windings



The above figure highlights the <u>phase-A winding</u>, showing the magnetic field that results from a <u>positive</u> winding current.

(The "<u>positive</u>" field points from a'→a up through the rotor region).

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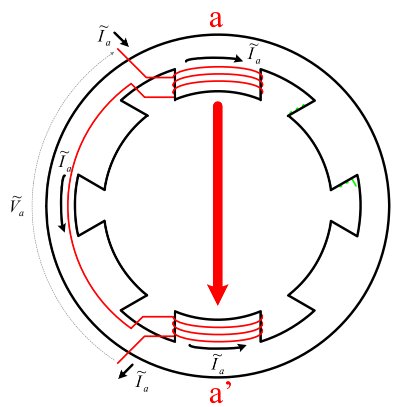
3Φ Stator Windings



Similarly, the above figure shows the magnetic field that results from a <u>negative</u> phase-A winding current.

(The "<u>negative</u>" field points from a→a' down through the rotor region).

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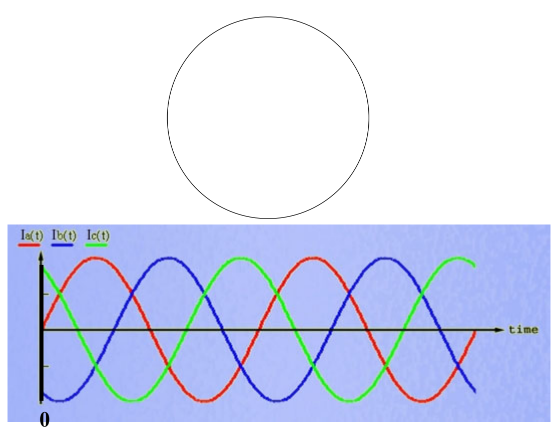
Rotating Stator Field

The net (resultant) field created by the three stator windings can be determined by summing the three individual field vectors at various points in time.



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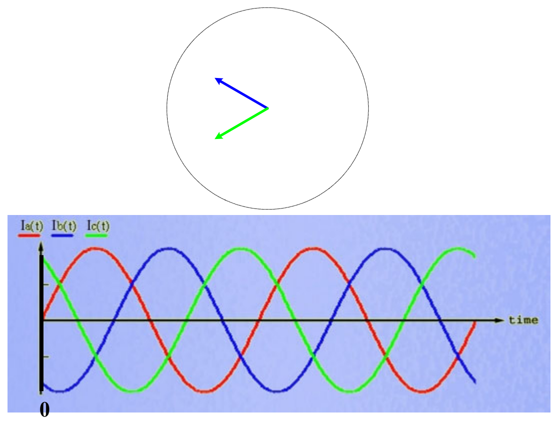
Rotating Stator Field

At time t = 0: $i_a = 0$, $i_b = -0.866$, $i_c = 0.866$

The magnitude and direction of the three field components, are shown below (referenced to the center of the rotor region):

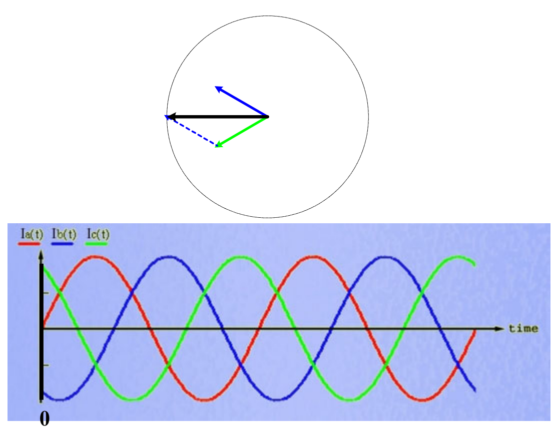


29

Rotating Stator Field

At time t = 0: $i_a = 0$, $i_b = -0.866$, $i_c = 0.866$

The resultant field is the vector-sum of the three individual components:

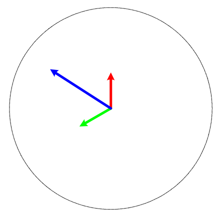


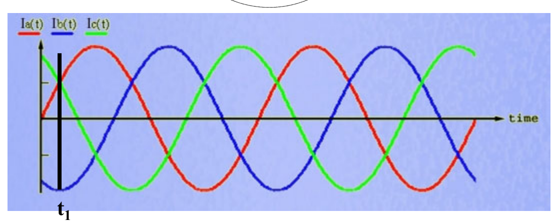
30

Rotating Stator Field

At time $t = t_1$: $i_a = 0.5$, $i_b = -1.0$, $i_c = 0.5$

The magnitude and direction of the three field components, are shown below (referenced to the center of the rotor region):



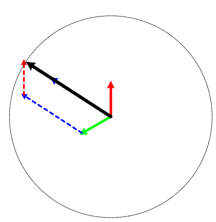


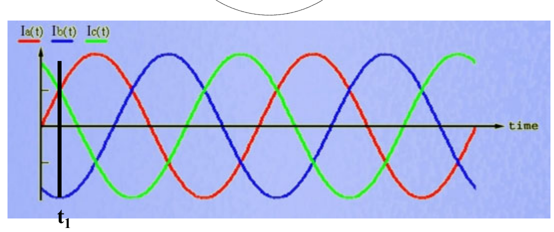
31

Rotating Stator Field

At time $t = t_1$: $i_a = 0.5$, $i_b = -1.0$, $i_c = 0.5$

The resultant field is the vector-sum of the three individual components:



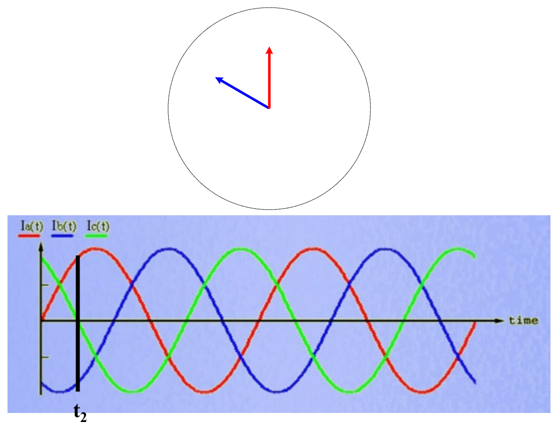


32

Rotating Stator Field

At time $t = t_2$: $i_a = 0.866$, $i_b = -0.866$, $i_c = 0.0$

The magnitude and direction of the three field components, are shown below (referenced to the center of the rotor region):

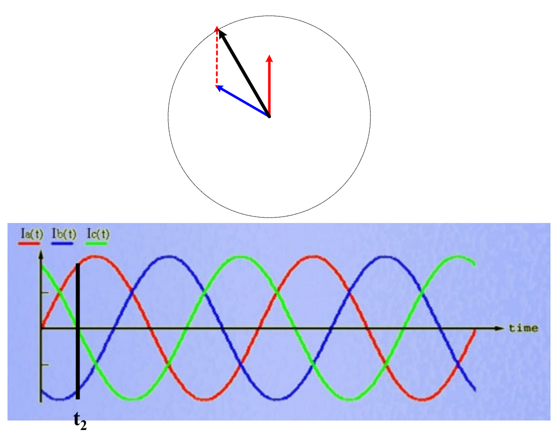


33

Rotating Stator Field

At time $t = t_2$: $i_a = 0.866$, $i_b = -0.866$, $i_c = 0.0$

The resultant field is the vector-sum of the three individual components:



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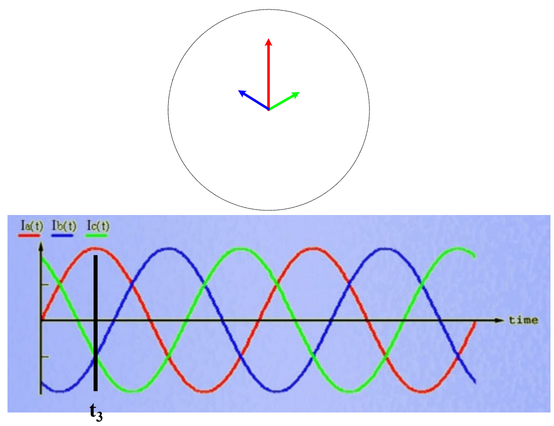
Rotating Stator Field

At time $t = t_3$: $i_a = 1.0$, $i_b = -0.5$, $i_c = -0.5$

The magnitude and direction of the three field components, are shown below (referenced to the center of the rotor region):



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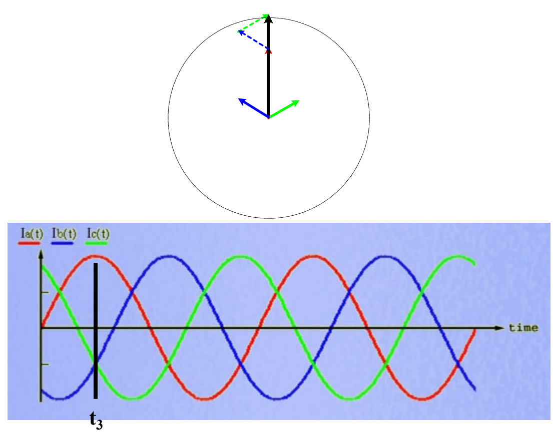
Rotating Stator Field

At time $t = t_3$: $i_a = 1.0$, $i_b = -0.5$, $i_c = -0.5$

The resultant field is the vector-sum of the three individual components:



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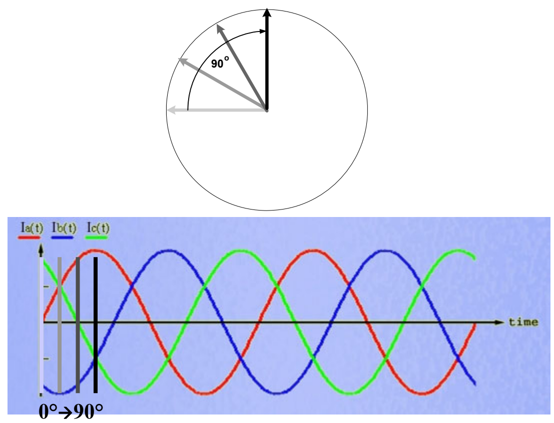
Rotating Stator Field

As time varies from $t=0 \rightarrow t=t_3$ (i.e. – from 0°→90° or ¼ cycle of the sinusoidal currents), the resultant field maintains a <u>constant magnitude</u> while its <u>vector direction rotates</u> clockwise by 90°.



