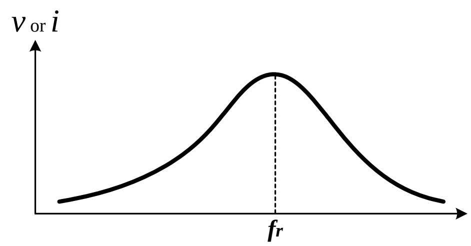
ECET 2111

Circuits II

Resonant
(Tuned)
Circuits

Resonant Circuits

A **Resonant Circuit** is an R-L-C circuit with a frequency response similar to that shown in the figure below, in which the response of the circuit is maximum at the resonant frequency, f_r .



Resonance

Resonance is a condition that occurs within an R-L-C circuit when the reactive power "consumed" by the inductive elements is equal to the reactive power "produced" by the capacitive elements.

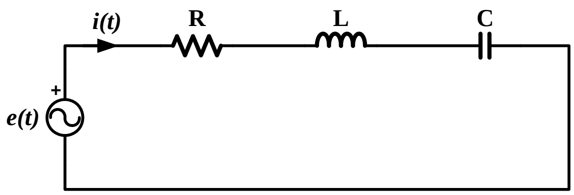
Note – if the reactive powers are equal, then the energy that the inductive elements are absorbing (or releasing) at any point in time will be equal to the energy that the capacitive elements are releasing (or absorbing) at that same instant.

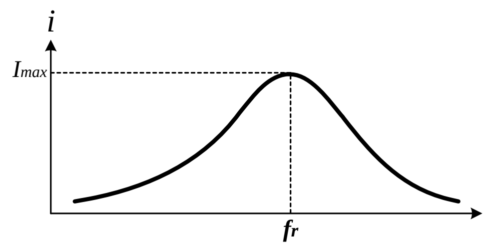
The **resonant frequency**, f_r , is the source frequency at which resonance occurs.

Series-Resonant Circuits

A <u>Series-Resonant Circuit</u> is an R-L-C circuit that contains a resistor, an inductor, and a capacitor that are connected in-series with each other.

In a series-resonant circuit, the current will be maximum when the source is operating at the resonant frequency.



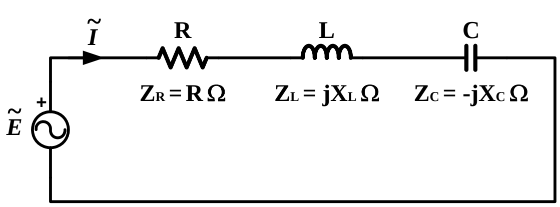


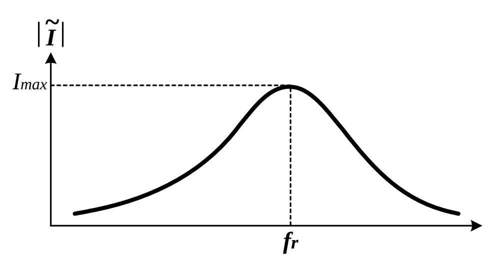
Series-Resonant Circuits

If the circuit elements are expressed as impedances, then the total series-impedance of the circuit is:

$$Z_T = R + j(X_L - X_C)\,\Omega$$

where: $X_L = \omega \cdot L$ $\qquad X_C = \dfrac{1}{\omega \cdot C}$





Series-Resonance

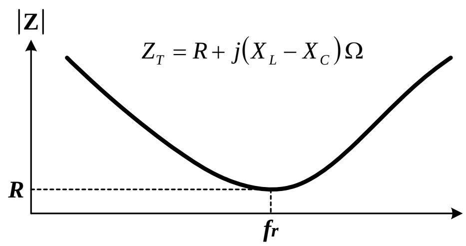
Resonance occurs in a series-resonant circuit when $X_L = X_C$.

