Transformers

(Part B)

Practical Transformers

ECET 3500 – *Survey of Electric Machines*

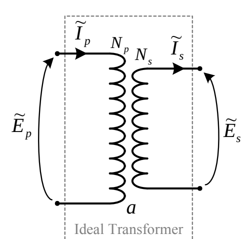
Ideal Transformer Model

Although the circuit model shown below can be used to define the operation of an **ideal transformer**, the "real world" operational characteristics of a transformer cannot be predicted by this model because it does not account for the internal losses and/or other non-ideal qualities of a practical transformer.

For this reason, we will develop a model for a practical transformer.

$$a = \frac{N_p}{N_s} \equiv \textit{turns ratio}$$

$$\frac{\tilde{E}_p}{\tilde{E}_s} = a \qquad \frac{\tilde{I}_p}{\tilde{I}_s} = \frac{1}{a}$$



Practical Transformer Model

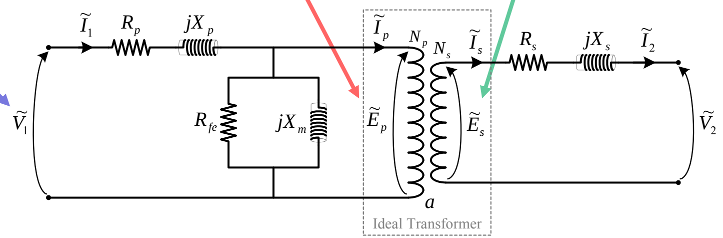
A circuit model for a **practical transformer** can be developed by expanding the ideal transformer model to include additional circuit elements that account for the "losses" that occur when operating an actual transformer.

The following circuit model, which is typically referred to as either the **Cantilever Equivalent Circuit** or the **Steinmetz Equivalent Circuit**, is often used to predict the operation of a practical transformer.

$\tilde{E}_p$ is the voltage across the primary winding of the "ideal" transformer that is embedded within the practical transformer model.

$\tilde{E}_s$ is the voltage across the secondary winding of the "ideal" transformer that is embedded within the practical transformer model.

$\tilde{V}_1$ is the voltage across the actual primary terminals of the practical transformer.



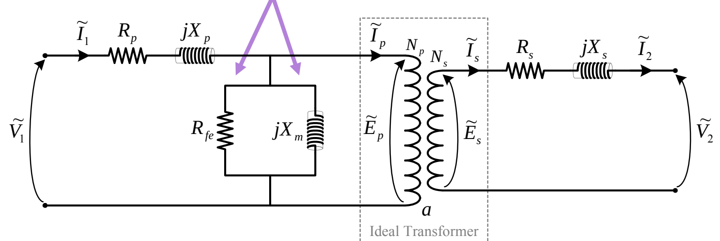
$\tilde{V}_2$ is the voltage across the actual secondary terminals of the practical transformer.

Practical Transformer Model

Within the model for a **practical transformer**:

- $\boldsymbol{R_p}$ and $\boldsymbol{R_s}$ account for the primary and secondary **winding resistances**
- $\boldsymbol{X_p}$ and $\boldsymbol{X_s}$ account for the primary and secondary winding **leakage reactances**
- $\boldsymbol{R_{fe}}$ accounts for the **Hysteresis/Eddy-Current losses** within the core
- $\boldsymbol{X_m}$ accounts for the **magnetization current** drawn into the primary winding

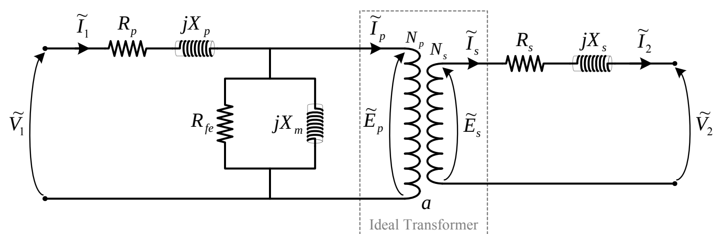
R_{fe} and X_m only appear on the primary side of the ideal transformer model because they are associated with loss components that relate to the mutually-linked, time-varying flux created within the transformer's core by the source that is connected to the primary winding.