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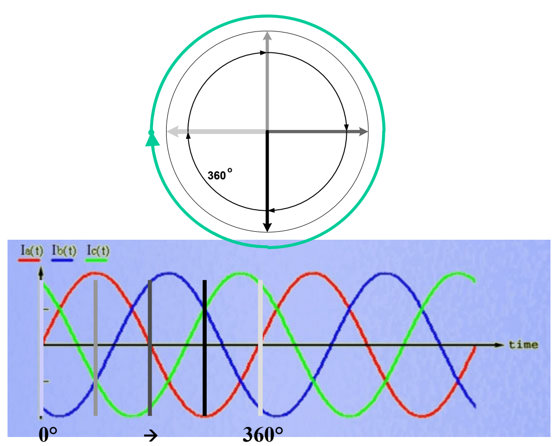
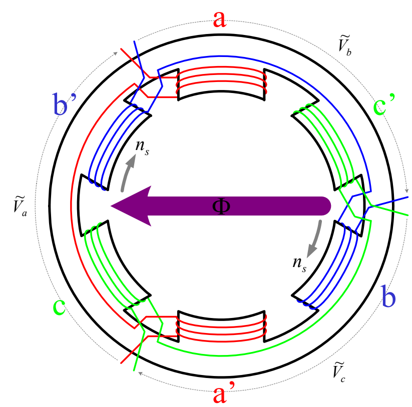
Rotating Stator Field

If the same logic is applied over one cycle of progression, the resultant field will have a constant magnitude and its vector direction will rotate 360° clockwise (i.e. – one complete revolution).



38

Rotating Stator Field



Thus, the three individual (winding) fields combine to form a net “stator field” field that is <u>constant in magnitude</u> but <u>rotates in direction</u>.

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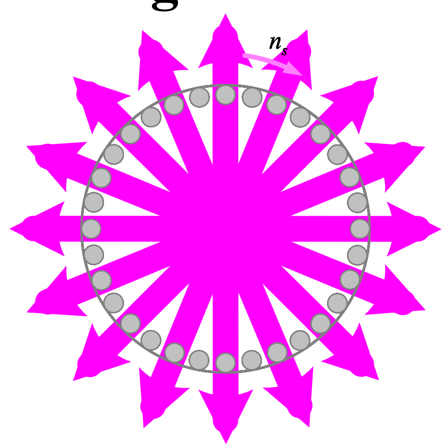
Rotating Stator Field



And since the “stator field” passes through the rotor region, the squirrel-cage rotor conductors will be exposed to a time-varying (rotational) magnetic field.

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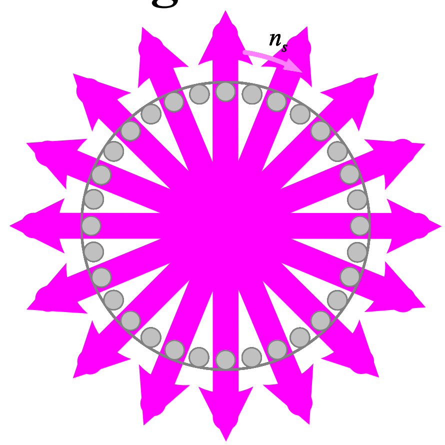
Rotating Stator Field



But let's characterize the "stator field" before we investigate the field's interaction with the rotor conductors.

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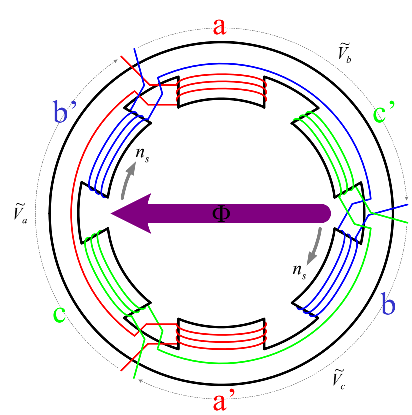
Synchronous Speed



The rotational speed, n_s, of the "stator field" defines the <u>synchronous speed</u> of the machine… (the speed at which the rotating rotor conductors and the stator field are synchronized).

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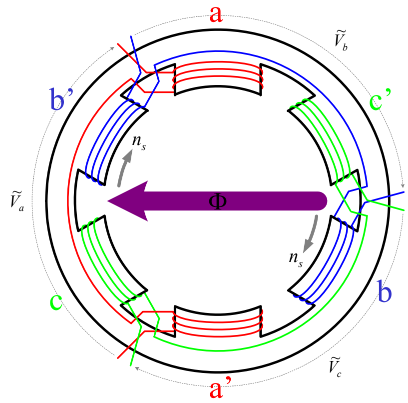
Synchronous Speed



$$n_s = \frac{120 \cdot f_{elec}}{\# \, poles}$$

The synchronous speed, n_s, is a function of both the source frequency and the number of poles* of the machine.

[* – the # of poles is a constructional feature of the machine]

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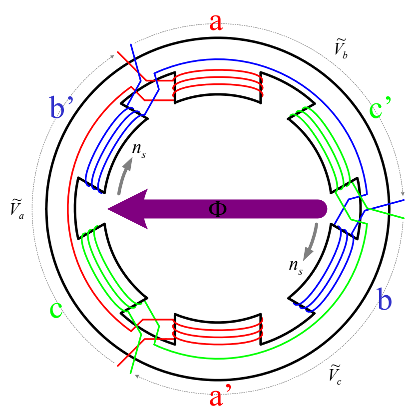
Two-Pole Stator Construction



$$n_s = \frac{120 \cdot f_{elec}}{\# \, poles}$$

The stator shown above is called a “two-pole” stator due to the nature of the resultant stator field.

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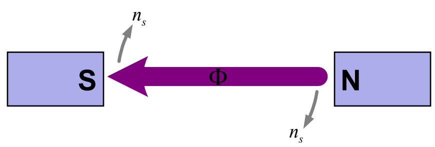
Two-Pole Stator Construction



It is called a "two-pole" design because the resultant field is similar to the field that would be created by the opposing poles of two permanent magnets (i.e. – one N pole & one S pole).

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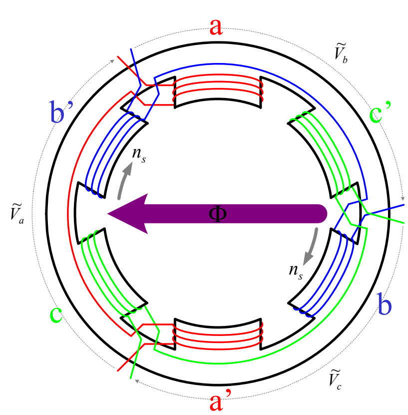
Two-Pole Stator Construction



The two-pole stator-field rotates one complete revolution per cycle of the stator excitation.

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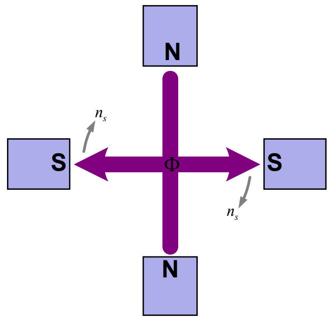
Four-Pole Stator Construction

Note that the winding configurations for the 4-pole and higher-order stator designs will not be shown in this presentation



$$n_s = \frac{120 \cdot f_{elec}}{\# \, poles}$$

Note that higher-order stators also exist, such as a <u>four-pole</u> design that will result in the stator-field shown above.

A four-pole stator-field has 2x the number of poles but only rotates ½ revolution per cycle of the stator excitation.

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Stator Field & Rotor Interaction

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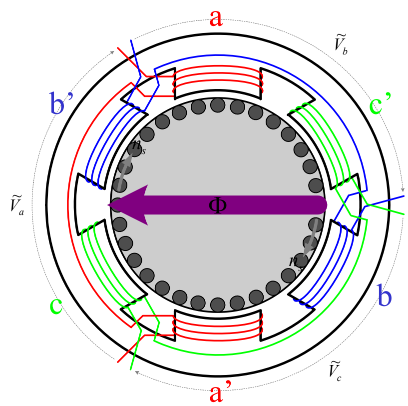
Stator Field – Rotor Interaction



What happens when the squirrel-cage rotor conductors are exposed to the rotating stator field?

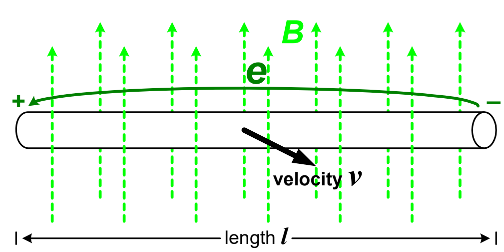
49

Faraday's Law of Induction

Applied to Linear Conductors

Based upon Faraday's Law, a <u>voltage</u> is induced across a conductor if the conductor is moving orthogonally through a magnetic field, the <u>magnitude</u> of which is defined by:

$$e = B \cdot l \cdot v$$

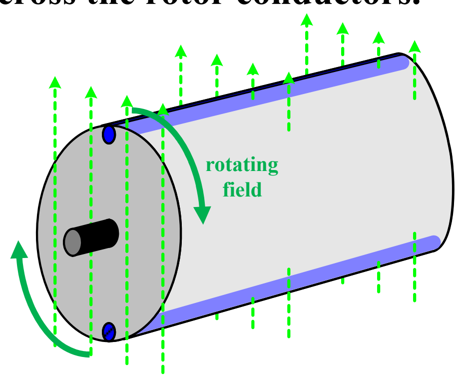


50

Faraday's Law of Induction

Rotating Stator Field & Stationary Rotor Conductors

Similarly, if a pair of conductors are embedded under the surface of a rotor that is placed within the region through which the "stator field" is rotating, then the field lines will be cutting-across the rotor conductors.



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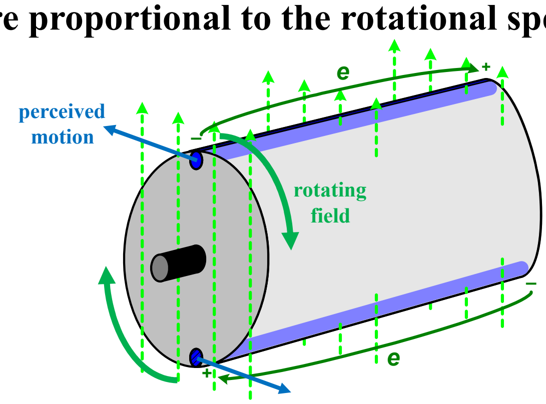
Faraday's Law of Induction

Rotating Stator Field & Stationary Rotor Conductors

Even though it is the field that is rotating, from the conductors' perspective, they are moving orthogonally through the field lines, resulting in voltages being induced across the conductors that are proportional to the rotational speed of the field.