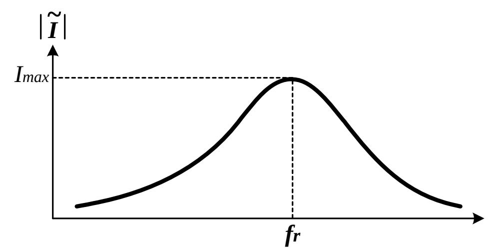
Thus, at resonance, the total series-impedance will be:

$$Z_T = R\,\Omega$$

the magnitude of which will be at its minimum value, resulting in a current that is in-phase with the source voltage and at its maximum possible magnitude, I_{max}.

T





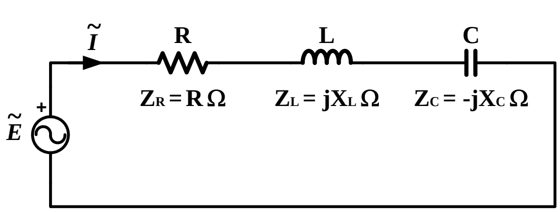
Series-Resonance Impedance

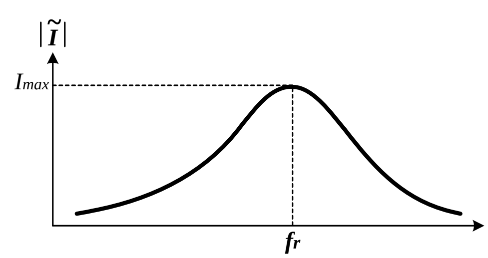
Since the total series-impedance at resonance is:

$$Z_T = R\,\Omega$$

if $\tilde{E} = E\angle\phi°$, then the maximum current magnitude will be:

$$I_{\max} = \left|\tilde{I}\right| = \left|\frac{\tilde{E}}{R}\right| = \frac{E}{R}$$



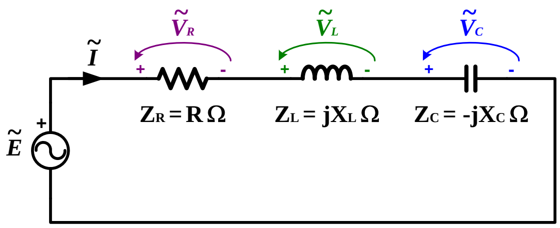


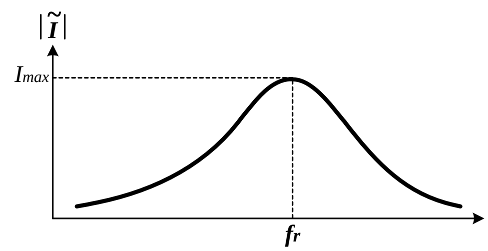
Series-Resonance Maximum Current

Additionally, the inductor and capacitor voltages will be:

$$\tilde{V}_L = \tilde{I} \cdot (jX_L) \qquad \tilde{V}_C = \tilde{I} \cdot (-jX_C)$$

Note that the inductor voltage will be out-of-phase with the capacitor voltage by 180°.



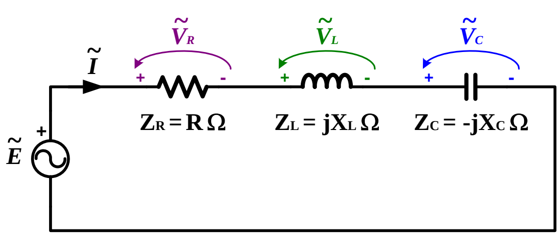


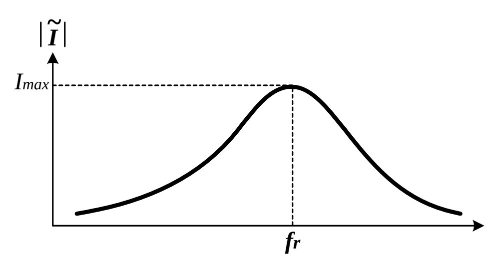
Series-Resonant Frequency

The <u>series-resonant frequency</u> (at which resonance occurs) can be determined by solving for frequency when X_L equals X_C.

$$X_L = X_C \longrightarrow \omega \cdot L = \frac{1}{\omega \cdot C}$$

$$\omega = \sqrt{\frac{1}{L \cdot C}} \longrightarrow f_r = \frac{1}{2\pi}\sqrt{\frac{1}{L \cdot C}}$$