Practical Transformer Model

Note – The circuit shown below is used to model the steady-state operation of transformers that have relatively low operating frequencies. (I.e. – **60Hz** or "power" transformers)

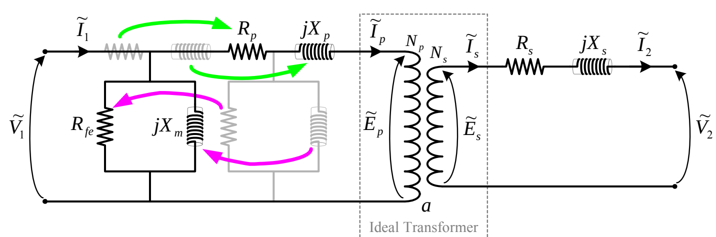
A different model may be required for transformers that operate at higher frequencies in order to properly account for other "losses" that are typically neglected at 60Hz.



Practical Transformer Model

In order to reduce the model's complexity, the parallel elements $\boldsymbol{R_{fe}}$ and $\boldsymbol{X_m}$ can be moved to the "left-side" of the series elements $\boldsymbol{R_p}$ and $\boldsymbol{X_p}$.

Note – This change should only introduce a small decrease in the overall accuracy of the model because $\boldsymbol{R_{fe}}$ and $\boldsymbol{X_m}$ are typically much larger in magnitude than $\boldsymbol{R_p}$ and $\boldsymbol{X_p}$.



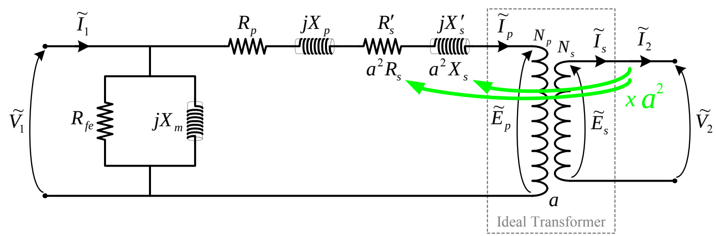
Practical Transformer Model

Additionally, the series elements R_s and X_s can be referred (moved) to the primary-side of the ideal transformer provided that their values are each multiplied by a^2.

Note – as defined in the figure: $R'_s = a^2 R_s$

$X'_s = a^2 X_s$

The "prime" denotes that the impedance was referred to the primary-side of the ideal transformer.



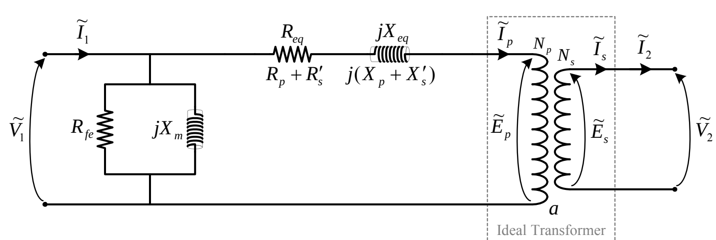
Practical Transformer Model

Then, the series resistive elements R_p and R'_s can then be combined into an equivalent resistive element R_{eq}, and the series reactive elements X_p and X'_s can be combined into an equivalent reactive element X_{eq}:

$$R_{eq} = R_p + R'_s$$

$$X_{eq} = X_p + X'_s$$

where: R_{eq} and X_{eq} account for the overall resistance and leakage reactance of the transformer's windings.



Example Problem

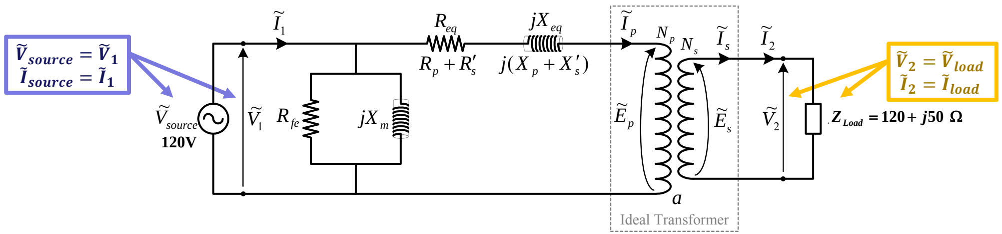
Given a **500VA**, **60Hz**, **240V–120V practical transformer** with the following equivalent circuit parameters:

$$R_{LV} = 0.20\Omega,\ X_{LV} = 0.40\Omega,\ R_{HV} = 0.80\Omega,\ X_{HV} = 1.20\Omega$$

$$R_{fe(LV)} = 480\Omega,\ X_{M(LV)} = 160\Omega$$ ← (both referred to the LV-winding)

If the transformer is used to step-up the voltage from a **120 volt** source while supplying a load, Z_{Load} **=(120+j50)**Ω…

Determine the **actual load voltage**, $\tilde{V}_2$, and the overall **source current**, $\tilde{I}_1$.