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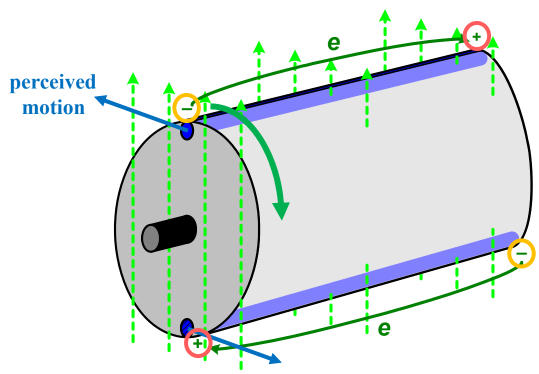
Faraday's Law of Induction

Rotating Stator Field & Stationary Rotor Conductors

Note that the polarities of the induced voltages will be opposite due to their perceived opposing directions of motion through the field lines.



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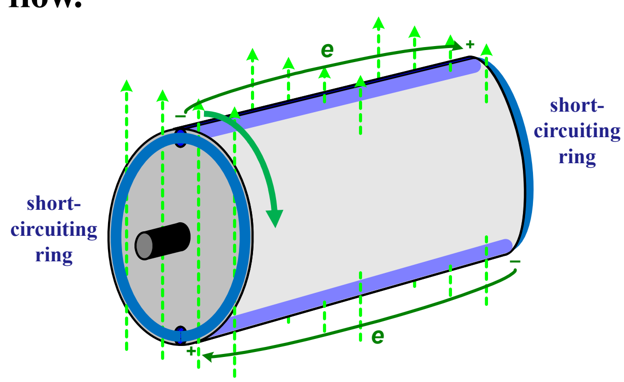
Stator Field – Rotor Interaction

Rotor Conductor Currents

In the case of the squirrel-cage rotor, the ends of the rotor conductors are shorted together by a pair of rings mounted on the ends of the rotor, providing a closed-loop path for current flow.



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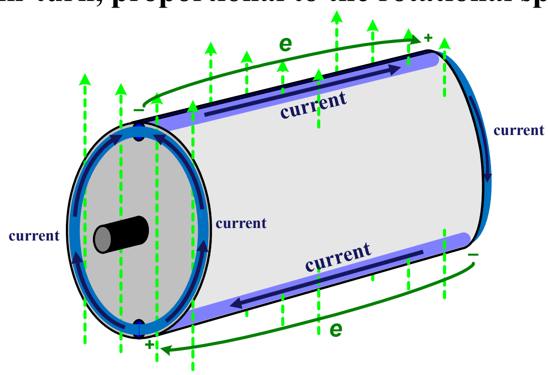
Stator Field – Rotor Interaction

Rotor Conductor Currents

Since the rotor conductor voltages sum around the closed-loop conductive paths, <u>currents</u> will be induced in the rotor conductors that are proportional to the conductor voltages (which are, in-turn, proportional to the rotational speed of the field).



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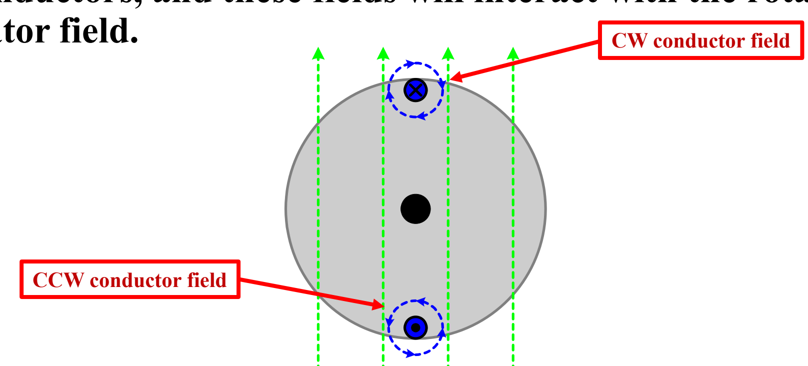
Stator Field – Rotor Interaction

Localized Rotor Conductor Fields

As previously discussed in Part A of the Magnetics presentation, if currents are flowing through each of the conductors, then localized magnetic fields will be induced around each of the conductors, and these fields will interact with the rotating stator field.



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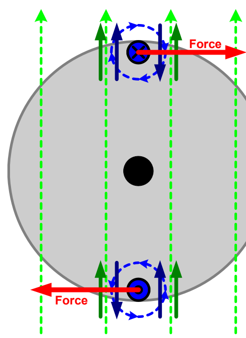
Stator Field – Rotor Interaction

Forces Developed on Rotor Conductors

Based upon the field interactions, a force will be developed on the upper conductor pointing to the right, and an equal force will be developed on the lower conductor pointing to the left.

$$F = B \cdot l \cdot I$$



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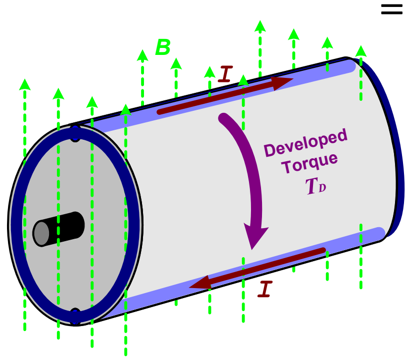
Stator Field – Rotor Interaction

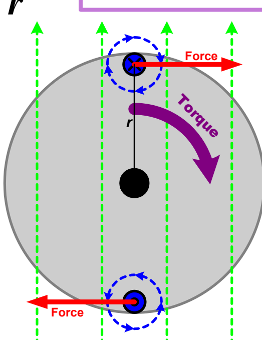
Developed Torque

Furthermore, the opposite-pointing forces result in a net <u>torque</u> (rotational force) being developed upon the rotor in the clockwise direction:

$$T_D = 2 \cdot F \cdot r$$
$$= 2 \cdot B \cdot l \cdot I \cdot r$$

T_D ≡ **<u>Developed Torque</u>** – the total torque (rotational force) that the machine develops.





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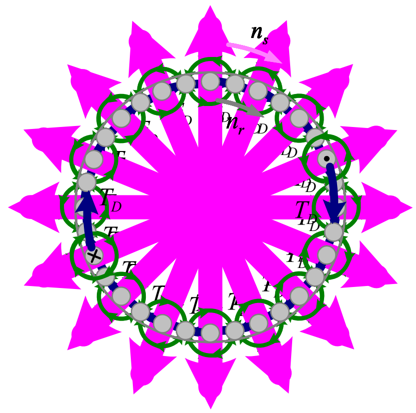
Rotating Stator Field



Note that, since the field is constantly rotating, the opposing conductors upon which a torque is developed will vary with the instantaneous position of the stator field.

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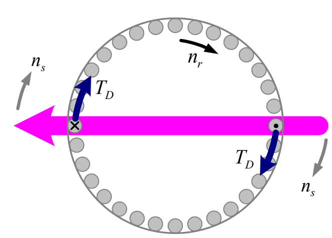
Stator Field – Rotor Interaction

Rotor Acceleration

T_{Load} ≡ **Total Load Torque** – the total torque applied to the shaft of the machine that opposes its rotation.

Note that T_{Load} includes both the stopping force provided by the mechanical load bolted to the shaft and any friction, windage, or other loss forces experienced by the rotor.



T_{accel} ≡ **Acceleration Torque** – the amount of torque available to accelerate the rotor and its attached mechanical load.

$$T_{accel} = T_D - T_{load}$$

If T_{accel} is positive, speed will increase.
If T_{accel} is negative, speed will decrease.
If T_{accel} is zero, speed will remain constant.

Provided that the developed torque, T_D, is greater than the total load torque, T_{Load}, then a positive acceleration torque, T_{accel}, will be available to accelerate the rotor in the same direction as the rotating stator field.

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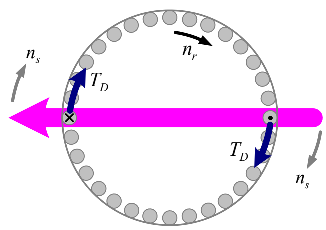
Stator Field – Rotor Interaction

Rotor Acceleration



And if there is a positive acceleration torque, T_{accel}, and the rotor is free to rotate, then the rotor will begin to accelerate the rotor in the same direction as the rotating stator field.

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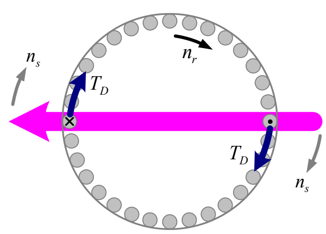
Stator Field – Rotor Interaction

Rotational Effects on Developed Torque



$$n_{effective} = n_s - n_r$$

Note that the developed torque, T_D, is proportional to the rate at which the stator field cuts across the rotor conductors.

Thus, as rotor begins to rotate, the effective speed at which the field-lines cut across the rotor bars decreases, resulting in a decrease in the developed torque, T_D.

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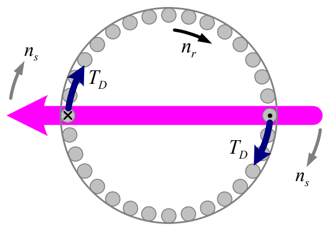
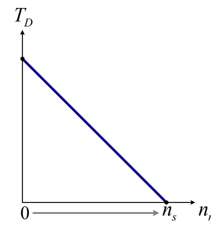
Stator Field – Rotor Interaction

Rotational Effects on Developed Torque





$$n_{effective} = n_s - n_r$$

Since the developed torque, T_D, is proportional to the effective speed, $n_{effective}$, at which the stator field cuts across the rotor bars, <u>the developed torque will decrease linearly as the rotor speed increases</u>, eventually decreasing to zero when the rotor is rotating at synchronous speed.

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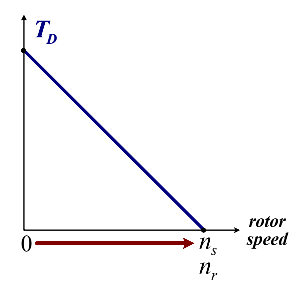
Stator Field – Rotor Interaction

No-Load Operation

If the rotor is rotating at synchronous speed ($n_r = n_s$), then the field lines will no longer be passing by the rotor conductors and no torque will be developed.



Under steady-state No-Load conditions:

$n_r \rightarrow n_s$

$T_D \rightarrow 0$

$T_{accel} = T_D - T_{load} \rightarrow 0$

Under "<u>no-load</u>" conditions (T_{load} =0), the rotor will accelerate until it reaches <u>synchronous speed</u> ($n_r = n_s$), at which point the developed and the acceleration torques equal zero, and the motor maintains steady-state rotation at synchronous speed.