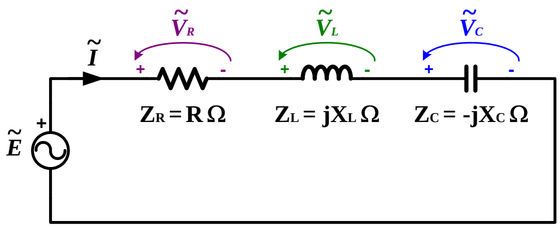


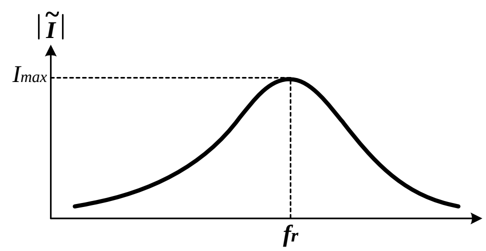
Quality Factor

The <u>quality factor</u>, Q_s, of a series-resonant circuit is defined by:

$$Q_s = \frac{\text{Reactive Power}}{\text{Real Power}}$$

Thus: $$Q_s = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega \cdot L}{R} \quad \text{or} \quad Q_s = \frac{I^2 X_C}{I^2 R} = \frac{X_C}{R} = \frac{1}{\omega \cdot R \cdot C}$$





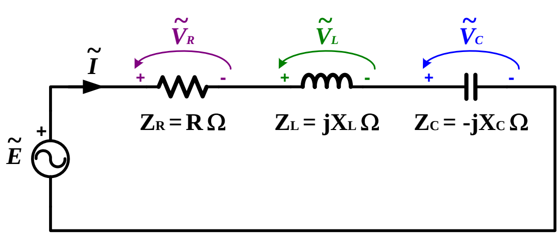
Quality Factor

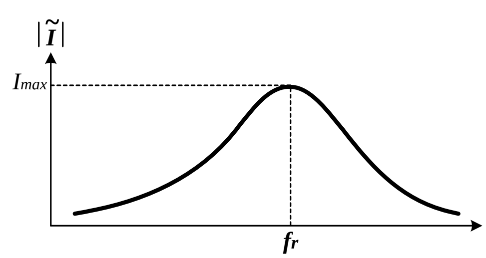
Note that, since:

$$\omega = \sqrt{\frac{1}{L \cdot C}}$$

<u>quality factor</u>, Q_s, will equal to:

$$Q_s = \frac{\omega \cdot L}{R} = \sqrt{\frac{1}{L \cdot C}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



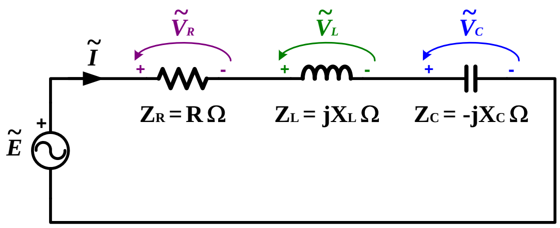


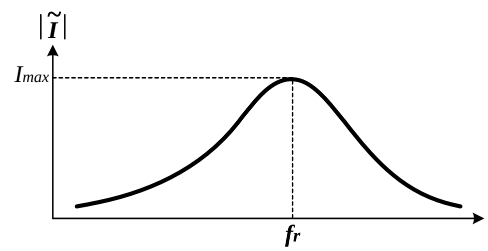
Inductor and Capacitor Voltages

Additionally, note that the magnitude of the inductor and capacitor voltages at the resonant frequency will be:

$$\left|\tilde{V}_L\right| = \left|\tilde{I}\cdot(jX_L)\right| = \frac{E}{R}\cdot X_L = E\cdot\frac{X_L}{R} = E\cdot Q_s$$

$$\left|\tilde{V}_C\right| = \left|\tilde{I}\cdot(-jX_C)\right| = \frac{E}{R}\cdot X_C = E\cdot\frac{X_C}{R} = E\cdot Q_s$$