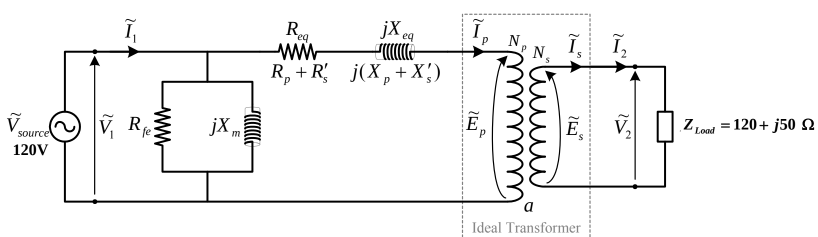
Determining the Model Parameters

To begin the problem, the transformer's **operational turns-ratio** must be determined…

Since the transformer is being used to **step-up** the **120V** source voltage:

- **the 120V winding is the primary winding** ← **LV winding**
- **the 240V winding is the secondary winding** ← **HV winding**

Therefore, the **operational turns-ratio** is: $$a = \frac{120\,\text{V}}{240\,\text{V}} = \frac{1}{2}$$



Determining the Model Parameters

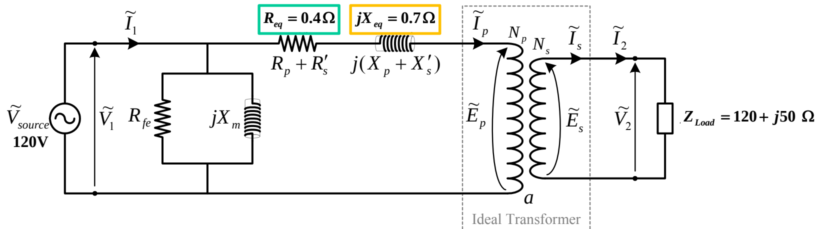
Additionally, since the **low-voltage winding** is the **primary winding**, the circuit parameters relating to the HV and LV sides may be redefined as:

$$\boldsymbol{R_p = R_{LV} = 0.20\Omega,} \quad \boldsymbol{X_p = X_{LV} = 0.40\Omega,}$$
$$\boldsymbol{R_s = R_{HV} = 0.80\Omega,} \quad \boldsymbol{X_s = X_{HV} = 1.20\Omega,}$$

from which $\boldsymbol{R_{eq}}$ and $\boldsymbol{X_{eq}}$ can be determined:

$$\boldsymbol{R_{eq} = R_p + a^2 R_s = 0.20 + \left(\tfrac{1}{2}\right)^2 (0.80) = 0.40\,\Omega}$$

$$\boldsymbol{X_{eq} = X_p + a^2 X_s = 0.40 + \left(\tfrac{1}{2}\right)^2 (1.20) = 0.70\,\Omega}$$



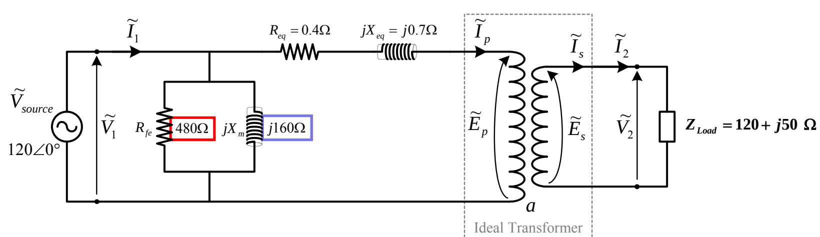
Determining the Model Parameters

Note that, as defined in the problem statement, $\boldsymbol{R_{fe}}$ and $\boldsymbol{X_m}$ were both referred to the LV winding:

$$\boldsymbol{R_{fe(LV)} = 480\Omega} \qquad \boldsymbol{X_{M(LV)} = 160\Omega}$$

Since the LV winding is the primary winding, both of those impedances can be directly imported into the model.