An induction motor cannot accelerate past its synchronous speed without the application of an external rotational force.

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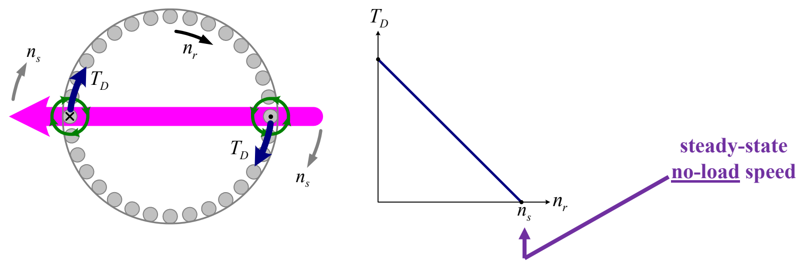
Ideal Motor – No-Load Operation

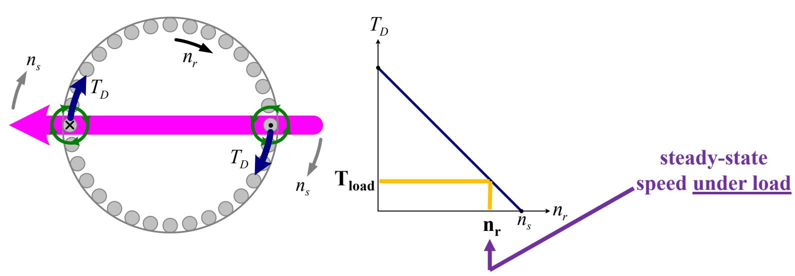


The torque-speed curve for an "ideal" induction machine is shown above for rotor speeds ranging from zero→n_s.

Under no-load conditions, the motor will accelerate to and run steady-state at its synchronous speed

65

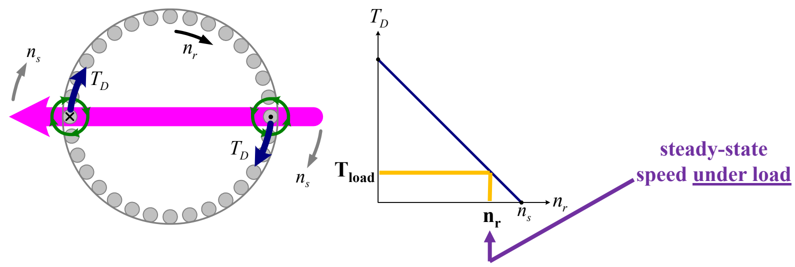
Ideal Motor – Operation Under Load



If the rotor is subjected to a load torque after reaching synchronous speed under no-load conditions, then the rotor will slow down to the speed at which the developed torque, T_D, equals to the load torque, T_{Load}.

66

Ideal Motor – Operation Under Load



Or, if the rotor is subjected to a load torque at startup, then the rotor will only be able to accelerate up to the speed at which the developed torque, T_D, equals to the load torque, T_{Load}.

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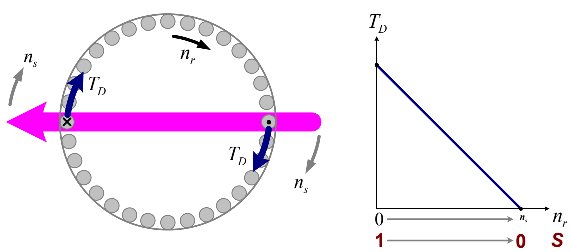
Stator Field – Rotor Interaction

Slip



$$slip \equiv s = \frac{n_s - n_r}{n_s}$$

Note – Rotor speed is often expressed in terms of slip, which provides a measure of how much slower the rotor is rotating compared to the speed of the stator field.

$$slip \equiv s = \frac{n_s - n_r}{n_s}$$

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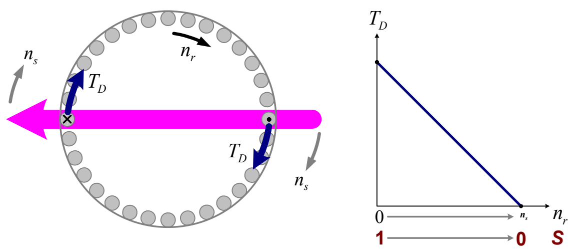
Stator Field – Rotor Interaction

Slip



$$slip \equiv s = \frac{n_s - n_r}{n_s}$$

As rotor speed varies from zero to synchronous speed ($n_r = 0 \rightarrow n_s$), the slip decreases linearly from one to zero ($s = 1 \rightarrow 0$).

Thus, slip and developed torque both vary in a similar manner.

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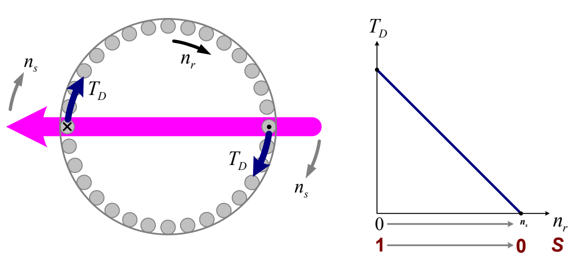
Stator Field – Rotor Interaction

Slip



T_{BR} ≡ **Blocked-Rotor Torque** – the torque developed by the machine when the rotor isn't moving ($n_r = 0$).

Note: Blocked-Rotor Torque may also be referred to as Locked-Rotor Torque and/or Starting Torque.

Since both slip and developed torque decrease linearly as rotor speed varies from zero to synchronous speed, T_D can be expressed in terms of the blocked-rotor torque, T_{BR}, and slip:

$$T_D = T_{BR} \cdot s$$

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Modeling
the
Induction Machine

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Induction Machine Modeling Concepts

The interaction between a 3Φ Induction Machine's stator windings and rotor conductors is similar to the interaction between a transformer's primary and secondary windings:

- **Time-varying voltages are applied to a set of stator (primary) windings.**
- **Each stator winding creates a time-varying flux within the machine's rotor region, the sum of which can be expressed a constant-magnitude "stator" field whose directional vector rotates in time.**
- **The (time-varying) rotating "stator" field induces a voltage across the rotor conductors (secondary windings).**

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Modeling the 3Φ Induction Machine

Note that, although the 3Φ Induction Machine has three stator windings, we will begin the modeling process by looking at the contribution of a single stator winding to the overall operation of the machine.

I.e. – we will create a 1Φ Equivalent Circuit for the 3Φ, Y-connected, Induction Machine

We can do this because, as a balanced load connected to a balanced 3Φ supply, the voltages and currents of the other two phases may be derived from the results of the single-phase circuit solution.

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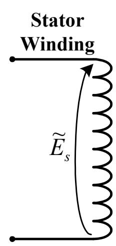
Creating the 1Φ Equivalent Circuit

A time-varying voltage is applied to the stator winding, resulting in a rotating stator field.

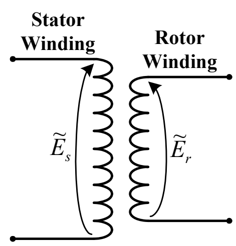


74

Creating the 1Φ Equivalent Circuit

A time-varying voltage is applied to the stator winding, resulting in a rotating stator field.

The rotating stator field induces voltages across the rotor bars.



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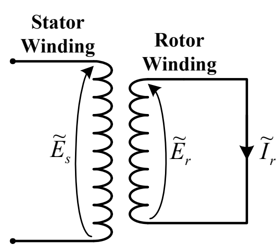
Creating the 1Φ Equivalent Circuit

A time-varying voltage is applied to the stator winding, resulting in a rotating stator field.

The rotating stator field induces voltages across the rotor bars.

Since the ends of the rotor bars are shorted together, the induced voltages cause currents to flow in the rotor bars.



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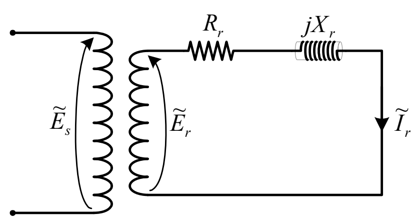
Rotor Bar Impedance Concerns

If the rotor bars are assumed to be ideal, then an infinite current would flow in the rotor conductors.

Thus, to accurately model the stator-rotor interaction, we must consider the impedance of the rotor bars, in terms of which the rotor current may be defined:

$$\tilde{I}_r = \frac{\tilde{E}_r}{R_r + jX_r}$$



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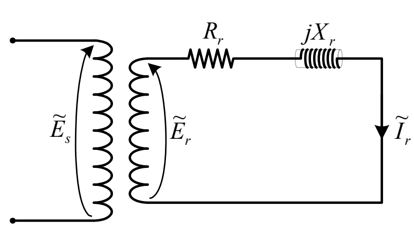
Rotor Voltage – Speed Interaction

In order to analyze the circuit as if it contains an ideal transformer, the ratio of the stator and rotor "winding" voltages across the must be constant:

$$a = \frac{\tilde{E}_s}{\tilde{E}_r}$$

If so, this would allow the rotor impedances to be referred to the stator side of the ideal windings.



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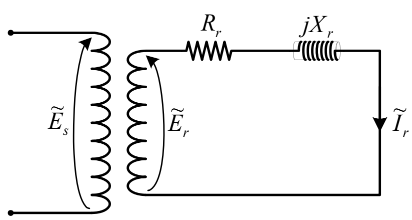
Rotor Voltage – Speed Interaction

But, the <u>magnitude</u> of the rotor conductor voltages varies as the machine accelerates, decreasing linearly as rotor speed increases from zero to synchronous speed:

$$\left|\tilde{E}_r\right| \rightarrow 0 \quad as \quad n_r \rightarrow n_s$$

And the <u>frequency</u> of the rotor voltages varies similarly:

$$f_r \rightarrow 0 \quad as \quad n_r \rightarrow n_s$$



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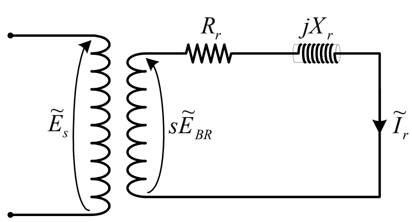
Rotor Voltage – Speed Interaction

The rotor voltage – speed relationship may be expressed in terms of the magnitude, E_{BR}, and the frequency, f_{BR}, of the rotor voltages under blocked-rotor conditions ($n_r = 0$) if rotor speed is expressed in terms of slip, as follows:

$$E_r = s \cdot E_{BR}$$

$$f_r = s \cdot f_{BR}$$



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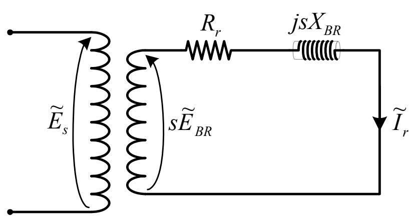
Rotor Voltage – Speed Interaction

It should be noted that the frequency, f_{BR}, of the rotor voltages will equal to the frequency of the stator voltage, under f_s, blocked-rotor conditions ($n_r = 0$), therefore:

$$f_r = s \cdot f_s$$

Because of this, the leakage reactance will also vary with speed:

$$X_r = \omega_r L_r = 2\pi f_r L_r = 2\pi(s \cdot f_s)L_r = s \cdot 2\pi f_s L_r = s \cdot X_{BR}$$



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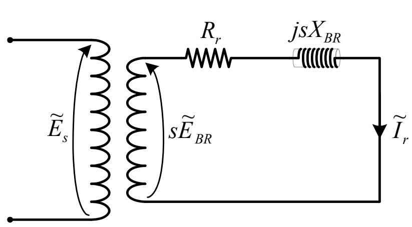
Developed Torque

One of the goals of creating this model is to be able to use it to predict the torque that the machine develops under various operating conditions.