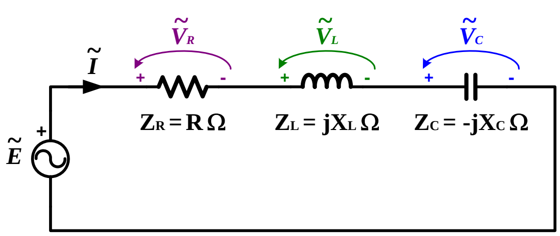


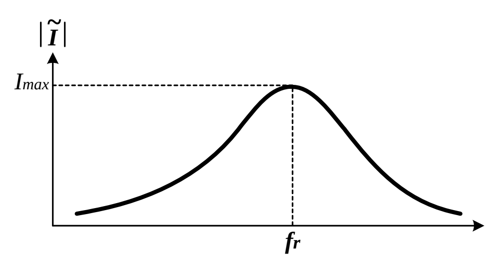


Selectivity

The term <u>Selectivity</u> is used to characterize the range of frequencies for which currents will pass through or flow in a series-resonant circuit.

Since the current magnitude decays as frequency varies away from the resonant frequency, arbitrary cutoffs are chosen in order to define the circuit's selectivity.



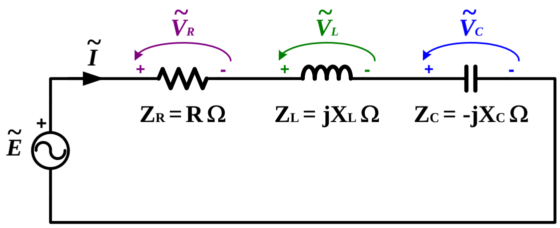


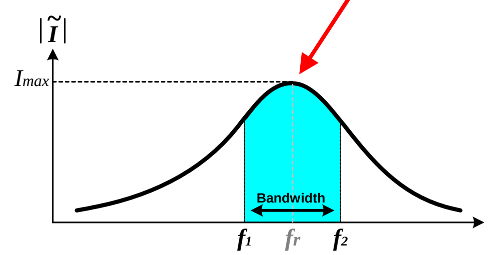
Bandwidth

The Bandwidth of a series-resonant circuit is the range of frequencies for which the real power delivered to the resistor is greater than or equal to one-half of the peak power value.

$$P_R \geq \frac{1}{2} P_{\max}$$

Note that peak power occurs at resonant frequency when the current is at a maximum.





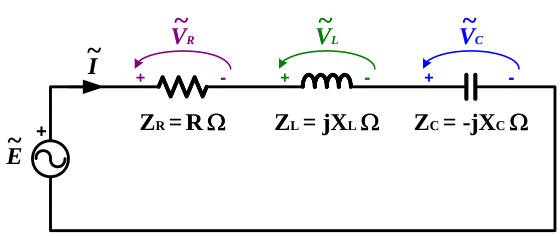
Half-Power Cutoff Frequencies

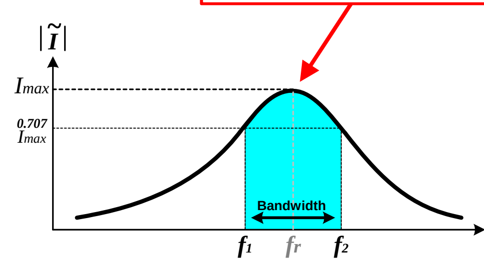
The Half-Power Cutoff Frequencies are the frequencies at which the real power delivered to the resistor is equal to one half of the peak power value.

Note that one-half peak power occurs whenever:

$$\left|\tilde{I}\right| = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$

Note that peak power occurs at resonant frequency when the current is at a maximum.