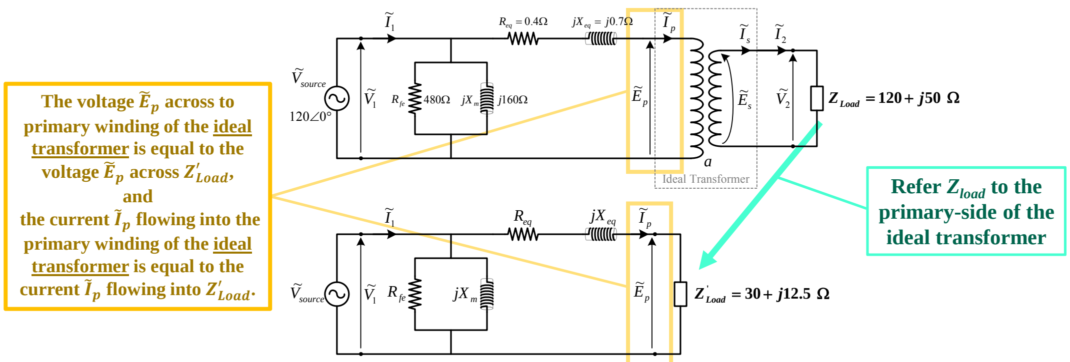
Thus, the following figure shows the circuit model for the practical transformer with all of the parameters included:



Referring the Load to the Primary-Side

In order to simplify the overall circuit, the "**ideal transformer**" portion of the model can be removed by referring the load to the primary-side of the ideal transformer, such that:

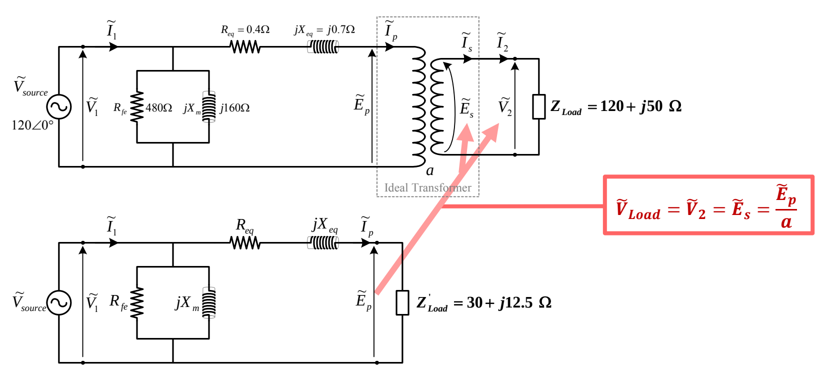
$$Z'_{Load} = a^2 Z_{Load} = \left(\tfrac{1}{2}\right)^2 (120 + j50) = (30 + j12.5)\Omega$$



Solving for the Load Voltage

Note that, although $\tilde{E}_s$ is the voltage across the actual load Z_{Load} in the original circuit, $\tilde{E}_p$ is the voltage across Z'_{Load} in the simplified circuit.

Thus, the **actual load voltage**, $\tilde{E}_s$, can be determined by first solving for $\tilde{E}_p$ in the simplified circuit and then using the turns-ratio equation to get $\tilde{E}_s$.



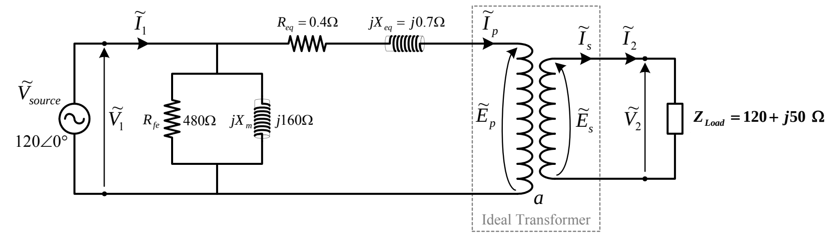
Solving for the Load Voltage

Since Z'_{Load} is in series with both R_{eq} and jX_{eq} in the simplified circuit, the voltage $\tilde{E}_p$ can be determined using a voltage-divider equation:

$$\tilde{E}_p = \tilde{V}_1 \cdot \left(\frac{Z'_{Load}}{R_{eq} + jX_{eq} + Z'_{Load}} \right) = 120\angle 0° \cdot \left(\frac{30 + j12.5}{0.4 + j0.7 + 30 + j12.5} \right) = 117.7\angle -0.85°\ volts$$