Assuming that the magnitude of the stator voltages (and thus the stator field) remains constant, the torque developed by the motor will be proportional to the square of the magnitude of the current, I_r, that flows in the rotor bars.



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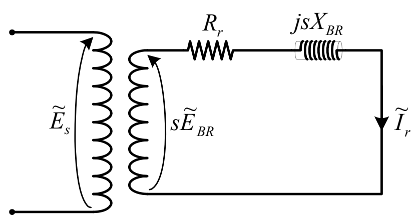
Manipulating the Rotor Circuit

The rotor current, I_r, when expressed in terms of the blocked-rotor values and slip, may defined as:

$$\tilde{I}_r = \frac{\tilde{E}_r}{R_r + jX_r} = \frac{s\tilde{E}_{BR}}{R_r + jsX_{BR}}$$

If the goal is to determine torque, which can be calculated from rotor current, then we can manipulate the model as needed provided that it still allows us to calculate the rotor current.



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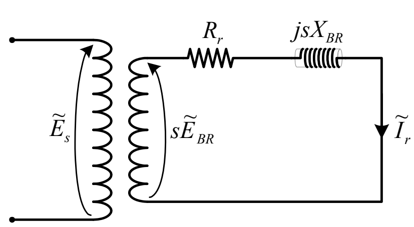
Manipulating the Rotor Circuit

Note – the speed dependence of the rotor "winding" voltage (expressed in terms of slip) prevents us from referring the rotor impedances to the "stator side" of the model.

We could remove this dependence if we divide the rotor voltage by slip, but this would also affect the rotor current.

$$\tilde{I}_r = \frac{\tilde{E}_r}{R_r + jX_r} = \frac{s\tilde{E}_{BR}}{R_r + jsX_{BR}}$$



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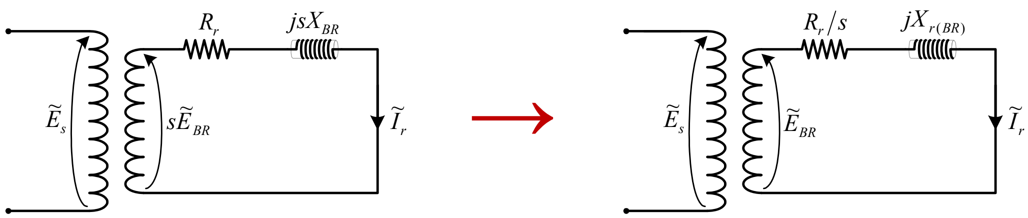
Manipulating the Rotor Circuit

But, the overall solution for the rotor current will not change provided that we divide <u>both</u> the rotor voltage and the rotor impedances by slip:

$$\tilde{I}_r = \frac{\tilde{E}_r}{R_r + jX_r} = \frac{s\tilde{E}_{BR}\cdot\frac{1}{s}}{\left(R_r + jsX_{BR}\right)\cdot\frac{1}{s}} = \frac{\tilde{E}_{BR}}{R_r\cdot\frac{1}{s} + jX_{BR}}$$



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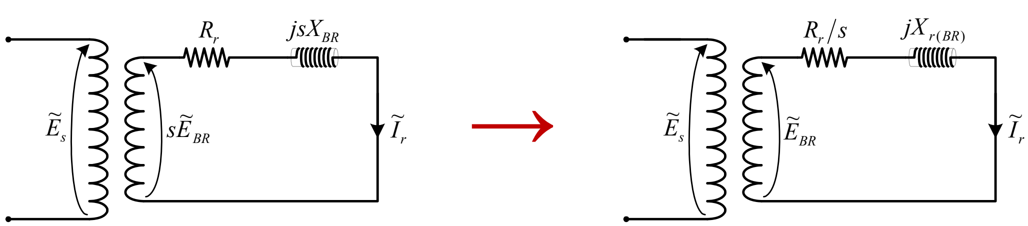
Manipulating the Rotor Circuit

When making this change, the slip terms cancel for both the rotor voltage and the rotor reactance.

Since the rotor voltage no longer varies with speed, we can treat the stator and rotor "windings" as we would an ideal transformer.



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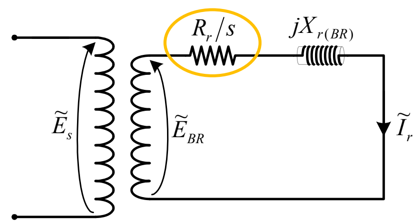
Manipulating the Rotor Circuit

Note that we are now left with a rotor resistance that varies inversely with slip.

Although this may seem problematic since the resistance of the rotor conductors should not vary when the rotor begins to rotate, it turns out that we can account for this issue by making a simple adjustment to the rotor circuit.



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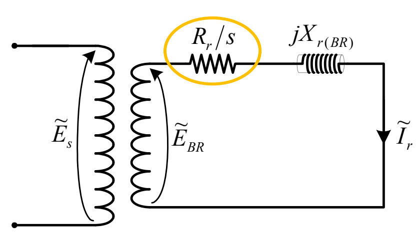
Manipulating the Rotor Circuit

In the model, the term R_r/s must account for all of the power transferred from the stator windings into the rotor circuit.

This includes both the electric power dissipated by the rotor resistance and the electric power that is being converted to a mechanical form whenever the motor is developing a torque at some rotational speed.



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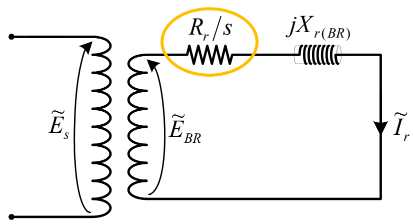
Manipulating the Rotor Circuit

We can account for these two independent power components by considering the resistance R_r/s as the series-equivalent of two distinct resistances, one the relates to the power loss in the rotor and the other that relates to the electrical power that is converted to a mechanical form, such that:

$$\frac{R_r}{s} = R_r + R_r\left(\frac{1-s}{s}\right)$$



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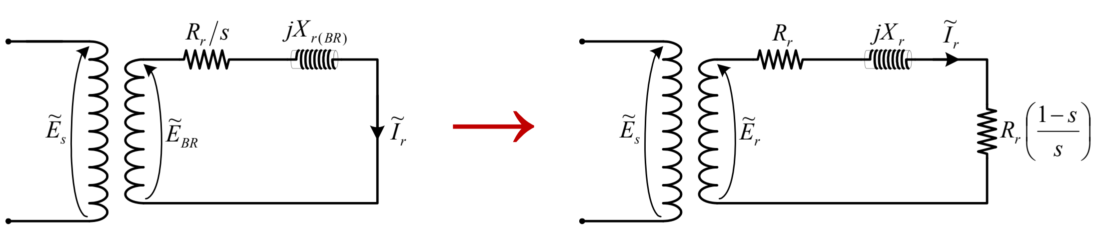
Manipulating the Rotor Circuit

Note that, when replacing the term R_r/s with the two series resistances, the resistance R_r is typically placed next to the reactance jX_r, while the remaining resistance is placed to the right to help differentiate it from the rotor impedances.

$$\frac{R_r}{s} = R_r + R_r\left(\frac{1-s}{s}\right)$$



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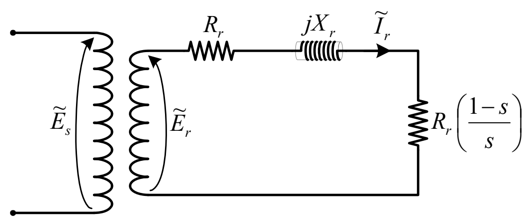
Mechanical Power

Since the resistance R_r accounts for the power dissipated by the rotor conductors, the remaining resistance,

$$R_r\left(\frac{1-s}{s}\right)$$

must account for the electrical power that is converted to a mechanical form.

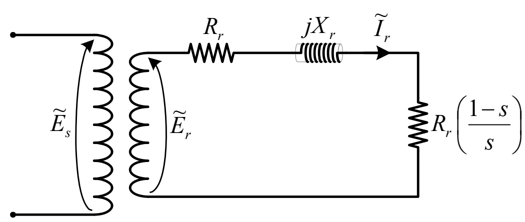


91

Modeling the Stator Losses

Now that we've developed the circuit model the rotor portion of the machine, we must also account for the losses that result from the AC-supplied stator windings.

Note that these stator losses are very similar in nature to the losses that result from the AC-supplied primary winding of a practical transformer.

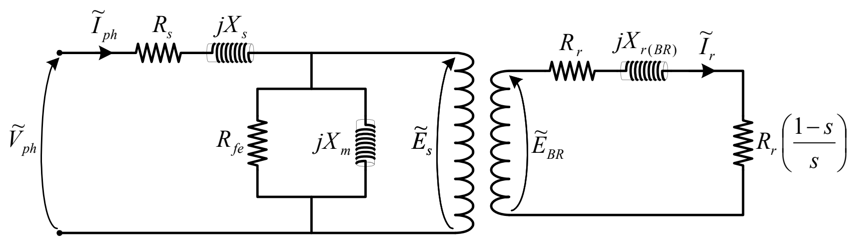


92

Modeling the Stator Losses

First, R_s and jX_s are added to the stator-side of the model to account for the resistance and leakage reactance of the stator windings.

The loss components, R_{fe} and jX_m, are then added to account for the magnetization effects and the power losses that occur in the core structure of the induction machine.