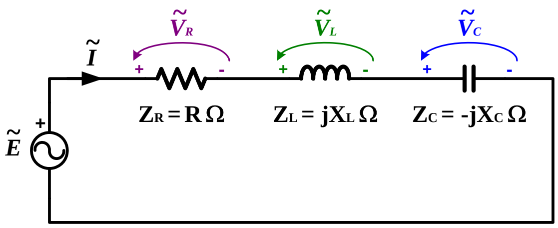


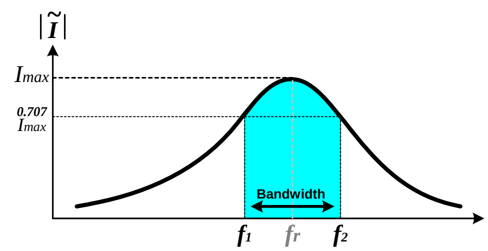


Half-Power Cutoff Frequencies

The <u>lower</u> and <u>upper</u> half-power cutoff frequencies, f_1 and f_2, can be calculated from:

$$f_1 = \frac{1}{2\pi}\left[\frac{-R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] \text{ Hz} \qquad f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] = \text{ Hz}$$

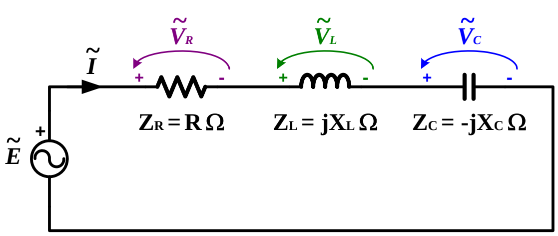


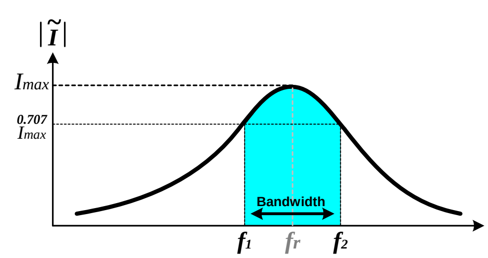


Half-Power Cutoff Frequencies

Note that the <u>resonant frequency</u>, f_r, is directly related to the lower and upper half-power cutoff frequencies, f_1 and f_2:

$$f_r = \sqrt{f_1 \cdot f_2}$$





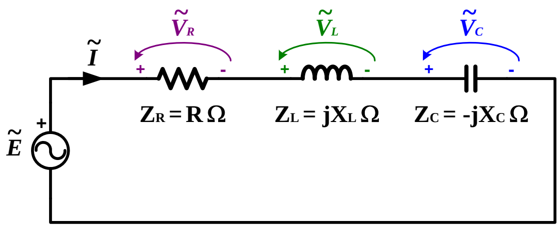
Bandwidth

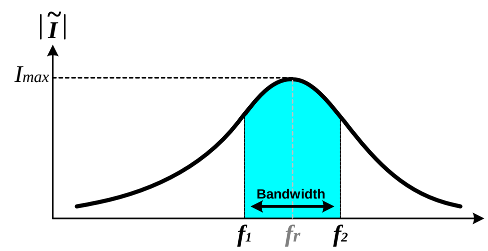
<u>Bandwidth</u> can be defined in terms of the cutoff frequencies as:

$$BW = f_2 - f_1$$

Additionally, <u>bandwidth</u> is equal to:

$$BW = \frac{f_r}{Q_S}$$

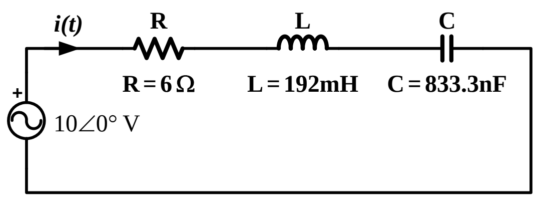


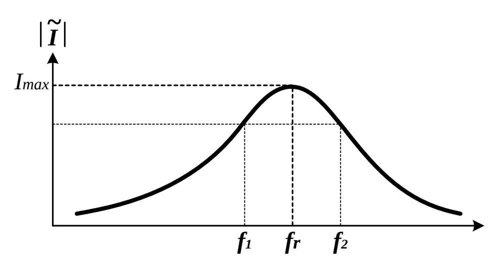


Quality Factor Example

Given the circuit shown below, determine:

- The **Resonant Frequency** of the circuit.
- The **Lower and Upper Cutoff Frequencies** of the circuit.
- The **Bandwidth** and the **Quality Factor** of the circuit.
- The resistor, inductor, and capacitor **voltage magnitudes** at the cutoff frequency.