The **load voltage** can then be solved by utilizing the turns-ratio equation:

$$\tilde{V}_{load} = \tilde{V}_2 = \tilde{E}_s = \frac{\tilde{E}_p}{a} = \frac{117.7\angle -0.85°}{1/2} = 235.4\angle -0.85°\ volts$$



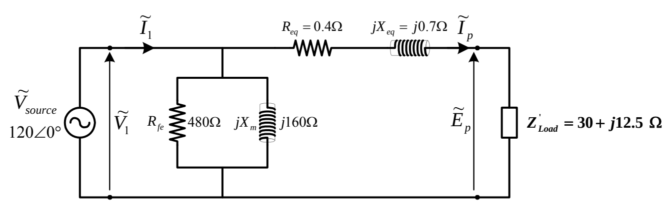
Solving for the Source Current

In order to solve for the **source current** $\tilde{I}_1$, the input impedance "looking" into the primary terminals of the practical transformer must be determined:

$$Z_{in} = R_{fe} \| X_m \| \left(R_{eq} + jX_{eq} + Z'_{load} \right) = \left(\frac{1}{480} + \frac{1}{j160} + \frac{1}{30.4 + j13.2} \right)^{-1} = 24.4 + 15.0\Omega$$

after which the **source current**, $\tilde{I}_1$, can be determined by:

$$\tilde{I}_{source} = \tilde{I}_1 = \frac{\tilde{V}_1}{Z_{in}} = \frac{120\angle 0°}{24.4 + j15.0} = 4.19\angle -31.6°\ amps$$

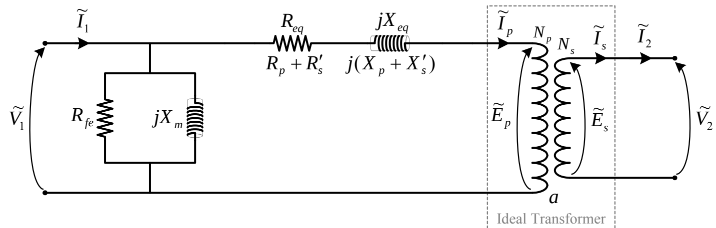


Determining the Model Parameters

The model parameters for a specific transformer are often unknown because they aren't typically provided by the manufacturer.

But, the parameters can be determined by performing a simple pair of tests on the transformer:

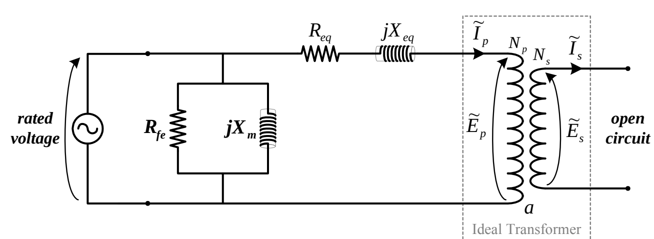
- The **Open-Circuit Test**, and
- The **Short-Circuit Test**.



The Open-Circuit Test

The **core-loss parameters** $\boldsymbol{R_{fe}}$ and $\boldsymbol{X_m}$ can be determined by performing an **Open-Circuit Test** on the transformer.

The **Open-Circuit Test** is typically performed by applying rated voltage to the primary winding of the transformer while leaving the secondary winding open-circuited.



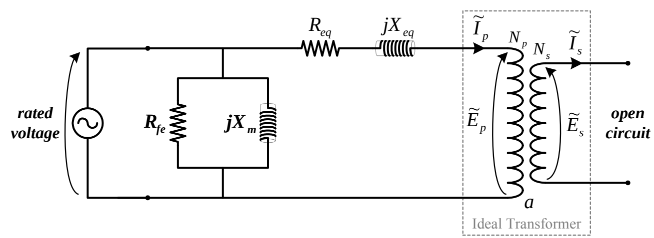
The Open-Circuit Test

If the secondary is <u>open-circuited</u>, then the current $\tilde{I}_s$ must be zero:

$$\tilde{I}_s = 0$$

And, if the current $\tilde{I}_s$ is zero, then the current $\tilde{I}_p$ flowing into the primary winding of the ideal transformer must also be zero:

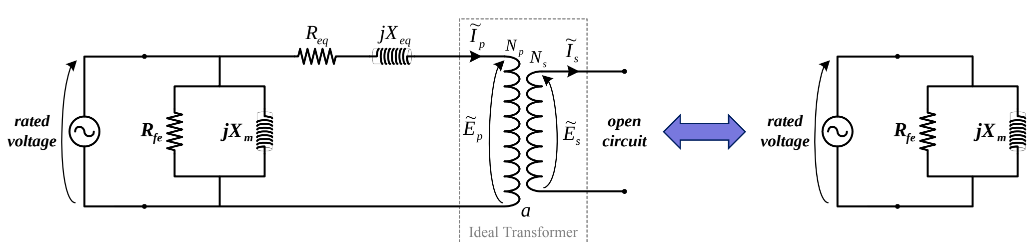
$$\tilde{I}_p = \frac{\tilde{I}_s}{a} = \frac{0}{a} = 0$$



The Open-Circuit Test

If the current $\tilde{I}_p$ is zero, then no current will be flowing through the series elements R_{eq} and X_{eq}.

Thus, the only active portion of the circuit under **open-circuit conditions** is the parallel combination of branches containing the core-loss elements R_{fe} and X_m.



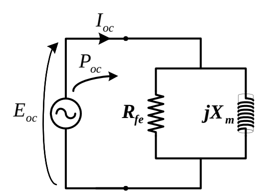
The Open-Circuit Test

Apply rated voltage E_{oc} to the primary-winding of the transformer with the secondary-winding open-circuited.

Measure the current I_{oc} and the real power P_{oc} supplied by the source.

The values of the **core-loss elements** can be determined from:

$$R_{fe} = \frac{E_{oc}^2}{P_{oc}} \qquad X_m = \frac{E_{oc}^2}{\sqrt{(E_{oc} \cdot I_{oc})^2 - P_{oc}^2}}$$

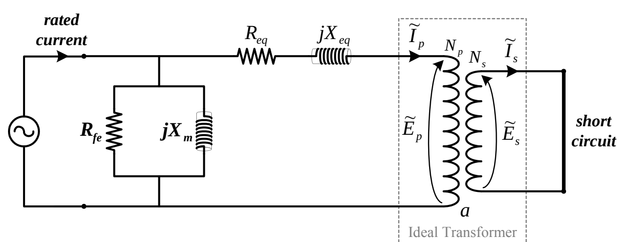


The Short-Circuit Test

The **winding-loss parameters** R_{eq} and X_{eq} can be determined by performing a **Short-Circuit Test** on the transformer.

The **Short-Circuit Test** is typically performed by supplying rated current to the primary winding of the transformer while the secondary winding is short-circuited.

****** Warning – Only a small primary voltage is required in order to ******
*********** reach rated current under short-circuit conditions ***********

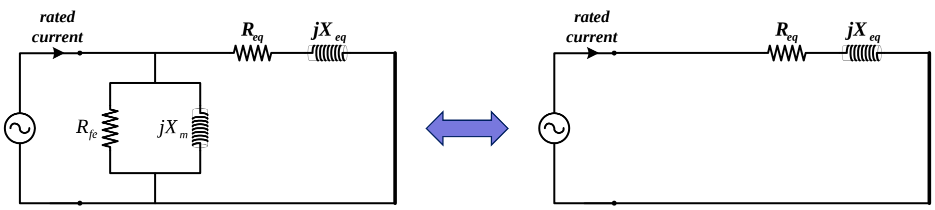


The Short-Circuit Test

Under short-circuit conditions, the ideal transformer portion of the model can be replaced by an ideal wire since:

$$Z'_{Load} = a^2 Z_{Load} = a^2(0) = 0\ \Omega$$

And, since the impedance magnitudes of R_{fe} and X_m are typically much larger than those of R_{eq} and X_{eq}, the elements R_{fe} and X_m can be neglected without affecting the accuracy of the model.



The Short-Circuit Test

Supply rated current I_{sc} to the primary-winding of the transformer with the secondary-winding short-circuited.

Measure the voltage E_{sc} and the real power P_{sc} supplied by the source.

The values of the **winding-loss elements** can be determined from:

$$R_{eq} = \frac{P_{sc}}{I_{sc}^2} \qquad X_{eq} = \frac{\sqrt{(E_{sc} \cdot I_{sc})^2 - P_{sc}^2}}{I_{sc}^2}$$