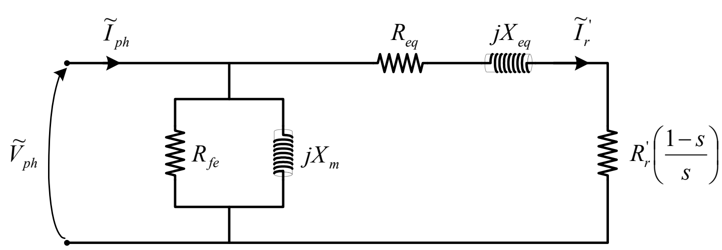
93

Simplifying the Model

The resultant model can then be simplified and reduced in the same manner as the model for the practical transformer, resulting in the following 1Φ equivalent circuit for a practical, 3Φ, induction machine:

such that: $$R_{eq} = R_s + R_r'$$

$$X_{eq} = X_s + X_r'$$



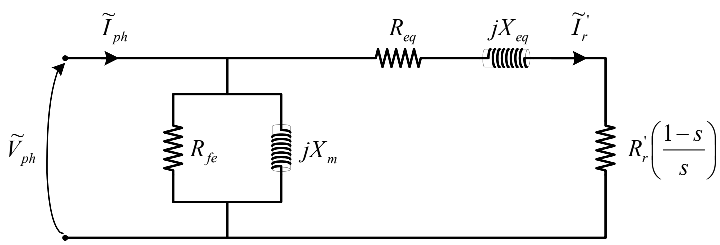
94

1Φ Equivalent Circuit

for a 3Φ Induction Machine

The following <u>1Φ Equivalent Circuit</u> is often used to model the operation of a Y-connected, 3Φ, Induction Machine.

If it is assumed that the machine is supplied by a balanced 3Φ source, then the voltages and currents of the other phases can be derived from the results of the 1Φ circuit solution.

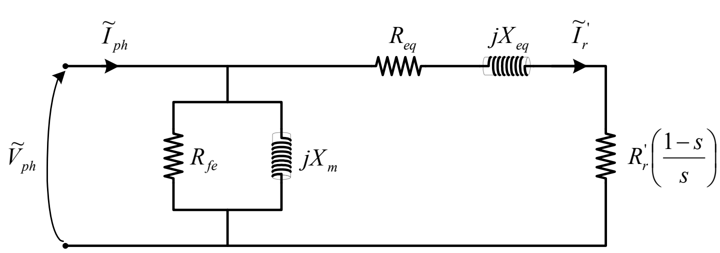


95

1Φ Equivalent Circuit

for a 3Φ Induction Machine

***R_{fe}* and *X_m* account for the magnetization effects due to the rotating (time-varying) magnetic field created by the stator windings within the core material that forms the physical structure of the machine.**

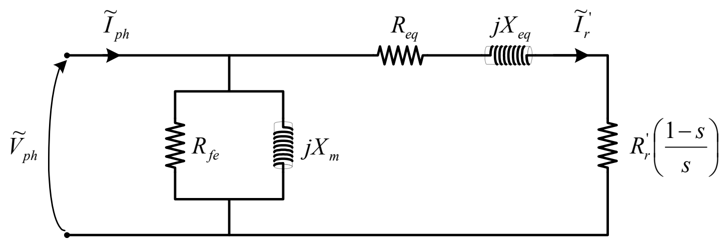


96

1Φ Equivalent Circuit

for a 3Φ Induction Machine

R_{eq} and X_{eq} account for the impedance of the stator windings combined with the effective per-phase impedance of the rotor conductors (referred to the stator side of the model).



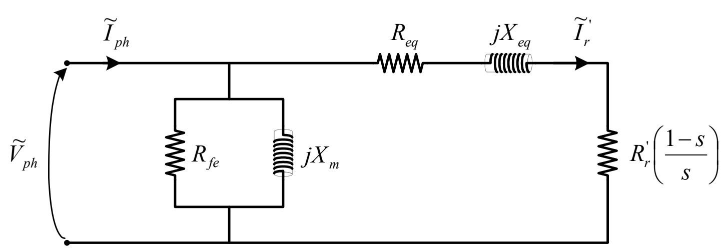
97

1Φ Equivalent Circuit

for a 3Φ Induction Machine

The remaining resistance, $R_r'\left(\frac{1-s}{s}\right)$, relates to the mechanical load that the machine is driving.

This resistance varies with slip, appearing as a short-circuit at a slip of one (s=1) and an open-circuit at a slip of zero (s=0).

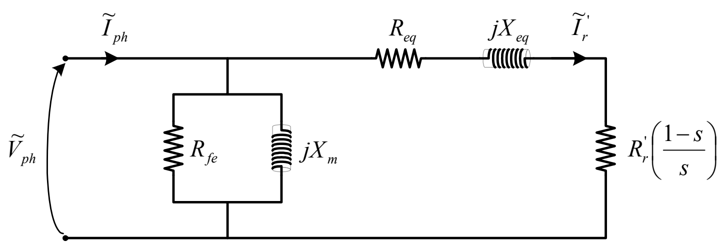


98

Mechanical Power

The power consumed by the resistance $R_r^{'}\left(\frac{1-s}{s}\right)$ equals to the per-phase mechanical power produced by the machine.

(I.e. – the electrical power converted to a mechanical form)

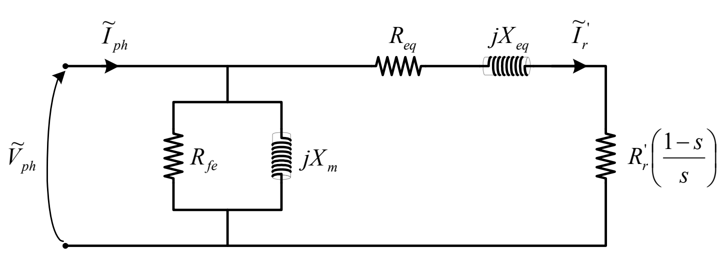


99

Mechanical Power

Assuming balanced operation, the total mechanical power produced by the machine, P_{mech}, will equal to three times (3x) the power consumed in one phase by $R_r^{'}\left(\frac{1-s}{s}\right)$.

$$P_{mech} = 3 \cdot \left|\tilde{I}_r^{'}\right|^2 \cdot R_r^{'}\left(\frac{1-s}{s}\right) \quad Watts$$



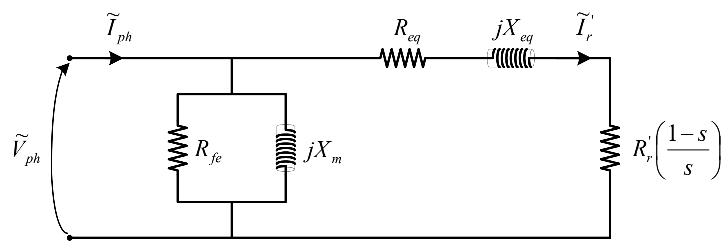
100

Developed Torque

Since $T_{D(lb\cdot ft)} = \dfrac{5252 \cdot P_{mech(hp)}}{n_{r(rpm)}}$ **and 1** $hp = 746\ watts$,

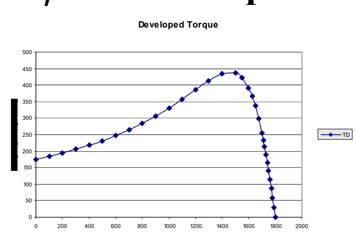
the total torque, T_D**, developed by the machine can be defined from the solution for mechanical power as:**

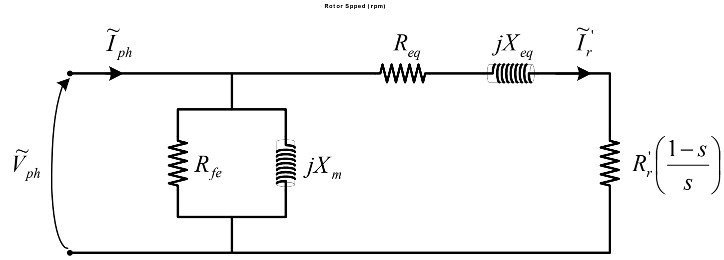
$$T_D = \frac{21.12 \cdot \left|\tilde{I}_r'\right|^2 \cdot R_r'}{s \cdot n_s} \quad lb \cdot ft$$



Developed Torque

If T_D **is plotted as a function of speed over the range** $n_r = 0 \rightarrow n_s$, **the resultant** T_D **vs** n_r **curve for a practical machine will resemble:**





Sample Problem

Given a **3Φ, 10hp, 230V, 6-pole, 60 Hz, 1138 rpm,** Y-connected, squirrel-cage, induction motor with the following parameters:

$\mathbf{R_s = 0.3\ \Omega}$, $\mathbf{X_s = 0.4\ \Omega}$, $\mathbf{R'_r = 0.3\ \Omega}$, $\mathbf{X'_r = 0.6\ \Omega}$,
$\mathbf{R_{fe} = 100\ \Omega}$, $\mathbf{X_m = 24\ \Omega}$, $\mathbf{P_{mechlosses} = 190\ W}$

If the motor is rotating at a speed of **1180 rpm** while supplied with **rated voltage** and driving an **unknown load,**

Determine: the amount of **torque,** $\boldsymbol{T_{shaft}}$, that is developed by the motor in order to drive its mechanical load, and

the total **real power,** $\boldsymbol{P_{elec}}$, that the 3Φ source supplies to the induction machine.