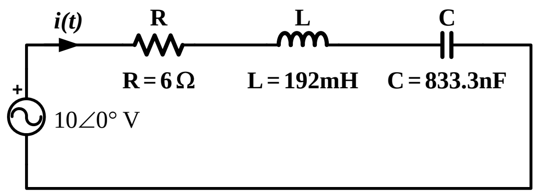


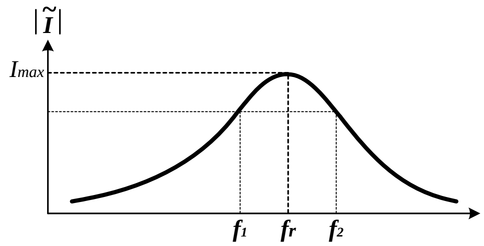


Quality Factor Example

Determine the resonant frequency:

$$f_S = \frac{1}{2\pi}\sqrt{\frac{1}{L\cdot C}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.192)\cdot(833.3\cdot 10^{-9})}} = 398\text{ Hz}$$



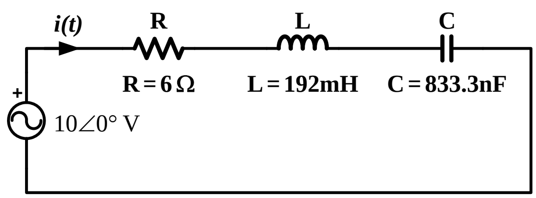


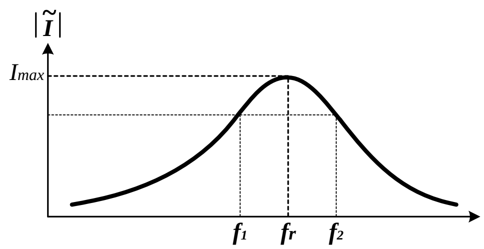
Quality Factor Example

Determine the lower and upper cutoff frequencies:

$$f_1 = \frac{1}{2\pi}\left[\frac{-R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] = \frac{1}{2\pi}\left[\frac{-6}{2\cdot 0.192} + \frac{1}{2}\sqrt{\left(\frac{6}{0.192}\right)^2 + \frac{4}{0.192\cdot 8333\cdot 10^{-9}}}\right] = 395.4\text{ Hz}$$

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] = \frac{1}{2\pi}\left[\frac{6}{2\cdot 0.192} + \frac{1}{2}\sqrt{\left(\frac{6}{0.192}\right)^2 + \frac{4}{0.192\cdot 8333\cdot 10^{-9}}}\right] = 400.4\text{ Hz}$$



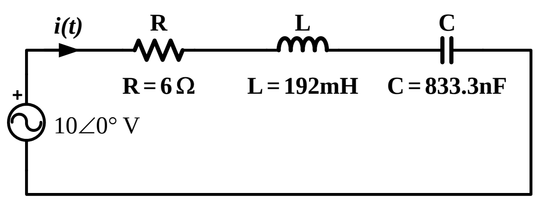


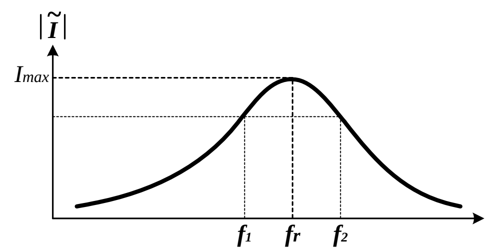
Quality Factor Example

Determine the <u>bandwidth</u> and the <u>quality factor</u>:

$$BW = f_2 - f_1 = 400.4 - 395.4 = 5 \text{ Hz}$$

$$Q_S = \frac{f_S}{BW} = \frac{398}{5} \approx 80$$





Quality Factor Example

Determine <u>voltage magnitudes</u> at the cutoff frequency:

$$\left|\tilde{V}_L\right| = E \cdot Q_s = 10 \cdot 80 = 800 \text{ V}$$

$$\left|\tilde{V}_C\right| = E \cdot Q_s = 10 \cdot 80 = 800 \text{ V}$$

Note that these voltages are <u>much larger</u> than the source voltage!