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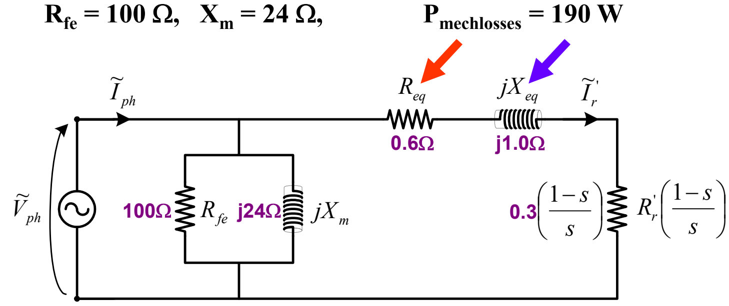
Populate the Model

Given a **3Φ, 10hp, 230V, 6-pole, 60 Hz, 1138 rpm,** Y-connected, squirrel-cage, induction motor with the following parameters:

$\mathbf{R_s = 0.3\ \Omega}$, $\mathbf{X_s = 0.4\ \Omega}$, $\mathbf{R'_r = 0.3\ \Omega}$, $\mathbf{X'_r = 0.6\ \Omega}$,
$\mathbf{R_{fe} = 100\ \Omega}$, $\mathbf{X_m = 24\ \Omega}$, $\mathbf{P_{mechlosses} = 190\ W}$



$$R_{eq} = R_s + R'_r = 0.3 + 0.3 = 0.6\ \Omega \qquad X_{eq} = X_s + X'_r = 0.4 + 0.6 = 1.0\ \Omega$$

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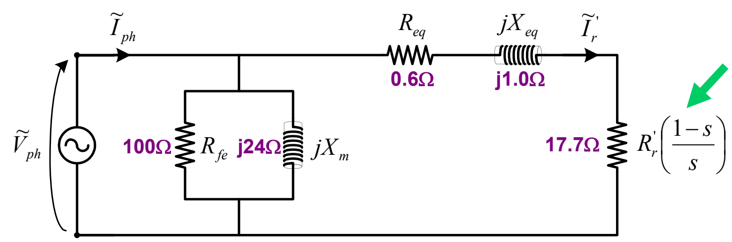
Populate the Model

Given a **3Φ, 10hp, 230V, 6-pole, 60 Hz, 1138 rpm,** Y-connected, squirrel-cage, induction motor with the following parameters:

R_s = 0.3 Ω, X_s = 0.4 Ω, R'_r = 0.3 Ω, X'_r = 0.6 Ω,
R_{fe} = 100 Ω, X_m = 24 Ω, $P_{mechlosses}$ = 190 W



$$n_s = 1200\ rpm \qquad s = \frac{n_s - n_r}{n_s} = \frac{1200 - 1180}{1200} = 0.01\overline{66} \qquad R_r'\left(\frac{1-s}{s}\right) = 0.3\left(\frac{1-0.01\overline{66}}{0.01\overline{66}}\right) = 17.7\ \Omega$$

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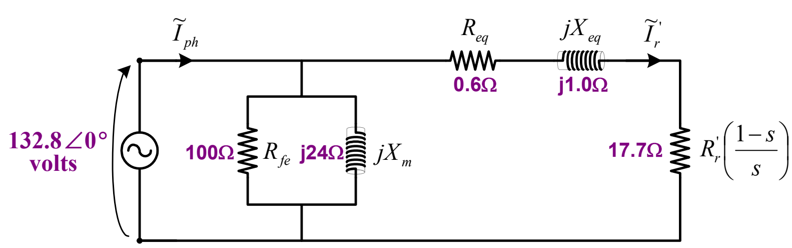
Define the Supply Voltage

Given a **3Φ, 10hp, 230V, 6-pole, 60 Hz, 1138 rpm,** Y-connected, squirrel-cage, induction motor with the following parameters:

R_s = 0.3 Ω, X_s = 0.4 Ω, R'_r = 0.3 Ω, X'_r = 0.6 Ω,
R_{fe} = 100 Ω, X_m = 24 Ω, $P_{mechlosses}$ = 190 W

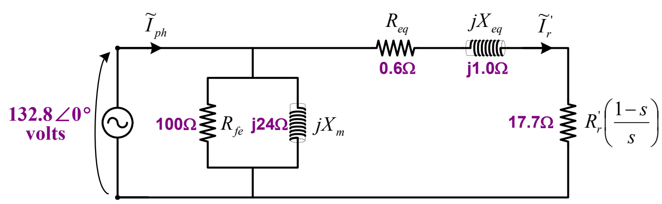


$$V_{ph} = \frac{V_{line}}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.8V$$

106

Solve for Mechanical Power

To solve for the **shaft torque, T_{shaft}**, first determine the total **mechanical power** produced by the machine, P_{mech}.



$$\tilde{I}_r' = \frac{\tilde{V}_{ph}}{R_{eq} + jX_{eq} + R_r'\left(\frac{1-s}{s}\right)} = \frac{132.8\angle 0^\circ}{0.6 + j1.0 + 17.7} = 7.246\angle -3.13^\circ$$

$$P_{mech} = 3 \cdot \left|\tilde{I}_r'\right|^2 \cdot R_r'\left(\frac{1-s}{s}\right) = 3 \cdot \left|7.246\right|^2 \cdot 17.7 = 2788\ Watts$$

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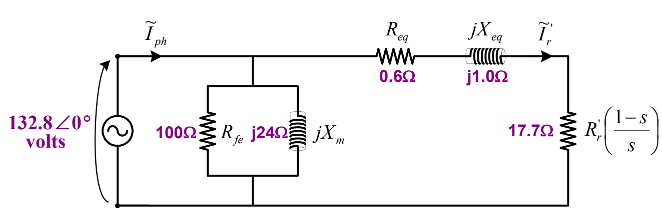
Solve for Shaft Torque

Then, **subtract** the **mechanical losses** from the total **mechanical power** in order to find P_{shaft}, convert the result to horse-power, and then determine T_{shaft} from the shaft power value:



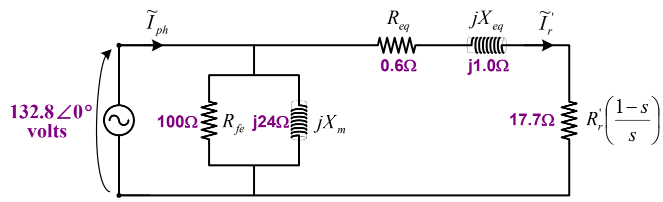
$$P_{shaft} = P_{mech} - P_{mech.losses} = P_{mech} - 190 = 2598\ Watts = \frac{2598\ Watts}{746\ \frac{Watts}{hp}} = 3.482\ hp$$

$$\boxed{T_{shaft} = \frac{5252 \cdot P_{shaft}}{n_r} = \frac{5252 \cdot 3.482}{1180} = 15.5\ lb \cdot ft}$$

108

Determine the Input Current

To solve for the total **real power** supplied by the source to the machine, P_{elec}, first **determine** the **input impedance** of the machine and use the result to **solve** for the **line current:**



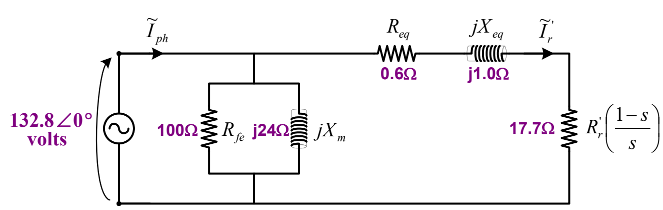
$$Z_{in} = R_{fe} \parallel jX_m \parallel \left(R_{eq} + jX_{eq} + R_r'\left(\frac{1-s}{s}\right)\right) = \left(\frac{1}{100} + \frac{1}{j24} + \frac{1}{18.3 + j1.0}\right)^{-1} = 10.4 + j7.26\,\Omega$$

$$\tilde{I}_{line} = \frac{\tilde{V}_{ph}}{Z_{in}} = \frac{132.8\angle 0^\circ}{10.4 + j7.26} = 10.4\angle -34.7^\circ\ A$$

109

Determine the Source Power

Then, determine the total **complex power** produced by the source, $S_{3\Phi}$, the real part of which equals the **real power** supplied by the source to the motor:



$$S_{source} = 3 \cdot \tilde{V}_{ph} \cdot \tilde{I}_{line}^* = 3 \cdot (132.8\angle 0^\circ) \cdot (10.4\angle +34.7^\circ) = 3411 + j2361 = P_{source} + jQ_{source}$$

$$\boxed{P_{source} = 3411 \quad Watts}$$

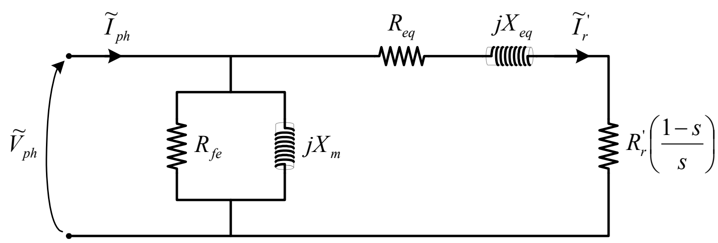
110

Determining the Model Parameters

The model parameters for an induction motor are often unknown because they aren't typically provided by the manufacturer.

But the parameters can be determined by performing three simple tests on the motor:

- a **No-Load Test**,
- a **Locked-Rotor Test**, and
- a **Stator Resistance Test.**

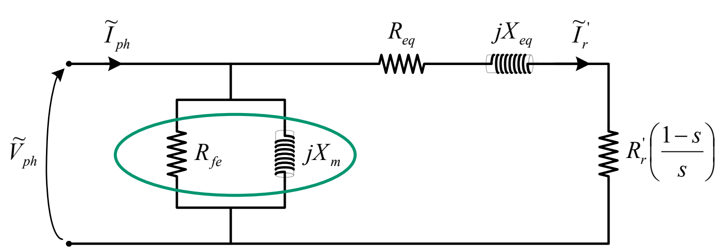


111

The No-Load Test

The **core-loss parameters R_{fe}** and **X_m** can be determined by performing a **No-Load Test** on an induction motor.

The **No-Load Test** is performed by applying **rated voltage** to the stator windings of the motor and **measuring** the magnitude of the **lines currents** and the **real power** supplied to the motor while leaving the rotor de-coupled from its mechanical load (i.e. – no load).



112

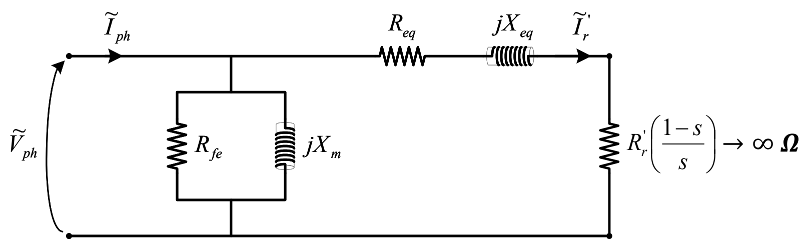
The No-Load Test

Under ideal no-load conditions, the rotor will rotate at the motor's synchronous speed ($\boldsymbol{n_r} = \boldsymbol{n_s}$), and thus:

$$\boldsymbol{slip \equiv s = \frac{n_s - n_r}{n_s} = \frac{n_s - n_s}{n_s} = 0}$$

and:

$$\boldsymbol{R_r\left(\frac{1-s}{s}\right) = R_r\left(\frac{1-0}{0}\right) \to \infty\ \Omega} \text{ (open circuit)}$$



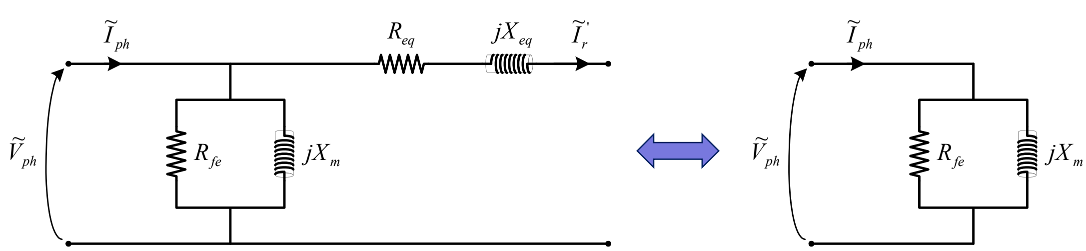
113

The No-Load Test

Thus, under no-load conditions, the rotor conductor current $\boldsymbol{\tilde{I}'_r}$ must be zero:

$$\boldsymbol{\tilde{I}'_r = 0}$$

resulting in the parallel combination of branches containing the core-loss elements $\boldsymbol{R_{fe}}$ and $\boldsymbol{X_m}$. being the only active part of the circuit.



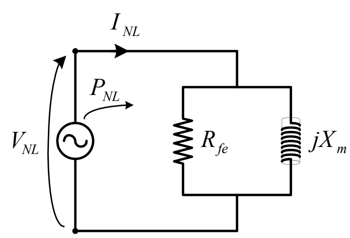
114

The No-Load Test

Apply **rated voltage V_{NL}** to the stator-windings of the motor under no-load conditions and measure the **current I_{NL}** and the **real power P_{NL}** supplied to each phase of the stator.

The values of the **core-loss elements** can be determined from:

$$R_{fe} = \frac{V_{NL}^2}{P_{NL}} \qquad X_m = \frac{V_{NL}^2}{\sqrt{(V_{NL} \cdot I_{NL})^2 - P_{NL}^2}}$$

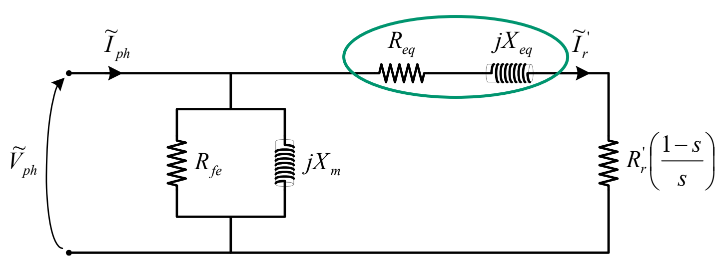


115

The Locked-Rotor Test

The **winding-loss parameters R_{eq}** and **X_{eq}** can be determined by performing a **Locked-Rotor Test** on the motor.

The **Locked-Rotor Test** is performed by supplying a **voltage** to the stator windings of the motor and **measuring** the magnitude of the **lines currents** and the **real power** supplied to the motor while the rotor is locked in place (i.e. – the rotor can't rotate).



116

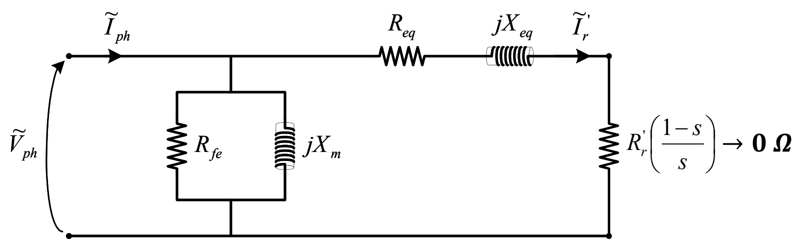
The Locked-Rotor Test

Under locked-rotor conditions, the rotational speed of the rotor will be zero ($n_r = 0$), and thus:

$$slip \equiv s = \frac{n_s - n_r}{n_s} = \frac{n_s - 0}{n_s} = 1$$

and:

$$R_r\left(\frac{1-s}{s}\right) = R_r\left(\frac{1-1}{1}\right) = 0\ \Omega \quad \text{(short circuit)}$$



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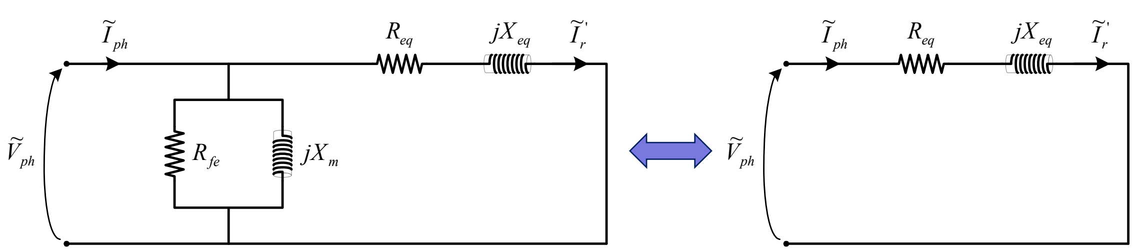
The Locked-Rotor Test

And since the impedance magnitudes of R_{fe} and X_m are typically much larger than those of R_{eq} and X_{eq}, the elements R_{fe} and X_m can be neglected under locked-rotor conditions without greatly affecting the accuracy of the model.



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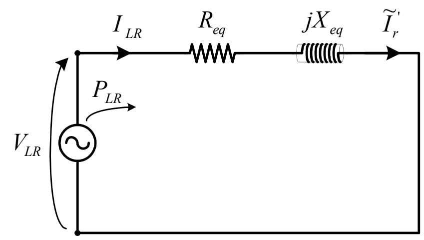
The Locked-Rotor Test

Apply a **voltage** $\boldsymbol{V_{LR}}$ to the stator-windings of the motor under locked-rotor conditions and measure the **current** $\boldsymbol{I_{RL}}$ and the **real power** $\boldsymbol{P_{RL}}$ supplied to each phase of the stator.

The values of the **winging-loss elements** can be determined from:

$$R_{eq} = \frac{P_{LR}}{I_{LR}^2} \qquad X_{eq} = \frac{\sqrt{(V_{LR} \cdot I_{LR})^2 - P_{LR}^2}}{I_{LR}^2}$$



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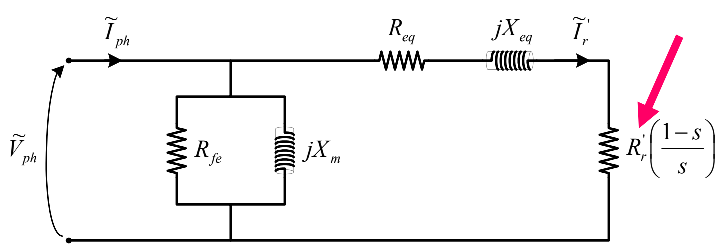
Determining the Model Parameters

The **No-Load** and **Locked-Rotor Tests** allowed us to solve for the parameters:

$$\boldsymbol{R_{fe}, X_m, R_{eq},} \text{ and } \boldsymbol{X_{eq}}$$

similar to the Open-Circuit and Short-Circuit Tests for a transformer, but there's one remaining parameter that we must also determine in order to utilize the equivalent circuit.

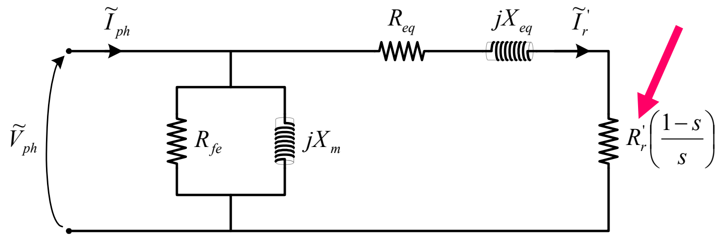


120

Determining the Model Parameters

The parameter R_r' represents the **resistance** of the **rotor-conductor circuit** referred to the stator side of the model.

But the resistance of the rotor circuit, which consists of multiple sets of rotor conductors, whose ends are shorted together, cannot be determined directly because there is no practical way to perform test only on the rotor circuit.



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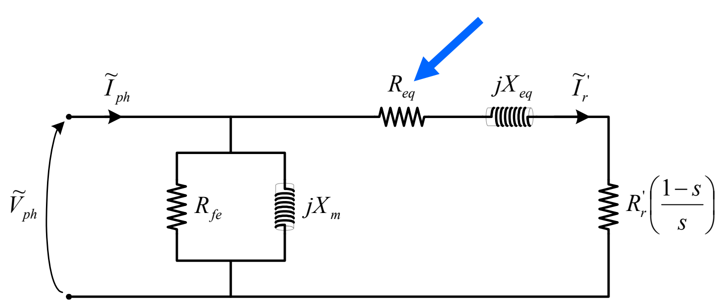
Determining the Model Parameters

On the other hand, the previously determined parameter R_{eq} is the sum of the **stator winding** and **rotor-conductor resistances**:

$$R_{eq} = R_s + R_r'$$

and the stator winding resistance can be measured directly.



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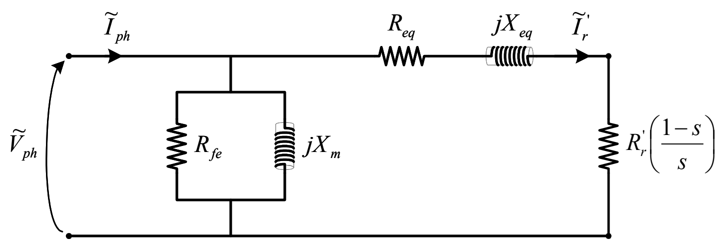
The Stator Resistance Test

Using an Ohmmeter, **measure** the **resistance** of the **stator windings** and then use the result to determine the rotor-circuit resistance:

$$R_r' = R_{eq} - R_s$$

Note that, since an ohmmeter measures DC resistance, a scaling factor of 1.25 may be applied to the measured stator winding resistance value for larger induction motors in order to account for the increased resistance under AC conditions due to skin-effect.

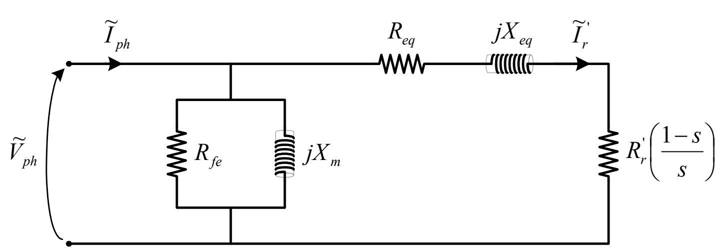
This factor will not be applied to any measurements taken in the Q-215 lab due to the small size of the Lab Volt induction motors.



123

Determining the Model Parameters

Now that all of the model parameters have been determined, the equivalent circuit can be utilized in order to predict the modeled induction motor's operation at any rotational speed (or slip) if the supply voltage is known.



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